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Two-Body Dirac Equation Approach to the Deuteron

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Abstract

The two-body Dirac(Breit) equation with potentials associated to one-boson-exchanges with cutoff masses is solved for the deuteron and its observables calculated. The 16-component wave-function for the $J^\pi = 1^+$ state contains four independent radial functions which satisfy a system of four coupled differential equations of first order. This system is numerically integrated, from infinity towards the origin, by fixing the value of the deuteron binding energy and imposing appropriate boundary conditions at infinity. For the exchange potential of the pion, a mixture of direct plus derivative couplings to the nucleon is considered. We varied the pion-nucleon coupling constant, and the best results of our calculations agree with the lower values recently determined for this constant. The present treatment differs from the more conventional ones in that non-relativistic reductions up to the order c^{-2} are not used.

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1 Introduction

The study of two interacting Dirac (spin $1/2$) particles is an age old problem , initiated with Breit [1] in 1929. The subject has been applied to problems ranging from the hydrogen-like atoms to the structure of mesons, the NN scattering and the deuteron [2, 3]. The exact field-theoretical approach to the problem was first formulated by Salpeter and Bethe [4] but

is not always used in applications, not only because it is difficult to solve, but also because it does not reduce to the ordinary one-body Dirac equation in the ladder approximation [5].

In the present paper, we shall work with a two-body Dirac equation which, while not fully covariant, exhibits several desirable features such as invariance under spatial rotations and reflections, the correct nonrelativistic limit and reduction to the one-body Dirac equation when one of the masses goes to infinity. Our principal aim here is to apply it to the deuteron $J^\pi = 1^+$ state and calculate its observables. We note that general methods to reduce the Breit equation appeared recently, with interactions restricted to the five Lorentz-type terms corresponding to direct, *i.e.* , nonderivative couplings, besides Breit-like terms introduced to account for retardation effects [6]. Here these direct coupling terms are associated to one-boson-exchange (OBE) interactions corresponding to the exchange of other mesons besides the pion. The exchange of the pion, as is well known [7], plays a fundamental role, being responsible for the long tail of the system. However, for the pion-nucleon interaction, one needs to include also a term corresponding to derivative coupling, which makes an extension of the work of Ref.[6] necessary. This will be done in some detail and we will show that the derivative pseudoscalar coupling amounts to introduce, in the framework of Ref. [6], two new shape functions, $\Omega_S(r)$ and $\Omega_T(r)$ and two new potentials, $V_{11}(r)$ and $V_{12}(r)$. Furthermore, the modifications needed to take into account the mixed coupling arising from the superposition of direct and derivative couplings for the pion are also discussed.

This work is organized as follows. Section 2 is devoted to the main definitions and to the notation related to the two-body Dirac equation. The special case of the derivative pseudoscalar interaction is the subject of Section 3. The reduction of the two-body Dirac equation, when derivative pseudoscalar terms are included, is given in Section 4. Section 5 discusses the mixed coupling of the pion to the nucleon. The specific OBE interaction used in our calculation is presented in Section 6, while the numerical integration of the radial equations is discussed in Section 7, together with the calculation of the deuteron observables. Finally, Section 8 contains our main results and conclusions.

2 Two-Body Dirac Equation

The two-body Dirac equation for two spin $1/2$ particles of masses m_1 and m_2 has the following form in the center of mass frame in natural units ($\hbar = c = 1$):

$$[-i \boldsymbol{\alpha}_1 \cdot \nabla + \beta_1 m_1 + i \boldsymbol{\alpha}_2 \cdot \nabla + \beta_2 m_2 + H_{12}(\mathbf{r})] \Psi(\mathbf{r}) = E \Psi(\mathbf{r}), \quad (1)$$

where $(\boldsymbol{\alpha}_1, \beta_1)$ and $(\boldsymbol{\alpha}_2, \beta_2)$ are the Dirac matrices acting on particles 1 and 2, respectively, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ is the relative separation between the two particles and we have made use of $\nabla_1 = -\nabla_2 = \nabla$. The instantaneous local interaction H_{12} has the general form

$$H_{12}(\mathbf{r}) = - \sum_n \Omega_n(r) \omega_n, \quad (2)$$

where $\Omega_n(r)$ are shape functions depending only on $r = |\mathbf{r}|$ and ω_n are spin-dependent operators.

The standard practice is to restrict the summation in Eq.(2) to the five Lorentz-type terms related to direct, i.e., nonderivative, couplings, namely:

$$\begin{aligned} \omega_1 &= \beta_1 \beta_2 \quad (\text{scalar}), \\ \omega_2 &= \frac{1}{2} (1 - \boldsymbol{\alpha}_1 \cdot \boldsymbol{\alpha}_2) \quad (\text{vector}), \\ \omega_3 &= \frac{1}{2} \beta_1 \beta_2 (\boldsymbol{\Sigma}_1 \cdot \boldsymbol{\Sigma}_2 + \boldsymbol{\alpha}_1 \cdot \boldsymbol{\alpha}_2) \quad (\text{tensor}), \\ \omega_4 &= \frac{1}{2} (\boldsymbol{\Sigma}_1 \cdot \boldsymbol{\Sigma}_2 - \Gamma_1 \Gamma_2) \quad (\text{pseudovector}), \\ \omega_5 &= \beta_1 \beta_2 \Gamma_1 \Gamma_2 \quad (\text{pseudoscalar}), \end{aligned} \quad (3)$$

where

$$\boldsymbol{\Sigma} = \frac{1}{2i} \boldsymbol{\alpha} \times \boldsymbol{\alpha} \quad \text{and} \quad \Gamma = \gamma_5 = \frac{1}{6i} \boldsymbol{\alpha} \times \boldsymbol{\alpha} \cdot \boldsymbol{\alpha}.$$

Exceptionally, one also includes Breit-type terms to take approximately into account the effects of retardation. In Ref.[6] one can see in detail how to deal with all such interactions.

3 Derivative Pseudoscalar Term

The scheme presented in the previous section may not always suffice. To see this, suppose one is interested in applications to the two-nucleon system. A simple model for the nuclear force would be the one-pion-exchange potential(OPEP). There are two possibilities for the interaction Lagrangian $\mathcal{L}_{\pi NN}$ behind this model, namely, in standard covariant notation,

$$\mathcal{L}_{\pi NN}^{\text{ps}} = -ig_\pi \bar{\psi} \gamma_5 \boldsymbol{\pi} \cdot \boldsymbol{\tau} \psi, \quad (4)$$

which corresponds to direct coupling, and

$$\mathcal{L}_{\pi NN}^{\text{pv}} = -\frac{f_\pi}{m_\pi} \bar{\psi} \gamma_5 \gamma^\mu \partial_\mu \boldsymbol{\pi} \cdot \boldsymbol{\tau} \psi, \quad (5)$$

corresponding to derivative coupling of the pion field $\boldsymbol{\pi}$ (isovector, pseudoscalar) to the nucleon field ψ . As is well known, under certain conditions, but not in general, the two couplings are equivalent as long as the coupling constants are related by

$$\frac{f_\pi}{m_\pi} = \frac{g_\pi}{2m}, \quad (6)$$

where m_π and m are the pion and nucleon masses, respectively.

If one chooses the direct coupling, the corresponding interaction for the two-body Dirac equation will be given by the term having $n = 5$ in Eq.(2) with

$$\Omega_5(r) = -\frac{g_\pi^2}{4\pi} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\exp(-m_\pi r)}{r}. \quad (7)$$

However, numerical calculations have shown that this does not lead to any reasonable results for the deuteron. To understand why this is so, consider the nonrelativistic reduction of this interaction up to order v^2/c^2 [8]. One gets, besides the usual nonrelativistic expression for OPEP, namely

$$-(1/4m^2)(\boldsymbol{\sigma}_1 \cdot \nabla)(\boldsymbol{\sigma}_2 \cdot \nabla)\Omega_5(r),$$

also a repulsive core term of the form $(1/4m)\Omega_5(r)^2$ which spoils the numerical results. A simple way to overcome this difficulty is to write for the interaction [3]

$$H_{12}^{\text{ps}}(r) = -\Omega_5(r) \omega_5 - \tilde{\Omega}_1(r) \omega_1,$$

with

$$\tilde{\Omega}_1(r) = \frac{\lambda}{4m} \Omega_5(r)^2.$$

This, for $\lambda = 1$, cancels out the repulsive core term up to that order, giving rise to reasonable results.

If one chooses, instead, the derivative coupling, one has for the Lagrangian describing the nucleon and the pion fields

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi + \frac{1}{2} (\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} - m_\pi^2 \boldsymbol{\pi} \cdot \boldsymbol{\pi}) - \frac{f_\pi}{m_\pi} \bar{\psi} \gamma_5 \gamma^\mu \partial_\mu \boldsymbol{\pi} \cdot \boldsymbol{\tau} \psi. \quad (8)$$

Performing the standard calculations – and making the instantaneous approximation, which consists in neglecting the time component of the four-momentum transfer with respect to its space components and to m_π , one gets the effective Hamiltonian

$$H = \int d^3\mathbf{r}_1 \psi^\dagger(\mathbf{r}_1) (-i\boldsymbol{\alpha}_1 \cdot \nabla_1 + \beta m) \psi(\mathbf{r}_1) + \frac{1}{2} \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \psi^\dagger(\mathbf{r}_1) \psi^\dagger(\mathbf{r}_2) H_{12}^{pv}(\mathbf{r}_1 - \mathbf{r}_2) \psi(\mathbf{r}_2) \psi(\mathbf{r}_1), \quad (9)$$

where

$$H_{12}^{pv}(\mathbf{r}) = -(\boldsymbol{\Sigma}_1 \cdot \nabla)(\boldsymbol{\Sigma}_2 \cdot \nabla) G^{pv}(r) \quad (10)$$

with

$$G^{pv}(r) = -\frac{1}{4\pi} \left(\frac{f_\pi}{m_\pi} \right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{exp(-m_\pi r)}{r}. \quad (11)$$

The two-body Dirac equation is obtained if one replaces the Hamiltonian given in Eq.(9) by its first-quantized counterpart for the two particle case. The corresponding interaction will, therefore, be given by Eq.(10), which can be put in a form similar to Eq.(2) as

$$H_{12}^{pv}(\mathbf{r}) = -\Omega_S(r) \omega_S - \Omega_T(r) \omega_T, \quad (12)$$

where we have introduced the new spin-dependent operators

$$\begin{aligned} \omega_S &= \boldsymbol{\Sigma}_1 \cdot \boldsymbol{\Sigma}_2, \\ \omega_T &= \frac{3(\boldsymbol{\Sigma}_1 \cdot \mathbf{r})(\boldsymbol{\Sigma}_2 \cdot \mathbf{r})}{r^2} - \boldsymbol{\Sigma}_1 \cdot \boldsymbol{\Sigma}_2, \end{aligned} \quad (13)$$

and the shape functions

$$\begin{aligned} \Omega_S(r) &= \frac{1}{3} \left(\frac{d^2 G^{pv}}{dr^2} + \frac{2}{r} \frac{dG^{pv}}{dr} \right), \\ \Omega_T(r) &= \frac{1}{3} \left(\frac{d^2 G^{pv}}{dr^2} - \frac{1}{r} \frac{dG^{pv}}{dr} \right). \end{aligned} \quad (14)$$

4 Reduction of the Two-Body Dirac Equation

We shall follow here the same procedure and notation presented in Ref.[6], the only difference being that we consider a general interaction of the form

$$H_{12}(r) = -\sum_{n=1}^5 \Omega_n(r) \omega_n - \Omega_S(r) \omega_S - \Omega_T(r) \omega_T. \quad (15)$$

The last two terms, which correspond to derivative pseudoscalar coupling, had not been included in that reference. As we shall see below, they will force us to explicitly introduce two new potentials, namely $V_{11}(r)$ and $V_{12}(r)$.

Recalling what has been done in Ref.[6], one starts by writing the spinor $\Psi(\mathbf{r})$ in Eq.(1) as a 4×4 matrix with the first index referring to particle 1 and the second to particle 2. One then introduces a modified spinor $\Phi(\mathbf{r})$ defined by

$$\Psi(\mathbf{r}) = -\frac{i}{2} (\Sigma_y)_2 \Phi(\mathbf{r}) \quad (16)$$

and decomposes it in the basis of 16 Dirac matrices as

$$\begin{aligned} \Phi(\mathbf{r}) &= I(\mathbf{r}) + \mathbf{A}(\mathbf{r}) \cdot \beta \boldsymbol{\alpha} - A_0(\mathbf{r}) \beta + \mathbf{G}(\mathbf{r}) \cdot \boldsymbol{\Sigma} + \mathbf{F}(\mathbf{r}) \cdot \boldsymbol{\alpha} \\ &+ J(\mathbf{r}) \Gamma + \mathbf{U}(\mathbf{r}) \cdot \beta \boldsymbol{\alpha} \Gamma - U_0(\mathbf{r}) \beta \Gamma. \end{aligned} \quad (17)$$

The two-body Dirac equation(Eq.(1)) then gives for its components the following set of equations:

$$[E + V_1(r)]I + 2i\nabla \cdot \mathbf{F} + 2mA_0 = 0, \quad (18)$$

$$[E + V_2(r) + V_9(r) \left(\frac{\mathbf{r}}{r} \times \right)^2] \mathbf{A} + 2\nabla \times \mathbf{U} - 2\mu \mathbf{F} = 0, \quad (19)$$

$$[E + V_3(r)]A_0 + 2mI = 0, \quad (20)$$

$$[E + V_4(r) - V_{10}(r) \left(\frac{\mathbf{r}}{r} \times \right)^2] \mathbf{G} + 2i\nabla \cdot J - 2m\mathbf{U} = 0, \quad (21)$$

$$[E + V_5(r) - V_{11}(r) \left(\frac{\mathbf{r}}{r} \times \right)^2] \mathbf{F} + 2i\nabla I - 2\mu \mathbf{A} = 0, \quad (22)$$

$$[E + V_6(r)]J + 2i\nabla \cdot \mathbf{G} + 2\mu U_0 = 0, \quad (23)$$

$$[E + V_7(r) + V_{12}(r) \left(\frac{\mathbf{r}}{r} \times \right)^2] \mathbf{U} + 2\nabla \times \mathbf{A} - 2m\mathbf{G} = 0, \quad (24)$$

$$[E + V_8(r)]U_0 + 2\mu J = 0, \quad (25)$$

which are a generalized version of those first obtained by Moseley and Rosen [9]. Here $m = (m_1 + m_2)/2$, $\mu = (m_1 - m_2)/2$ and the potentials $V_i(i = 1, \dots, 12)$ are related to the shape functions in Eq.(15) by

$$\begin{aligned} V_1 &= \Omega_1 + 2\Omega_2 - 3\Omega_3 - 2\Omega_4 + \Omega_5 - 3\Omega_S, \\ V_2 &= -\Omega_1 + \Omega_2 + \Omega_4 + \Omega_5 + \Omega_S - 4\Omega_T, \\ V_3 &= \Omega_1 - \Omega_2 - \Omega_4 - \Omega_5 - 3\Omega_S, \\ V_4 &= \Omega_1 + \Omega_3 + \Omega_5 + \Omega_S - 4\Omega_T, \\ V_5 &= -\Omega_1 - \Omega_3 - \Omega_5 + \Omega_S - 4\Omega_T, \\ V_6 &= -\Omega_1 + 2\Omega_2 + 3\Omega_3 - 2\Omega_4 - \Omega_5 - 3\Omega_S, \\ V_7 &= \Omega_1 + \Omega_2 + \Omega_4 - \Omega_5 + \Omega_S - 4\Omega_T, \end{aligned} \quad (26)$$

$$\begin{aligned}
V_8 &= -\Omega_1 - \Omega_2 - \Omega_4 + \Omega_5 - 3\Omega_S, \\
V_9 &= -6\Omega_T, \\
V_{10} &= 6\Omega_T, \\
V_{11} &= 6\Omega_T, \\
V_{12} &= -6\Omega_T.
\end{aligned}$$

The next step is to separate the radial and angular variables in each of the components in Eq.(17) corresponding to a state with total angular momentum J, M_J . For the components $S \equiv I, A_0, J$ and U_0 we write

$$S(\mathbf{r}) = s_J(r) Y_{JM_J}(\vartheta, \varphi), \quad (27)$$

with $s = i, a, j$ and u , respectively, and for the components of $\mathbf{V} = \mathbf{A}, \mathbf{G}, \mathbf{F}$ and $\mathbf{U}$ we write

$$\mathbf{V}(\mathbf{r}) = [v_J^{(1)}(r)\mathbf{L} + v_J^{(2)}(r)r\nabla + v_J^{(3)}(r)\frac{\mathbf{r}}{r}] Y_{JM_J}(\vartheta, \varphi), \quad (28)$$

with $v = a, g, f$ and u , respectively, and $\mathbf{L} = -i\mathbf{r} \times \nabla$. One then introduces the auxiliary radial functions

$$\begin{aligned}
w_1(r) &= ir^2 f_J^{(3)}(r), \\
w_2(r) &= -ij_J(r), \\
w_3(r) &= ru_J^{(1)}(r), \\
w_4(r) &= ira_J^{(2)}(r), \\
w_5(r) &= r^2 g_J^{(3)}(r), \\
w_6(r) &= -ij_J(r), \\
w_7(r) &= ira_J^{(1)}(r), \\
w_8(r) &= ru_J^{(2)}(r),
\end{aligned} \quad (29)$$

in terms of which the merely algebraic relations resulting from the set of equations Eqs.(18-25) allow one to write the other radial components as

$$a_J = \frac{2m}{E + V_3} w_2, \quad (30)$$

$$u_J = -\frac{2i\mu}{E + V_8} w_6, \quad (31)$$

$$a_J^{(3)} = -\frac{2i}{(E + V_2)r^2} [\mu w_1 + J(J+1)w_3], \quad (32)$$

$$u_J^{(3)} = \frac{2}{(E + V_7)r^2} [mw_5 - J(J+1)w_7], \quad (33)$$

$$f_J^{(1)} = -\frac{2i\mu}{(E + V_5 + V_{11})r} w_7, \quad (34)$$

$$f_J^{(2)} = \frac{2i}{(E + V_5 + V_{11})r} (w_2 - \mu w_4), \quad (35)$$

$$g_J^{(1)} = \frac{2m}{(E + V_4 + V_{10})r} w_3, \quad (36)$$

$$g_J^{(2)} = \frac{2}{(E + V_4 + V_{10})r} (w_6 + mw_8). \quad (37)$$

The remaining relations give rise to the radial equations

$$\begin{aligned}
\frac{dw_1}{dr} &= C_{12} w_2 + C_{14} w_4, \\
\frac{dw_2}{dr} &= C_{21} w_1 + C_{23} w_3, \\
\frac{dw_3}{dr} &= C_{32} w_2 + C_{34} w_4, \\
\frac{dw_4}{dr} &= C_{41} w_1 + C_{42} w_2,
\end{aligned} \quad (38)$$

and

$$\begin{aligned}
\frac{dw_5}{dr} &= C_{56} w_6 + C_{58} w_8, \\
\frac{dw_6}{dr} &= C_{65} w_5 + C_{67} w_7, \\
\frac{dw_7}{dr} &= C_{76} w_6 + C_{78} w_8, \\
\frac{dw_8}{dr} &= C_{85} w_5 + C_{87} w_7,
\end{aligned} \quad (39)$$

whose coefficients are rational expressions involving $m, \mu, J, E, V_i(r)$ and r .

As discussed in [6] one identifies two classes of solutions for the two-body Dirac equation. Class I corresponds to states with parity $\eta_1 \eta_2 (-)^J$, where η_1 and η_2 are the intrinsic parities of particles 1 and 2. They satisfy the radial equations Eq.(38) and have $w_5 = w_6 = w_7 = w_8 = 0$. Class II corresponds to states with parity $\eta_1 \eta_2 (-)^{J+1}$, have $w_1 = w_2 = w_3 = w_4 = 0$ and satisfy equations Eq.(39). The latter ones are, therefore, the equations relevant for the deuteron and we limit ourselves here to give the explicit expressions for their coefficients:

$$\begin{aligned}
C_{56} &= \frac{2J(J+1)}{E + V_4 + V_{10}} - \frac{[(E + V_6)(E + V_8) - (2\mu)^2]r^2}{2(E + V_8)}, \\
C_{58} &= \frac{2mJ(J+1)}{E + V_4 + V_{10}},
\end{aligned}$$

$$\begin{aligned}
C_{65} &= \frac{(E + V_4)(E + V_7) - (2m)^2}{2(E + V_7)r^2}, \\
C_{67} &= \frac{2mJ(J+1)}{(E + V_7)r^2}, \\
C_{76} &= \frac{2m}{E + V_4 + V_{10}}, \\
C_{78} &= -\frac{(E + V_4 + V_{10})(E + V_7 - V_{12}) - (2m)^2}{2(E + V_4 + V_{10})}, \\
C_{85} &= \frac{2m}{(E + V_7)r^2}, \\
C_{87} &= -\frac{2J(J+1)}{(E + V_7)r^2} + \frac{(E + V_2 - V_9)(E + V_5 + V_{11}) - (2\mu)^2}{2(E + V_5 + V_{11})}.
\end{aligned} \tag{40}$$

5 Mixed Coupling

It is important to notice that the results of the previous section cannot be used without modifications if one assumes mixed coupling for the pion to the nucleon, that is, an interaction Lagrangian of the form

$$\mathcal{L}_{\pi NN}^{mix} = \bar{\mu}\mathcal{L}_{\pi NN}^{ps} + (1 - \bar{\mu})\mathcal{L}_{\pi NN}^{pv}, \tag{41}$$

where $\bar{\mu}$ is a mixing parameter between 0 and 1 and the coupling constants are assumed to satisfy Eq.(6). The reason is that, at the level of the two-body Dirac equation, there appears a cross term in the interaction, which is now given by

$$H_{12}^{mix}(\mathbf{r}) = H_{12}^{ps}(\mathbf{r}) + H_{12}^{pv}(\mathbf{r}) + H_{12}^{ps\,pv}(\mathbf{r}), \tag{42}$$

where the first two terms are the same as in sections 2 and 3 with the replacements

$$g_\pi \longrightarrow \bar{\mu}g_\pi; \quad f_\pi \longrightarrow (1 - \bar{\mu})f_\pi$$

and the third one is

$$H_{12}^{ps\,pv}(\mathbf{r}) = i \left[(\beta\Gamma)_1 \frac{\Sigma_2 \cdot \mathbf{r}}{r} - \frac{\Sigma_1 \cdot \mathbf{r}}{r} (\beta\Gamma)_2 \right] \Lambda(r) \tag{43}$$

with

$$\Lambda(r) = -\frac{\bar{\mu}(1 - \bar{\mu})f_\pi g_\pi}{4\pi m_\pi} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{d}{dr} \left(\frac{\exp(-m_\pi r)}{r} \right). \tag{44}$$

This introduces extra terms proportional to $\Lambda(r)$ in equations Eqs.(18-25) which are not merely in the form of extra contributions to V_i . As a consequence, the form itself of the radial equations Eq.(38) and Eq.(39) changes, even though the separation into two classes of solutions remains.

More specifically, if one adds to the general interaction Eq.(15) an extra term of the form Eq.(43), then Eq.(18) and Eq.(25) remain unchanged, while Eqs.(19)-(24) become

$$\begin{aligned}
[E + V_2(r) + V_9(r) \left(\frac{\mathbf{r}}{r} \times \right)^2] \mathbf{A} + 2\Lambda(r) \frac{\mathbf{r}}{r} \times \mathbf{G} + 2\nabla \times \mathbf{U} - 2\mu \mathbf{F} &= 0, \\
[E + V_3(r)] A_0 - 2i\Lambda(r) \frac{\mathbf{r}}{r} \cdot \mathbf{F} + 2mI &= 0, \\
[E + V_4(r) - V_{10}(r) \left(\frac{\mathbf{r}}{r} \times \right)^2] \mathbf{G} - 2\Lambda(r) \frac{\mathbf{r}}{r} \times \mathbf{A} + 2i\nabla \cdot J - 2m\mathbf{U} &= 0, \\
[E + V_5(r) - V_{11}(r) \left(\frac{\mathbf{r}}{r} \times \right)^2] \mathbf{F} + 2i\Lambda(r) A_0 \frac{\mathbf{r}}{r} + 2i\nabla I - 2\mu \mathbf{A} &= 0, \\
[E + V_6(r)] J - 2i\Lambda(r) \frac{\mathbf{r}}{r} \cdot \mathbf{U} + 2i\nabla \cdot \mathbf{G} + 2\mu U_0 &= 0, \\
[E + V_7(r) + V_{12}(r) \left(\frac{\mathbf{r}}{r} \times \right)^2] \mathbf{U} + 2i\Lambda(r) J \frac{\mathbf{r}}{r} + 2\nabla \times \mathbf{A} - 2m\mathbf{G} &= 0,
\end{aligned} \tag{45}$$

with the potentials $V_i (i = 1, \dots, 12)$ still given by Eq.(26). Keeping the same definitions for the auxiliary radial functions as in Eq.(29), one must then replace Eq.(30), Eq.(33), Eq.(36) and Eq.(37), respectively, by

$$\begin{aligned}
a_J &= \frac{2m}{E + V_3} \left(\frac{\Lambda}{mr^2} w_1 + w_2 \right), \\
u_J^{(3)} &= \frac{2}{(E + V_7)r^2} \left(mw_5 + \Lambda r^2 w_6 - J(J+1) w_7 \right), \\
g_J^{(1)} &= \frac{2m}{(E + V_4 + V_{10})r} \left(w_3 + \frac{\Lambda}{m} w_4 \right), \\
g_J^{(2)} &= \frac{2}{(E + V_4 + V_{10})r} (w_6 + \Lambda w_7 + mw_8),
\end{aligned} \tag{46}$$

while Eq.(31), Eq.(32), Eq.(34) and Eq.(35) remain as before. The radial equations change accordingly, and the ones relevant to the deuteron take now the form

$$\begin{aligned}
\frac{dw_5}{dr} &= \bar{C}_{55} w_5 + \bar{C}_{56} w_6 + \bar{C}_{57} w_7 + \bar{C}_{58} w_8, \\
\frac{dw_6}{dr} &= \bar{C}_{65} w_5 + \bar{C}_{66} w_6 + \bar{C}_{67} w_7, \\
\frac{dw_7}{dr} &= \bar{C}_{76} w_6 + \bar{C}_{77} w_7 + \bar{C}_{78} w_8, \\
\frac{dw_8}{dr} &= \bar{C}_{85} w_5 + \bar{C}_{86} w_6 + \bar{C}_{87} w_7 + \bar{C}_{88} w_8,
\end{aligned} \tag{47}$$

with the new coefficients given by

$$\bar{C}_{55} = \frac{2m\Lambda}{E + V_7},$$

$$\begin{aligned}
\bar{C}_{56} &= C_{56} + \frac{2r^2\Lambda^2}{E+V_7}, \\
\bar{C}_{57} &= 2J(J+1) \left(\frac{1}{E+V_4+V_{10}} - \frac{1}{E+V_7} \right) \Lambda, \\
\bar{C}_{58} &= C_{58}, \\
\bar{C}_{65} &= C_{65}, \\
\bar{C}_{66} &= -\frac{2m\Lambda}{E+V_7}, \\
\bar{C}_{67} &= C_{67}, \\
\bar{C}_{76} &= C_{76}, \\
\bar{C}_{77} &= \frac{2m\Lambda}{E+V_4+V_{10}}, \\
\bar{C}_{78} &= C_{78}, \\
\bar{C}_{85} &= C_{85}, \\
\bar{C}_{86} &= 2 \left(\frac{1}{E+V_7} - \frac{1}{E+V_4+V_{10}} \right) \Lambda, \\
\bar{C}_{87} &= C_{87} - \frac{2\Lambda^2}{E+V_4+V_{10}}, \\
\bar{C}_{88} &= -\frac{2m\Lambda}{E+V_4+V_{10}}.
\end{aligned} \tag{48}$$

6 OBE model for the NN interaction

In our applications to the deuteron, OPEP is not good enough and we shall take instead the NN interaction to be of the OBE type. We shall however restrict the exchanged bosons to the most important ones for nuclear structure calculations, namely the π , ρ and ω mesons plus a fictitious scalar-isoscalar σ boson, which simulates certain 2π and $\pi\rho$ exchanges in a more complete model [10]. For the three real mesons, the coupling constants g_b and masses m_b should be taken as close as possible to their experimental values, while the remaining parameters can be more freely adjusted to reproduce the deuteron observables. While for the pion, as explained previously, we allow for mixed coupling, for the remaining bosons we consider direct coupling only.

Within this model and following a procedure analogous to the one sketched above for the pion, we arrive at an interaction Hamiltonian of the form

$$\begin{aligned}
H_{12} &= -\Omega_1(r) \omega_1 - \Omega_2(r) \omega_2 - \Omega_5(r) \omega_5 - \Omega_S(r) \omega_S - \Omega_T(r) \omega_T \\
&+ i\Lambda(r) \left[(\beta\Gamma)_1 \frac{\mathbf{\Sigma}_2 \cdot \mathbf{r}}{r} - \frac{\mathbf{\Sigma}_1 \cdot \mathbf{r}}{r} (\beta\Gamma)_2 \right].
\end{aligned} \tag{49}$$

It is also important to regularize the interactions close to the origin, taking phenomenologically into account the extended nature of nucleons and mesons by means of form factors, for which we choose the so-called monopole parametrization [11]. The net result is to introduce for each boson b a cutoff mass Λ_b and replace the Yukawa functions as follows

$$\frac{\exp(-m_b r)}{r} \rightarrow C(r, m_b, \Lambda_b) \equiv \left(\frac{\exp(-m_b r)}{r} - \frac{\exp(-\Lambda_b r)}{r} \right) \frac{\Lambda_b^2}{\Lambda_b^2 - m_b^2}. \tag{50}$$

In summary, in place of Eq.(7), Eq.(11) and Eq.(44) we now have

$$\Omega_5(r) = -\frac{(\bar{\mu}g_\pi)^2}{4\pi} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 C(r, m_\pi, \Lambda_\pi), \tag{51}$$

$$G^{pv}(r) = -\frac{1}{4\pi} \left[\frac{(1-\bar{\mu})f_\pi}{m_\pi} \right]^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 C(r, m_\pi, \Lambda_\pi), \tag{52}$$

and

$$\Lambda(r) = -\frac{\bar{\mu}(1-\bar{\mu})f_\pi g_\pi}{4\pi m_\pi} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{d}{dr} C(r, m_\pi, \Lambda_\pi). \tag{53}$$

With these modifications, $\Omega_S(r)$ and $\Omega_T(r)$ are still given by Eqs.(14) and the remaining shape functions by

$$\Omega_1(r) = \frac{g_\sigma^2}{4\pi} C(r, m_\sigma, \Lambda_\sigma) + \frac{\lambda}{4m} \Omega_5(r)^2 \tag{54}$$

and

$$\Omega_2(r) = -2\frac{g_\omega^2}{4\pi} C(r, m_\omega, \Lambda_\omega) - 2\frac{g_\rho^2}{4\pi} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 C(r, m_\rho, \Lambda_\rho). \tag{55}$$

The origin of the second term in Eq.(54) has been discussed in Section 3 and it appears now, as a correction to the fictitious σ boson exchange.

7 Numerical Integration and Calculation of the Deuteron Observables

The deuteron has $J^\pi = 1^+$ and therefore its spinor can be written as [5]

$$\Psi_{J^\pi=1^+} = \frac{1}{\mathcal{N}} \begin{bmatrix} f_1(r)[^3D_1] + g_1(r)[^3S_1] & f_2(r)[^1P_1] + g_2(r)[^3P_1] \\ f_3(r)[^1P_1] + g_3(r)[^3P_1] & f_4(r)[^3D_1] + g_4(r)[^3S_1] \end{bmatrix} \tag{56}$$

where

$$\begin{aligned}
[^{2S+1}L_J]_{M_J} &= \sum_{M_S} \langle LM_J - M_S SM_S | JM_J \rangle Y_{L, M_J - M_S}(\vartheta, \varphi) \chi_{SM_S} = \\
&\begin{bmatrix} \langle LM_J - 1S1 | JM_J \rangle Y_{L, M_J - 1}(\vartheta, \varphi) & \frac{1}{\sqrt{2}} \langle LM_J S0 | JM_J \rangle Y_{L, M_J}(\vartheta, \varphi) \\ \frac{(-)^{S+1}}{\sqrt{2}} \langle LM_J S0 | JM_J \rangle Y_{L, M_J}(\vartheta, \varphi) & \langle LM_J + 1S - 1 | JM_J \rangle Y_{L, M_J + 1}(\vartheta, \varphi) \end{bmatrix}.
\end{aligned} \tag{57}$$

The functions $f_i(r)$ and $g_i(r)$ are given in terms of the auxiliary radial functions by

$$\begin{aligned}
f_1(r) &= \frac{1}{\sqrt{3}r} \left[-\left(1 + \frac{2m}{E+V_7}\right) \frac{w_5(r)}{r} + 2\left(\frac{1}{E+V_4+V_{10}} - \frac{\Lambda r}{E+V_7}\right) w_6(r) \right. \\
&\quad \left. + 2\left(\frac{\Lambda r}{E+V_4+V_{10}} + \frac{2}{E+V_7}\right) \frac{w_7(r)}{r} + \left(\frac{2m}{E+V_4+V_{10}} + 1\right) w_8(r) \right], \\
g_1(r) &= \frac{1}{\sqrt{6}r} \left[\left(1 + \frac{2m}{E+V_7}\right) \frac{w_5(r)}{r} + 2\left(\frac{2}{E+V_4+V_{10}} + \frac{\Lambda r}{E+V_7}\right) w_6(r) \right. \\
&\quad \left. + 4\left(\frac{\Lambda r}{E+V_4+V_{10}} - \frac{1}{E+V_7}\right) \frac{w_7(r)}{r} + 2\left(\frac{2m}{E+V_4+V_{10}} + 1\right) w_8(r) \right], \\
f_2(r) &= \frac{i}{\sqrt{2}} w_6(r), \quad g_2(r) = -i \frac{w_7(r)}{r}, \\
f_3(r) &= \frac{i}{\sqrt{2}} w_6(r), \quad g_3(r) = i \frac{w_7(r)}{r}, \\
f_4(r) &= \frac{1}{\sqrt{3}r} \left[\left(-1 + \frac{2m}{E+V_7}\right) \frac{w_5(r)}{r} + 2\left(\frac{1}{E+V_4+V_{10}} + \frac{\Lambda r}{E+V_7}\right) w_6(r) \right. \\
&\quad \left. + 2\left(\frac{\Lambda r}{E+V_4+V_{10}} - \frac{2}{E+V_7}\right) \frac{w_7(r)}{r} + \left(\frac{2m}{E+V_4+V_{10}} - 1\right) w_8(r) \right], \\
g_4(r) &= \frac{1}{\sqrt{6}r} \left[-\left(-1 + \frac{2m}{E+V_7}\right) \frac{w_5(r)}{r} + 2\left(\frac{2}{E+V_4+V_{10}} - \frac{\Lambda r}{E+V_7}\right) w_6(r) \right. \\
&\quad \left. + 4\left(\frac{\Lambda r}{E+V_4+V_{10}} + \frac{1}{E+V_7}\right) \frac{w_7(r)}{r} + 2\left(\frac{2m}{E+V_4+V_{10}} - 1\right) w_8(r) \right].
\end{aligned} \tag{58}$$

In these equations, as well as in the rest of this section and in the numerical calculations, we are neglecting, for simplicity, the neutron-proton mass difference, i. e., setting $\mu = 0$. This should not sensibly affect the numerical results.

The auxiliary radial functions $w_i(r)$ are obtained by numerically integrating the system of differential equations Eq.(39). To this end, one first observes that in the asymptotic region one has $V_i(r), \Lambda(r) \rightarrow 0$ and this system becomes

$$\begin{aligned}
\frac{d\bar{w}_5(r)}{dr} &= \left(\frac{4}{E} - \frac{Er^2}{2}\right) \bar{w}_6(r) + \frac{4m}{E} \bar{w}_8(r), \\
\frac{d\bar{w}_6(r)}{dr} &= \frac{E^2 - 4m^2}{2Er^2} \bar{w}_5(r) + \frac{4m}{Er^2} \bar{w}_7(r), \\
\frac{d\bar{w}_7(r)}{dr} &= \frac{2m}{E} \bar{w}_6(r) - \frac{E^2 - 4m^2}{2E} \bar{w}_8(r), \\
\frac{d\bar{w}_8(r)}{dr} &= \frac{2m}{Er^2} \bar{w}_5(r) + \left(-\frac{4}{Er^2} + \frac{E}{2}\right) \bar{w}_7(r),
\end{aligned} \tag{59}$$

whose solution regular as $r \rightarrow \infty$ is

$$\begin{aligned}
\bar{w}_5(r) &= \left[(2ma_0 + Eb_0) \left(\frac{1}{\gamma^2 r} + \frac{1}{\gamma} \right) + \frac{Eb_0 r}{2} \right] \exp(-\gamma r), \\
\bar{w}_6(r) &= b_0 \gamma \left(\frac{1}{\gamma^2 r^2} + \frac{1}{r} \right) \exp(-\gamma r), \\
\bar{w}_7(r) &= a_0 \gamma \left(\frac{1}{\gamma r} + 1 \right) \exp(-\gamma r), \\
\bar{w}_8(r) &= - \left[\left(\frac{Ea_0}{2} + mb_0 \right) \left(\frac{1}{\gamma^2 r^2} + \frac{1}{\gamma r} \right) + \frac{Ea_0}{2} \right] \exp(-\gamma r),
\end{aligned} \tag{60}$$

with a_0, b_0 arbitrary constants and $\gamma = (m^2 - \frac{E^2}{4})^{1/2}$. Using these results one obtains for the large-large components in the asymptotic region the expressions

$$\begin{aligned}
\bar{f}_1(r) &= -\frac{(a_0 + b_0)(2m + E)}{2r\sqrt{3}} \left[3 \left(\frac{1}{\gamma^2 r^2} + \frac{1}{\gamma r} \right) + 1 \right] \exp(-\gamma r), \\
\bar{g}_1(r) &= \frac{(2m + E)(-2a_0 + b_0)}{2r\sqrt{6}} \exp(-\gamma r).
\end{aligned} \tag{61}$$

Choosing two convenient sets $(a_0^{(1)}, b_0^{(1)})$ and $(a_0^{(2)}, b_0^{(2)})$ of the arbitrary constants one obtains, following what was done in [12], two solutions of Eq.(59) that produce

$$\begin{aligned}
\bar{u}^{(1)} &= r\bar{g}_1^{(1)} = \exp(-\gamma r), \quad \bar{w}^{(1)} = r\bar{f}_1^{(1)} = 0, \\
\bar{u}^{(2)} &= r\bar{g}_1^{(2)} = 0, \quad \bar{w}^{(2)} = r\bar{f}_1^{(2)} = \left[3 \left(\frac{1}{\gamma^2 r^2} + \frac{1}{\gamma r} \right) + 1 \right] \exp(-\gamma r),
\end{aligned} \tag{62}$$

giving the result that for solution (1) the large-large components of $\Psi_{J=1+}$ behave as an S -wave while for solution (2) they behave as a D -wave.

Integrating the system Eq.(39) from a large value of r toward the origin, using as initial values Eq.(60) for each set $(a_0^{(i)}, b_0^{(i)})$ of arbitrary constants determined as indicated above, one finds two linearly independent solutions $w_i^{(1)}(r), w_i^{(2)}(r), (i = 5, 6, 7, 8)$. We then take a linear combination of $w_i^{(1)}(r)$ and $w_i^{(2)}(r)$,

$$w_i(r) = A_S w_i^{(1)}(r) + A_D w_i^{(2)}(r), \quad i = 5, 6, 7, 8 \tag{63}$$

and determine the ratio $A_D/A_S \equiv \eta$ by finding a hard core radius $r = r_0$ such that at this point

$$\begin{aligned}
u(r_0) &= A_S u^{(1)}(r_0) + A_D u^{(2)}(r_0) = 0, \\
w(r_0) &= A_S w^{(1)}(r_0) + A_D w^{(2)}(r_0) = 0.
\end{aligned} \tag{64}$$

Having obtained the auxiliary radial functions $w_i(r)$, one uses them in Eq.(58) and determines completely the deuteron spinor Eq.(56).

Now we proceed to find expressions for the deuteron observables. By imposing $\langle \Psi_{J^\pi=1+} | \Psi_{J^\pi=1+} \rangle = 1$, one finds that

$$\mathcal{N}^2 = \int \sum_{i=1}^4 [|f_i(r)|^2 + |g_i(r)|^2] r^2 dr, \quad (65)$$

where this and all the radial integrals to follow exclude the hard core region $r < r_0$. The root-mean square matter radius r_m , defined by

$$r_m = \frac{1}{2} \langle r^2 \rangle^{1/2} = \frac{1}{2} \left(\int Tr(\Psi_{J^\pi=1+}^\dagger \Psi_{J^\pi=1+} d\mathbf{r}) \right)^{1/2}, \quad (66)$$

is an $R(3)$ scalar and, therefore, its value is determined from Eq.(65) as

$$r_m = \frac{1}{2} \left[\frac{1}{\mathcal{N}^2} \int \sum_{i=1}^4 [|f_i(r)|^2 + |g_i(r)|^2] r^4 dr \right]^{1/2}. \quad (67)$$

For the quadrupole moment, one takes the usual definition,

$$Q = \frac{1}{4} \langle \Psi_{J^\pi=1+}(M_J = J) | (3z^2 - r^2) | \Psi_{J^\pi=1+}(M_J = J) \rangle = \frac{1}{4} \sqrt{\frac{16\pi}{5}} \langle \Psi_{J^\pi=1+}(M_J = J) | Y_{20}/r^2 | \Psi_{J^\pi=1+}(M_J = J) \rangle, \quad (68)$$

and finds

$$Q = \frac{1}{\sqrt{50}\mathcal{N}^2} \int \left\{ \text{Re}[f_1^*(r)g_1(r)] - \frac{1}{2\sqrt{2}} |f_1(r)|^2 + \frac{1}{\sqrt{2}} [-|f_2(r)|^2 + \frac{1}{2}|g_2(r)|^2 - |f_3(r)|^2 + \frac{1}{2}|g_3(r)|^2] + \text{Re}[f_4(r)^*g_4(r)] - \frac{1}{2\sqrt{2}} |f_4(r)|^2 \right\} r^4 dr. \quad (69)$$

Concerning the triplet scattering length a_t , we will compute it only approximately. We start from Eq.(10.21) in Ref. [13],

$$a_t \cong \frac{1}{\gamma[1 - \frac{1}{2}\gamma(3r_e)]}, \quad (70)$$

where $3r_e$ is the triplet effective range, together with Eq.(3) of Ref.[14], namely,

$$(1 + \eta^2)N^2 \cong \frac{1}{1 - \gamma(3r_e)} \quad (71)$$

where

$$N^2 = \frac{1}{2\gamma \int [|U(r)|^2 + |W(r)|^2] dr}, \quad (72)$$

and $U(r)$ and $W(r)$ are proportional to $u(r)$ and $w(r)$ with the property that $U(r) \rightarrow \exp(-\gamma r)$ as $r \rightarrow \infty$. Combining the two equations one obtains

$$a_t = \frac{2}{\gamma[1 + \frac{1}{(1+\eta^2)N^2}]}. \quad (73)$$

Since Eq.(71) and Eq.(72) are nonrelativistic expressions, we extend N^2 to the relativistic case by writing

$$N^2 = \frac{1}{2\gamma \int \sum_{i=1}^4 [|f_i(r)|^2 + |g_i(r)|^2] r^2 dr} = \frac{1}{2\gamma \mathcal{N}^2} \quad (74)$$

in place of Eq.(72).

8 Numerical Results and Conclusions

We have integrated the system of equations Eq.(39) for $J^\pi = 1^+$, taking for the interaction the OBE model described in Sec. 6 with two sets (A) and (B) of parameters inspired in Ref.[15] and listed in Table 1. We have varied the values of $\lambda, \bar{\mu}$ and $f_\pi^2/4\pi$ in the intervals (0, 2), (0, 1) and (0.070, 0.080), respectively, and the best results obtained are listed in Table 2, where

$$\Delta = \left[\frac{1}{4} \sum_{i=1}^4 \left(1 - \frac{\text{calculated value}}{\text{experimental value}} \right)_i^2 \right] \times 100\% \quad (75)$$

is a mean percent deviation for the computed observables, namely, $i = \eta, r_m, Q$ and a_t . The experimental values have been taken from references cited in Ref.[15]. Note that we have not corrected the experimental value of the deuteron quadrupole moment for the meson-current contributions, as done, for instance, in Refs.[16, 17]. However, this would lower its value to 0.277 fm^2 , bringing it, in fact, to closer agreement with our results.

In Fig. 1 we show the normalized large-large components $u(r)/\mathcal{N} = rg_1(r)$ and $w(r) = rf_1(r)/\mathcal{N}$ of the deuteron wave-function computed with the interaction parameters corresponding to the third entry in Table 2. The other entries give similar results.

It is interesting to notice that all the best results have occurred for $f_\pi^2/4\pi = 0.070$, except for one case, in which it occurred for a slightly larger value (0.072). This is lower than the formerly widely accepted value of about 0.080 for this coupling constant based on NN phase shift analyses [18, 19] and πN scattering data [20]. Starting with the extensive phase shift analysis of pp scattering by the Nijmegen group [21], later extended to np and $\bar{p}p$ scattering [22], which set its value close to 0.075, a controversy started as to the true value of this constant [23]. This controversy is not completely settled yet, but, anyway, the

Table 1: Coupling constants $g_b^2/4\pi$ and masses m_b for the bosons with spin J , parity P and isospin I included in the OBE models A and B. We used a single cutoff mass Λ_b for all bosons, equal to $1600MeV$ and $1610MeV$ for models A and B, respectively. For the pion-nucleon coupling, see text.

Bosons		Model A		Model B	
$J^P(I)$	Boson	$g_b^2/4\pi$	$m_b(MeV)$	$g_b^2/4\pi$	$m_b(MeV)$
$0^+(0)$	σ	5.51	516	5.51	523
$1^-(0)$	ω	9.85	782.8	9.02	782.8
$1^-(1)$	ρ	0.383	760	0.279	760
$0^-(1)$	π		138		138

Table 2: Best results obtained for the deuteron observables with λ smaller than, equal to and greater than 1 for the two models in Table 1. The last line lists the experimental values for comparison.

model	λ	$\bar{\mu}$	$f_\pi^2/4\pi$	r_0	η	r_m	Q	a_t	Δ
				(fm)		(fm)	(fm ²)	(fm)	(%)
A	0.61	0.20	0.070	0.542	0.0266	1.95	0.279	5.37	2.4
A	1.00	0.30	0.070	0.549	0.0270	1.93	0.277	5.32	3.4
A	1.42	0.40	0.070	0.505	0.0268	1.96	0.281	5.39	2.5
B	0.50	0.15	0.072	0.541	0.0268	1.96	0.283	5.41	2.4
B	1.00	0.15	0.070	0.536	0.0262	1.96	0.278	5.40	1.8
B	1.12	0.15	0.070	0.535	0.0262	1.96	0.278	5.40	1.8
Expt					0.0256	1.96	0.286	5.42	

lower value found by the Nijmegen group has been confirmed by independent analyses of both pion-nucleon [24] and nucleon-nucleon [25] scattering data. In fact, some authors have extracted values of $f_\pi^2/4\pi$ as low as 0.070 from both np scattering [26] and the $pp \rightarrow nn$ charge-exchange reaction [27]. It can, therefore, be said that the optimum value we found for $f_\pi^2/4\pi$ in the present calculation of deuteron observables conforms to the tendency for a lower value of this coupling constant according to its more recent determinations. In a sense this statement is not in disagreement with the deuteron calculations of Machleidt and collaborators [16, 17], since we do not use a tensor coupling for the ρ meson.

It is also important to mention that in our calculations it has been essential to include some amount of nonderivative coupling into the predominantly derivative πNN coupling, with mixing parameter ranging from $\bar{\mu} = 0.15$ to 0.40 . This is in agreement with the more sophisticated relativistic calculations of Gross *et al.* [15] with similar models for the interaction. A pure derivative coupling ($\bar{\mu} = 0$) does not lead to a bound deuteron in our calculation, while a pure nonderivative coupling ($\bar{\mu} = 1$) gives much poorer results [3]. Finally let us remark that a *sine qua non* condition to get reasonable results has been to include in all cases the quadratic scalar term [the second term in Eq.(54)] that compensates the repulsive core contribution arising from the direct πNN coupling.

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Figure caption: Normalized large-large components of the deuteron wave function