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We are investigating the direct detection of photon-photon elastic scattering at optical energies. In a first experiment using two high-intensity pulsed laser beams, we have explored the feasibility of the method, and in particular the rejection of back-ground noise. We obtained an upper limit of the photon-photon elastic scattering cross section at 95\% confidence level of $10^{-39}$ cm$^2$. This limit can be lowered by twenty orders of magnitude by stimulating the scattering by a third beam, and by using high repetition rate existing lasers.

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In the classical electromagnetic theory, light satisfies the superposition principle and scattering of light by light does not occur. In quantum electrodynamics (QED), photon-photon scattering is possible and involves a virtual electron-positron pair in an intermediate state. The QED treatment of light-by-light scattering is well known, and the phenomenon has been observed in the Delbrück elastic scattering of photon in the electrostatic field of a nucleus[1], and indirectly in its small contribution to the anomalous magnetic moments of the electron[2] and of the muon[3]. Both processes involve high energy virtual photons for which the amplitude is not particularly small. At low energy however, the cross section becomes extremely small. If observed, a deviation from QED would be of a particular interest, as diagrams including a fermion loop allow for gauge invariant tests of the fermion propagator[4]. Note also the large cross section predicted by the composite-photon theory or by the effective-photon model [5].

We are preparing an experiment for the direct observation of photon-photon elastic scattering, taking advantage of the high brilliance of high power pulsed lasers. In this paper, we present the results of a first experiment in which we have brought two laser beams to head-on collision, and we have searched for scattered photons at an angle. In this experiment we have studied with special care the alignment of the relative position of the two beams and the rejection of background noise which are the key points in the experiment. The obtained upper bound of the cross section is twenty five orders of magnitude above QED. The three-beam experiment which is now in preparation could set the limit at five orders of magnitude above QED.

The determination of the photon-photon scattering amplitude was first obtained by Euler in 1936 and confirmed later using Feynman rules [6]. At low energy \((h\nu<<m_e c^2)\) and for unpolarized light, the differential cross section is:

\[
\frac{d\sigma_{\gamma\gamma \rightarrow \gamma\gamma}}{d\Omega} = \frac{139}{32400\pi} \alpha^2 r_e^2 \left( \frac{h\nu}{m_e c^2} \right)^6 (3 + \cos^2 \theta)^2
\]

where \(\alpha\) is the fine structure constant, \(m_e\) the electron mass, \(r_e\) the electron classical radius, and \(h\nu\) the energy of each photon in the system of the center of mass. The total cross section in the low energy domain is:

\[
\sigma_{\gamma\gamma \rightarrow \gamma\gamma} = \frac{973}{10125\pi} \alpha^2 r_e^2 \left( \frac{h\nu}{m_e c^2} \right)^6
\]

that is in practical units:

\[
\sigma_{\gamma\gamma \rightarrow \gamma\gamma} [cm^2] = 0.73 \cdot 10^{-65} (h\nu[eV])^6
\]

In this domain, the cross section increases very rapidly with photon energy. The exact expression shows a maximum of \(\sigma_{\text{max}} = 1.6 \cdot 10^{-30} \text{ cm}^2\) for energies close to the electron mass \(h\nu = 1.5 m_e c^2\) and a decrease in \((h\nu)^{-2}\) in the high-energy limit \((h\nu>>m_e c^2)\) [7].
In this experiment we bring two laser beams in collision in vacuum and we search for elastically scattered photons. One of the beams is frequency doubled so that the scattered photons can be easily identified from the incoming photons by their different wavelength. From the energy and momentum conservation equations, we obtain the wavelengths of the scattered photons as functions of the observation angle \( \alpha_f \). In the case of a planar geometry and for two head-on colliding beams at frequencies \( \nu_1 \) and \( 2\nu_0 \), we obtain (see Fig.1):

\[
\begin{align*}
\lambda_1 &= \lambda_0 \frac{3 - \cos \alpha_f}{4},
\lambda_2 &= \lambda_0 \frac{3 - \cos \alpha_f}{5 - 3 \cos \alpha_f} \\
\text{and } \quad i\eta \alpha_2 &= 4 \sin \alpha_f / (5 \cos \alpha_f - 3).
\end{align*}
\]

The local luminosity \( \mathcal{L}(x,y,z,t) \) is related to the local photon density of the two incident beams \( \rho_a \) and \( \rho_b \) by \( \mathcal{L}(x,y,z,t) = 2\pi \rho_a(x,y,z,t) \rho_b(x,y,z,t) \). The integrated luminosity \( \mathcal{L} \) per laser shot is \( \mathcal{L} = \int \mathcal{L}(x,y,z,t) dx dy dz dt \). With laser beams having gaussian transverse and time profiles, the photon density is:

\[
\rho_{a,b}(x,y,z,t) = \frac{N_{a,b}}{(\pi / 2)^{3/2} w_{x,a} w_{y,a} w_{z,a} w_{z,b}} \exp \left[ - \frac{2x^2}{w_{x,a}^2} - \frac{2y^2}{w_{y,a}^2} - \frac{2(z - ct)^2}{w_{z,b}^2} \right]
\]

\( N_{a,b} \) is the total number of photons in beams \( a \) and \( b \) and \( w_{x,y} \) is the radius at 1/e² in intensity which can be expressed as a function of the waist at the focal point \( w_0 \) by \( w_{z,a,b}(z) = w_{z,a,b} [1 + (z / z_{R,a,b})^2]^{1/2} \), where \( z_{R,a,b} \) is the Rayleigh length \( (z_{R,a,b} = \pi w_0^2 / \lambda) \). The pulse duration at full-width at half-maximum (FWHM) \( \tau \) is related to \( w_z \) by \( w_z = c \tau / \sqrt{2 \ln 2} \). The sign in \((z \pm ct)\) takes opposite values for beams \( a \) and \( b \). After integration we obtain:

\[
\mathcal{L} = \frac{N_a N_b}{\pi^{3/2} w_0} \exp \left[ \frac{-u^2}{\mu^2} \right] du = \frac{N_a N_b e^{\mu^2 / \mu^2}}{\sqrt{\pi w_0^2}} \left[ 1 - \text{erf} \left( \frac{2z}{w_z} \right) \right]
\]

with:

\[
\begin{align*}
\mu &= \frac{2z}{w_z} \quad , \quad \mu &= \frac{2z}{w_z} \quad , \\
w_0^2 &= \frac{w_{a}^2 + w_{b}^2}{2} \quad , \\
w_z^2 &= \frac{w_{a}^2 + w_{b}^2}{2} \quad , \\
z \quad &= \frac{2}{z_{K,a}} \frac{2}{z_{K,b}} w_{a}^2 w_{b}^2 \frac{1}{w_{a}^2 + w_{b}^2}.
\end{align*}
\]

For short laser pulses (\( w_z < < z_R \) or \( \mu >> 1 \)) we find the usual expression of the luminosity in a collision of two parallel cylindrical beams: \( \mathcal{L}_{\text{short}} = N_a N_b / \pi w_0^2 \) and for long laser pulses (\( w_z > > z_R \) or \( \mu < < 1 \)) : \( \mathcal{L}_{\text{long}} = \mathcal{L}_{\text{short}} \cdot (2 \sqrt{\pi z_R} / w_z) \). The number of observed photons is \( N = \mathcal{L} / \sigma \eta \epsilon n \), where \( n \) is the number of laser shots, \( \eta \) the acceptance of the apparatus and \( \epsilon \) the detection efficiency. In absence of background noise, we get the limit detectable cross section \( \sigma = \sigma_{\text{limit}} \) for \( N = 1 \). For long laser pulses we obtain:

\[
\sigma_{\gamma \gamma \rightarrow \gamma \gamma \text{limit}} = \frac{1}{\mathcal{L}_\eta \eta \epsilon n} = \frac{w_{a}^2 w_{b}^2 \sqrt{\pi}}{\eta \epsilon n N_a N_b z_R}
\]
The experimental set-up is shown in Fig. 2. The laser pulse delivered by a Nd-YLF oscillator is amplified in the laser chain of LULI and separated in two similar beams at 1.053 μm with an output diameter of 65 mm and an energy of 70 ± 5 J. A KDP frequency doubling crystal generates a 30±5 J green beam at 0.526 μm. The measured pulse length at 1.053 μm is 600±20 ps which gives \( w_z = 15.3±0.5 \) cm. For our high energy pulses, the efficiency of the frequency doubler is saturated, and the green beam has about the same pulse length as its genitor. Two one-meter focal length bichromatic doublets \((f/10)\) focus the two beams onto a nominal interaction point precisely defined in the centre of the vacuum chamber by a retractable pinhole \((\Phi 30 \mu m)\). The longitudinal position of the waist of the two beams is adjusted by optimizing the transmission through the pinhole of a continuous infrared beam propagating along the laser chain. The accuracy \( \Delta z \) is better than ±250 μm. This is confirmed by a direct observation of breakdown in air generated by low energy shots just above threshold. For lateral alignment the pinhole is imaged onto two imaging systems along the laser axes. The lateral and vertical positions of the focusing lenses are then adjusted on low-energy shots to align the two focal spots. With this technique we estimate the accuracy at \( \Delta = \pm 20 \) μm. This also provides a control of the alignment in each high-energy shot. The same imaging systems are used to measure the focal spots. Two intensity distributions are shown in Fig. 3. From the profiles we obtain diameters (FWHM) of 45±5 μm for the infrared beam and 35±5 μm for the green beam, i.e. \( w_{oa} = 38±5 \) μm, \( w_{ob} = 30±5 \) μm and \( w_o = 35±5 \) μm. The central peak contains 60% of the total energy for the infrared beam (a) and 30% for the green beam (b), i.e. a number of photons \( N_a = (3.7±0.3)\cdot10^{20} \) and \( N_b = (8±1.3)\cdot10^{19} \).

The Rayleigh lengths \( z_{Ra,b} \) are obtained by imaging successive planes along the focusing axis. This is shown in Fig. 4 together with a fit assuming a gaussian profile. Actually the profile of the beam before focusing is not gaussian. It is enclosed in a \( \Phi 65 \) mm disc and presents intensity variations up to a factor two. The FWHM size of the Airy profile of a uniform, \( \Phi 65 \) mm, incident beam is 17μm, giving an equivalent waist of 14μm and a Rayleigh length of 590 μm in the gaussian model. The measured waist and Rayleigh length of the infrared beam are \( w_{oa} = 38±5 \) μm and \( z_{Ra} = 1.7±0.2 \) mm, respectively 2.7 times and 2.9 times the expected values. We conclude that the gaussian model at 2.8 times the diffraction limit is a reasonable description of the geometry of the central spot of the actual beam. For the green beam we obtain \( w_{ob} = 30±5 \) μm and \( z_{Rb} = 1.5±0.2 \) mm, i.e. about 5 times the diffraction limit. The two laser pulses are synchronized with a streak camera and a Nd-YLF beam with a pulse duration of 100 ps. The accuracy is better than \( \Delta t = \pm 20 \) ps.

The interaction point is imaged at 45° with two \( f/5 \) lenses, giving a unit magnification at the entrance of a spectrometer-phomultiplier combination (cf Table I). The acceptance \( \eta \) of the detection apparatus is equal to \( \eta = \left[ \frac{\theta_c}{4\gamma(1-\beta \cos \alpha_f)} \right]^2 = 3.8\cdot10^{-3} \), where \( \theta_c \) is the aperture of the imaging optics \( (\theta_c=0.2) \), and \( \beta \) is the velocity of the photon pair system in the laboratory frame \( (\beta=1/3) \). The spectrometer is centered at 0.604 μm and the photocathode of the
photomultiplier collects photons in the range 0.562 – 0.646 μm, larger than the spectral width of the accepted photons due to the finite aperture of the injection and collection optics, which is equal to 0.577 – 0.634 μm. In this energy domain, the efficiencies of the spectrometer and of the photocathode are respectively 40% and 8%. The photomultiplier signal is measured by a 100 MHz oscilloscope externally triggered by an electric signal synchronized with the laser pulse, with a jitter of 1 ns. The average one photo-electron signal is 10 mV.

The most difficult point in this experiment is the suppression of all noise sources. Photons from the laser beams at 1.053 and 0.526 μm scattered by the entrance windows, by the residual gas and reflected in the chamber are easily suppressed by colored-glass filters (KG3 and OG570 Schott). Photons generated near 604 nm in the residual gas are dominant at high pressure. Their number varies with the square of the residual pressure and becomes negligible near 10⁻³ mbar where less than 1 photon per nanosecond is detected. The experiment was performed at 10⁻⁵ mbar so that this effect does not give any measurable contribution. On the contrary a wide spectrum is generated when the green laser beam passes through the optics. This was confirmed by filtered photographs of the entrance window and of the focusing lens. As shown in Fig.2 a set of masks were placed inside the chamber to prevent these photons to reach the entrance of the spectrometer.

Typical signals measured on the oscilloscope are shown in Fig.5. The dotted line is obtained by inserting a small object at the interaction point to scatter the laser light. It determines the time of the crossing of the beams. When the two beams are brought to collision in vacuum, we only observe several noise photons arriving more than 5 ns after crossing. Over 7 shots, no signal was observed at crossing time. Using formula (1) we get an integrated luminosity for the 7 shots of 3.8·10⁴³ cm⁻² and a limit cross section of 2.2·10⁻⁴⁰ cm². As the luminosity involves a product of the photon densities of the two beams, we only considered the energy contained in the central peaks of the focal spots.

In presence of misalignment, the integrated luminosity reads:

\[
L_{L(\Delta)} = \frac{N_a N_b}{\pi^{3/2}} \int_{-\infty}^{+\infty} \frac{e^{-\frac{\Delta^2}{w_\sigma^2}}}{w_\sigma^2} \left( \frac{u + \frac{c \Delta}{2}}{\mu^2 + \frac{4u R \Delta}{\mu^2 w_\sigma^2} + \frac{4u R \Delta}{\mu^2 w_\sigma^2}} \right)^2 \text{d}u
\]

with \( R = (z_a^2 w_{oa}^2 - z_a^2 w_{ob}^2) / (z_a^2 w_{oa}^2 + z_a^2 w_{ob}^2) \), and \( \Delta^2 = \Delta_x^2 + \Delta_y^2 \).

As many parameters enter in the expression of the integrated luminosity, some of them with a large fluctuation, its distribution is not gaussian and it is not possible to obtain a good estimate by simple linearization. Instead, we have performed a simulation by a Monte-Carlo method, varying
each parameter with a gaussian distribution, with a RMS calculated from the FWHM uncertainties. The simulation provides an integrated luminosity of $L_{\gamma\gamma} = 3.5 \pm 1.3 \times 10^{42} \text{ cm}^{-2}$ (RMS) per shot, leading to $(2.5 \pm 0.3) \times 10^{43} \text{ cm}^{-2}$ over 7 shots. The main part of the (35%) loss in luminosity and of the fluctuation is due to the fluctuation of the relative transverse position of the beams. Therefore we believe that the limited accuracy of our description of the complex structure of the laser focal spot does not affect the result. As no scattered photon was observed, we obtain an upper limit of the cross section at a 95% confidence level from the simulated luminosity distribution.

$$\sigma_{\gamma\gamma \to \gamma\gamma}^{95\%CL} = 9.9 \times 10^{-64} \text{ cm}^{-2}.$$  

To our knowledge, only one previous attempt of direct low energy photon-photon collision has been performed, in 1930 [8]. Two sunlight beams passed through red filters and were focussed at the same point inside a dustfree air box. The intersection point was examined through a green filter by a dark-adapted eye. No scattered light was detected and this led to an upper limit of $\sigma_{\text{limit}} = 3.10^{20} \text{ cm}^{-2}$. We improve this result by nearly twenty order of magnitude. Note that a search for nearly massless, weakly coupled particles, by the study of the vacuum birefringence induced by a magnetic field, has a sensitivity to QED effects only three orders of magnitude above theory[9].

Our result is in contradiction with the composite photon theory which predicts a cross section of $10^{-22} \text{ cm}^{2}$[10]. At our center of mass photon energy of 1.7 eV, the QED cross section amounts to $1.6 \times 10^{-64} \text{ cm}^{2}$. Thus there are still twenty five orders of magnitude to reach the back-ground from QED. The use of high repetition rate lasers will bring an improvement by about four orders of magnitude. A much larger potential of improvement can be brought by stimulating the scattering by a third beam[11]. With a 1mJ beam provided by an optical parametric amplifier, we will gain sixteen orders of magnitude, bringing us at five orders of magnitude from QED. The actual observation of the QED effect would need a further increase in the energy of the laser beams.

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References


A. A. Varfolomeev, Sov. Phys. JETP 23 (1966) 681,


### Laser beam a

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<thead>
<tr>
<th>Parameter</th>
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<tr>
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<tr>
<td>Energy</td>
<td>70±5 J in central peak 60%</td>
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<tr>
<td>Diameter</td>
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<tr>
<td>Pulse duration</td>
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<td>Focal spot (FWHM)</td>
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<tr>
<td>Rayleigh length: $z_{Ra}$</td>
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### Laser beam b

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<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Wavelength</td>
<td>0.526 µm</td>
</tr>
<tr>
<td>Energy</td>
<td>30±5 J in central peak 30%</td>
</tr>
<tr>
<td>Diameter</td>
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<tr>
<td>Pulse duration</td>
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<td>Focal length</td>
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<tr>
<td>Focal spot (FWHM)</td>
<td>35±5 µm</td>
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<tr>
<td>Rayleigh length: $z_{Rb}$</td>
<td>1.5±0.2 mm</td>
</tr>
</tbody>
</table>

### Vacuum (residual pressure)

- $10^{-5}$ mbar

### Detection system at 45°

- **Spectrometer**
  - Focal length: 22 cm
  - Grating: 1200 grooves per mm
  - Dispersion: 26 Å/mm
  - Efficiency at 0.604 µm: $\varepsilon_{spectro}$: 40%

- **Photomultiplier**
  - Thorn EMI 9902B
  - PM voltage: 1400 V
  - Diameter: 32 mm
  - Efficiency at 0.604 µm: $\varepsilon_{PM}$: 8%

- **Filters**
  - (Schott)
    - KG3 (e= 2 mm) 83%
    - OG570 (e= 3 mm) < 0.001%
    - BG39 (e= 1 mm) 87%
  - Transmission at $\lambda=0.526$ µm: 79%
  - $\lambda=0.604$ µm: 91%
  - $\lambda=1.053$ µm: 0.06%
Figure Captions

FIG 1: Schematics of a head-on elastic collision of two photons at frequencies $\nu_0$ and $2\nu_0$.

FIG 2: Schematics of the experimental set-up. Two synchronized laser beams at $\lambda_0 = 1.053$ μm and $\lambda_0/2$ are focussed to a common focal spot. A photon-photon collision can give a scattered photon detected at $\alpha_1 = 45^\circ$ with $\lambda_1 = 0.604$ μm wavelength. We did not attempt to observe the other scattered photon at $\alpha_2 = 79^\circ$ with $\lambda_2 = 0.838$ μm.

FIG 3: Focal spots and profiles of the infrared beam (a) and of the green beam (b).

FIG 4: Experimental transverse waist $w_{x,y}$ versus $z$ for the infrared laser. $z=0$ is the focal point. The curve $w_{x,y} = w_{x,y} = w_{o_d}[1+(z/z_{R_d})^2]^{1/2}$ fits the experimental points with $w_{o_d} = 38$ μm and $z_{R_d} = 1.7$ mm.

FIG 5: Typical time-resolved signal from spectrometer-photomultiplier-oscilloscope system obtained at 0.604 μm (solid line), together with a calibration signal (dotted line). See text.
Fig. 1
Fig. 2