A Monte Carlo Study of the Acceptance to Scattered Events in a Depth Encoding PET Camera

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Abstract

We present a Monte Carlo study of the acceptance to scattered events in a Depth Encoding Large Aperture Camera (DELAC), a hypothetical PET scanner with the capacity to encode the depth-of-interaction (DOI) of incident $\gamma$-rays. The simulation is initially validated against the measured energy resolution and scatter fraction of the ECAT-953B scanner. It is then used to assess the response to scattered events in a PET camera made of position encoding blocks of the EXACT HR PLUS type, modified to have DOI resolution through a variation in the phototube pulse height. The detection efficiency for 511 keV $\gamma$-rays, as well as for those that scattered in the object or left only part of their energy in the block, is studied for several combinations of DOI sensitivities and block thicknesses. The scatter fraction predicted by the simulation for DELACs of various ring radii is compared to that of the ECAT-953B as a function of the energy threshold. The results indicate that the poorer discrimination of object scatters with depth sensitive blocks does not lead to a dramatic increase of the scatter fraction.

I. INTRODUCTION

The depth-of-interaction of $\gamma$-rays induces the so-called parallax error in PET and degrades the image resolution uniformity across the field-of-view (FOV) of the camera. The development of position encoding detectors with DOI resolution aims for a restoration of the spatial resolution uniformity and, as by-products, increased sensitivity and lower fabrication costs through a reduction of the tomograph ring radius. Better resolution uniformity and sensitivity at lower cost would hopefully enlarge the range of clinical applications for this modality.

Recent work at TRIUMF [1, 2] has focused on position encoding block detectors capable of simultaneously good DOI and transverse position resolutions. In our approach, DOI sensitivity [2] is acquired by inducing a gradient in the light collection efficiency of state-of-the-art position encoding multicrystal BGO detectors. This allows the DOI of $\gamma$-rays to be simply estimated from the variation in the phototube pulse height from the top to the back faces of the block. Recent results by our group [2], have indicated that good DOI resolution could be obtained with reduced light collection efficiency without spoiling the transverse position resolution of a prototype EXACT HR PLUS block [3, 4]. Based on the measured performances of that block, we also used Monte Carlo simulations to compare the expected point source position resolution in a conventional camera made of unmodified blocks and in a Depth Encoding Large Aperture Camera (DELAC). At the edge of the FOV, correction for the DOI was predicted to lead to a resolution that could be as good as 4.0 mm in a DELAC compared to 5.75 mm in a conventional camera of identical geometry. In addition, the dependence of the DOI correction upon the inverse of the ring radius was shown to result in a better resolution uniformity if the ring of the DELAC is made smaller than that of the current generation of wholebody PET tomographs. From these results, attraction for this approach seems justified. Furthermore, the use of a simple modification to detectors in commercial production offers the potential for rapid implementation in a full tomograph without significant increase in the fabrication cost of the detectors. However, the impact on the DELAC performance of $\gamma$-rays that Compton-scattered in the subject volume or the passive material of the tomograph, here termed “object scatters”, as well as that of $\gamma$-rays that only had Compton interactions in the block, termed “detector scatters”, raise several legitimate concerns and calls for an investigation. The rejection of low energy events based on pulse height thresholding depends on their depth distribution in DOI sensitive blocks. As a result, one may expect the scatter fraction to be unacceptably higher in the DELAC than in conventional tomographs. The potential increase will be even further accentuated by a reduction in the ring radius. To confirm the feasibility of a tomograph based on DOI encoding through the total pulse height, we present Monte Carlo studies that assess the combined effect of detection threshold, DOI sensitivity, block thickness, and ring radius on the acceptance to scattered events in a DELAC.
II. THE SIMULATION

An outline of the simulation used for these studies is available in [2]. It predicts the position and energy response of septaless multi-ring PET cameras using the measured crystal-by-crystal energy spectra and line-spread-functions (LSFs) of prototype position encoding BGO block detectors of the ECAT series as its main inputs. This approach ensures the reliability of the results while avoiding extensive modeling of the light transport in position encoding blocks. This was shown [2] to successfully predict the transaxial point-source-projections measured in the ECAT-953B, both at the center and at the edge of the FOV. The same simulation was further developed for predicting the measured energy response and scatter fraction in septaless tomographs and was validated against measurements on the ECAT-953B.

A. Modeling the Energy Resolution of the ECAT-953B

A realistic model of the energy response of position encoding multicrystal block detectors is crucial in predicting the scatter fraction of state-of-the-art PET cameras as a function of the lower energy discriminator threshold. The attention of this study has therefore first focused on a validation of the simulation against the inclusive single energy spectrum measured in a wholebody tomograph.

Such inclusive energy spectra were measured by Mazoyer et al. [5] and Spinks et al. [6] in order to check the global energy resolution and calibration linearity of the ECAT-953B. Figure 1 compares the energy distributions measured by these two groups for single 511 keV γ-rays to that predicted by the simulation. The agreement between the measured and simulated spectra is excellent over the full energy range. Its quality relies on a realistic modeling of the following effects that broaden the inclusive photopeak energy resolution:

- realistic crystal-by-crystal variations of the photopeak pulse height due to the individual crystal-to-PMT couplings;
- the influence of these variations on the energy and position estimates encoded by the block for events with multiple interactions;
- errors due to these variations in encoding the event's crystal address and in calibrating its energy; and finally,
- the contribution of events that Compton-scattered in the passive material surrounding the blocks.

B. Modeling the ECAT-953B’s Acceptance for Scattered Events

The scatter fraction measured in the ECAT-953B as a function of the lower energy discriminator level was further used in the validation of the simulation. Previous work by Michel et al. [7] reports the measured percentage of scattered events observed in the 953B without septa from a line source, 15 cm long, at the center of a cylindrical perspex phantom, of 21.6 cm diameter and 22 cm length, filled with 6.355 liters of water. Much care was paid to reproduce with the simulation the experimental conditions that impact on the reported results. In particular, scattering from the passive material composing the phantom: water, perspex and carbon holder; as well as scattering from the tomograph’s end-shields and detector ring casing were effectively modeled. Hardware constraints set on crystal pair addresses to limit the transaxial and axial FOV of the tomograph were also taken into account.

Figure 2(a) shows the simulated inclusive transaxial projection for lines-of-responses (LORs) from a line source at the center of the water phantom in the ECAT-953B without septa. A lower energy threshold of 150 keV is applied, and the events are sampled in 3.1 mm wide bins [6]. True events form the peak at the center of the projection. The shaded histogram gives the contribution of LORs for which at least one photon scattered in the phantom or tomograph passive material. The Monte Carlo history of each event was first used to evaluate the scatter fraction by counting the number of LORs for which at least one photon had scattered in passive material. To duplicate the experimental method of determining the scatter fraction, an
estimation was also carried out using only the inclusive transaxial projection. The number of scattered events was obtained by fitting exponential functions to the projection histogram, starting at 2.5 cm on either side of the peak and extending up to ±7.5 cm [6]. The fitted functions were then extrapolated under the peak and integrated from 0 to ±2.5 cm respectively, and the resulting integral added to the number of events observed in the FOV at projection values above ±2.5 cm.

Figure 2(b) compares the measured scatter fraction in the ECAT-953B as a function of the lower energy threshold [7] to that predicted by the simulation for a FOV of 22 cm diameter. For the simulated data, the scatter fraction is that estimated from the exponential fit to the scatter profile. The error bars on the simulation results reflect the variation of the scatter fraction due to the uncertainties in the slope and normalization of the exponential functions fitted to the projection profile. The scatter fraction evaluated using the events' Monte Carlo history agrees with the exponential fit results within an absolute uncertainty of 1% up to threshold values of 500 keV. Measured and simulated data match well up to a threshold of 350 keV. Compared to previous Monte Carlo predictions by Michel et al. [7], this simulation presents a better agreement with the measured scatter fraction, especially at low energy thresholds. In spite of a successful modeling of the measured single inclusive energy distribution, the simulation underestimates the measured scatter fraction by up to 5% above 350 keV.

III. Acceptance to Scattered Events in a DELAC

A. Detection of Scattered Events with Depth Sensitive Blocks

Given identical tomograph and phantom geometries, the scatter fraction using depth encoding detectors can only depart from that of a conventional camera through different detection efficiencies for trues, object or detector scatter. Conventionally, the efficiency to trues is maximized by properly choosing the thickness of the block detector, while the efficiencies to object and detector scatter are controlled by a discriminator threshold. In a block where a variation of the events' pulse height is induced to acquire a DOI sensitivity, the block thickness and the discriminator threshold have an intertwined impact on the detection efficiencies. To understand this impact, one must consider the depth distribution of each class of events as well as the depth variation of the pulse height in the block.

The simulation validated in the previous section was used to assess the difference in detection efficiencies to trues, object and detector scatter of various DOI sensitive block candidates compared to those of a conventional block design. All blocks were assumed to have the 36×38 mm cross-sectional area and 8×8 crystal matrix of the EXACT HR PLUS detector. The blocks were arranged in a two ring cylindrical tomograph of 41 cm radius. A realistic incident single γ-ray energy spectrum was obtained by generating events from the same line source plus water filled phantom used in the validation of the simulation against the measured scatter fraction in the ECAT-953B. It was assumed that blocks with thicknesses, \( z_{\text{block}} \), of 3 cm and 2 cm could be manufactured to achieve DOI sensitivities of -75%, -50%, +50% and +75%, where the DOI sensitivity is defined by [2]:

\[
S_{\text{DOI}} = \frac{P(z_{\text{block}}) - P(0)}{\max(P(z_{\text{block}}), P(0))}
\]

As expressed, \( S_{\text{DOI}} \) gives the percentage variation of the events' photopage pulse height, \( P \), from the minimum to maximum depth-of-interaction in the block. A conventional block is characterized by \( S_{\text{DOI}} \sim 0 \), while a DOI sensitivity of +75% (-75%) indicates that events interacting deep, at \( z_{\text{block}} \), will have a photopage pulse height 75% higher (lower) than ones that interact at the front face.

Figure 3 presents the resulting single γ-ray detection efficiency, averaged over the full geometrical acceptance of the tomograph, as a function of pulse height and depth-of-interaction in 3 cm thick blocks with \( S_{\text{DOI}} \) of +75% and -75% respectively. To ease the discussion, a value of 511 units was assumed as the minimum photopage pulse height.
for both blocks.

The figure shows well the difference in the pulse height
distributions of trues and scatters depending on the DOI
sensitivity of the block. As can be seen from Figure 3(a),
for $S_{DOI} = +75\%$, the photopeaks of the trues form a
ridge that extends linearly from pulse height values of 511
to 2044 for depths of zero to 30 mm. For $S_{DOI} = -75\%$, in
Figure 3(b) the slope changes sign, and the trues populate
photopeak pulse height values of 2044 to 511 over the same
depth range.

Object scattered events undergo a stronger attenuation
in BGO due to their reduced energy. As a result, they
tend to interact within the first centimeter of the block and
do deposit little energy. For $S_{DOI} = +75\%$, these events
form a sharp peak at pulse heights lower than 511 and
small depth values. A lower threshold on the pulse height
discriminates these events from the ridge defined by the
trues. On the contrary, for a negative DOI sensitivity, the
same shallow events form a much broader distribution that
extends toward pulse height values higher than 511. Their
contribution overlaps significantly with that of deeply in-
teracting unscattered events. Because the pulse height is
correlated to the DOI, the rejection of scattered events in
such detectors is not only determined by a lower energy
threshold, but also by the DOI sensitivity and thickness
of the block. This is directly evident from the previous
figure, where a lower threshold along the pulse height axis,
to reject the object scatter contribution, will cut a differ-
ent percentage of the trues for positive or negative DOI
sensitivity.

Figure 4 presents the relative efficiencies to detect true,
object and detector scattered photons as a function of the
pulse height threshold for various combinations of block
thicknesses and DOI sensitivities. The efficiencies were
obtained by counting with the simulation the number of
detected single $\gamma$-rays of each kind that were above thresh-
hold. For a given combination of DOI sensitivity and block
thickness, the sum of all three efficiencies was normalized
to unity for a lower threshold of zero. This choice of norm-
alization allows one to easily compare the relative differ-
ences between the 3 cm and 2 cm deep blocks, by factoring
out their absolute detection efficiencies.

Not surprisingly, the simulation indicates that the effi-
ciency to detect trues presents little dependence upon the
DOI sensitivity for pulse height thresholds lower than 400.
With increasing threshold, the loss in efficiency for trues
is the fastest for the blocks with $S_{DOI}$ values of zero and
+50%, which have the narrowest photopeak pulse height
distributions. The three centimeter deep blocks detect, in
proportion, more trues than the two centimeter ones.

The efficiencies to object and detector scatters show a
much stronger dependence upon the DOI sensitivity of the
blocks. Clearly, the lowest values are reached for a con-
ventional block with $S_{DOI}=0\%$. The negative DOI sensi-
tivities present the poorest rejection power to object scat-
ters. This is inherent to the shallow depth distribution of
these events, which translates to high pulse height values for the negative DOI blocks. The same tendency to interact shallow in the block results in very little difference in the relative detection efficiency to object scatters for the 3 and 2 cm thick blocks. The largest difference between these appears in the efficiencies to detector scatters. For a source at the center of the FOV, detector scatters are more likely to be caused by deep Compton interactions followed by the escape of the recoil photon through the back face of the block. As a result, the relative efficiencies to detector scatters are higher for 2 cm than 3 cm thick blocks. Furthermore, as deep Compton interactions yield a higher signal in the blocks with $S_{DOI} = +75\%$ and $+50\%$, they are more affected by these events.

For a depth encoding camera presenting a scatter fraction competitive to that of a conventional scanner, the best block candidate should present maximal efficiency to trues while minimizing the efficiency to scatters. Considering the impact of the DOI sensitivity on these efficiencies, a good figure of merit to compare all detector candidates on a quantitative basis is the ratio of trues to object plus detector scatter efficiencies for a nominal lower energy threshold of 400 keV. Table 1 compares that ratio, $\epsilon_t/\epsilon_s$, as well as the median depth resolution [2], $\Delta Z$, for all block candidates considered.

<table>
<thead>
<tr>
<th>DOI sensitivity</th>
<th>$+75%$</th>
<th>$+50%$</th>
<th>0%</th>
<th>$-75%$</th>
<th>$-50%$</th>
</tr>
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<tr>
<td>$\epsilon_t/\epsilon_s$</td>
<td>1.5</td>
<td>2.0</td>
<td>2.9</td>
<td>0.9</td>
<td>1.2</td>
</tr>
<tr>
<td>$\Delta Z$ (mm)</td>
<td>6</td>
<td>10</td>
<td>-</td>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 1: Comparison of the ratio of trues to object plus detector scatters efficiencies, as well as of the median depth resolution for all block candidates considered.

The results in Table 1 highlight the compromise between depth resolution and scatter rejection in the choice of a block candidate for a DELAC. Though the 2 cm thick block with $S_{DOI} = +75\%$ offers the best depth resolution, its capacity to discriminate scattered events is clearly limited with a true to scatter efficiency ratio of only 1.2. A better compromise between scatter rejection and depth resolution seems to lie in the 3 cm thick blocks with positive DOI sensitivities. The highest value of the true to scatter efficiency ratio, $\epsilon_t/\epsilon_s$, is reached for the 3 cm thick block with $S_{DOI} = +50\%$. This detector was also shown [2] to lead to a position resolution with a FWHM (FWTM) of 3.5(9.5) to 5.0(10.5) mm from the center to the edge of the FOV in a 41 cm radius DELAC. At the cost of slightly poorer scatter rejection and position resolution at the center of the FOV [2], the DOI sensitivity of the 3 cm block can be increased to $+75\%$ to improve the position resolution uniformity across a DELAC's FOV.

B. The Scatter Fraction in a DELAC

The 3 cm thick blocks with DOI sensitivities of $+50\%$ and $+75\%$ were selected as good candidates to study the resulting acceptance to scatters in a DELAC. The simulation was used to predict the scatter fraction of 41 cm radius DELACs made of these two block candidates. The transaxial FOV of the DELACs was limited by excluding coincidences occurring between two adjacent groups of six blocks. Two rings of blocks made the full axial acceptance of the simulated tomographs. Events were again generated from the line source plus water filled phantom used in the validation of the simulation presented in the previous section. Figure 5 presents the scatter fraction evaluated from the events' Monte Carlo history as a function of the lower energy threshold over a 22 cm wide FOV. The results are compared to the measured scatter fraction reported in [7] for the ECAT-953B in 3D mode.

![Graph showing scatter fraction as a function of lower energy threshold](image)

Figure 5: Variation of the scatter fraction as a function of the lower energy threshold predicted by the simulation for DELACs of 31 cm and 41 cm ring radii based on 36x38x30 mm blocks with $S_{DOI} = +50\%$ and $+75\%$ respectively. The simulated results are compared to the measured scatter fraction in the ECAT-953B in 3D mode.

The results readily indicate that the scatter fraction in a 41 cm ring radius DELAC remains smaller or comparable to that of the ECAT-953B when using DOI sensitive blocks. In that geometry, the loss in scatter rejection with the depth sensitive blocks is being canceled by the larger ring radius of the DELAC compared to that of the ECAT-953B. This stands true up to lower energy thresholds of
400 keV, above which the poor rejection of object scatters in the +75% DOI sensitive block significantly impacts on the scatter fraction. For a lower threshold of 350 keV, the simulation predicts a scatter fraction of 39% in a DELAC based on blocks with \( S_{DOI} = +50\% \). The gain in position resolution uniformity achieved with \( S_{DOI} = +75\% \) must be valued against an increase of 4% in the scatter fraction at that threshold.

The impact of a reduction of the DELAC ring radius on the scatter fraction was studied by also simulating cameras of 31 cm radius for both block candidates. The results are presented in Figure 5. At a threshold of 350 keV, decreasing the ring radius to 31 cm brings the scatter fraction to values of 50% and 55% for the 3 cm thick blocks with \( S_{DOI} = +50\% \) and \( S_{DOI} = +75\% \) respectively. Since in volume imaging the acceptance is dominated by the solid angle of the camera relative to the source position, the relationship between the scatter fraction and the DELAC ring radius is easily understood. For true events this acceptance is directly proportional to the solid angle of the scanner which, for a source at the center of a cylinder, simply scales like the inverse power of the ring radius. Had the Compton interaction been isotropic, the acceptance to object scatters would be proportional to the square of that to trues. However, because of the forward distribution of Compton scattering and energy thresholding, the acceptance to object scatters is proportional to \( 1/R \) raised to a power \( x \) between 1 and 2 [8]. As a result, the scatter fraction shows only a soft dependence upon \( R \) of the form
\[
\sim \frac{1}{(1 + R^{x-1})}.
\]
For \( R \) decreasing from 41 cm to 31 cm, the scatter fraction is accordingly expected to increase by a factor between 1.1 and 1.3, in good agreement with our results.

IV. DISCUSSION AND CONCLUSIONS

In this study, we assessed through simulations the impact of scattered photons on the performance of a depth encoding wholebody PET camera based on detectors with which the DOI of individual photons is estimated through the photopeak pulse height. The combined effect of detection threshold, DOI sensitivity, and block thickness on the detection efficiencies to trues, object scatters and detector scatters was first studied for a number of possible block candidates. This was achieved by using a realistic simulation of the energy and angular distributions of true and Compton-scattered events observed from a line source at the center of a cylindrical phantom filled with water.

Comparing the relative detection efficiencies to object scatters first allowed us to discard block designs with negative depth sensitivities. These block candidates initially seemed promising as they yield a high pulse height for shallow interactions. Though this feature assures a good energy resolution for the predominant shallow interactions of 511 keV photons, it was also shown to lead to a poor rejection of object scatters. This poor rejection puts them at a real disadvantage as in these blocks object scatters interacting shallow may incorrectly be interpreted as deep photoelectric interactions. When a correction is applied for their DOI, this depth miscoding effectively moves object scatters from the front towards the back face of the block, which degrades the transverse position resolution [2].

The largest relative difference between the 2 cm and 3 cm thick block candidates was noticed in the efficiency to detector scatters. This effect was noted in earlier work by Dahlbom et al. [9]. A significant relative increase in the fraction of these events is seen in the 2 cm thick block candidates with positive depth sensitivities. In 2 cm deep blocks, a large part of these events will have interactions under the saw cut recesses defining the crystal matrix. As position encoding becomes distorted for events interacting under the saw cuts [10], one may expect to see a loss in the transverse position resolution. Along with an obvious loss in absolute detection efficiency, this discourages the use of 2 cm deep blocks for a depth encoding camera.

As it offers a balanced compromise between good DOI resolution and scatter rejection, the 3 cm thick block with DOI sensitivity of +75% stands out as best candidate for a DELAC. The results indicate that its poorer rejection power for object scatters does not lead to a dramatic increase of the camera's scatter fraction. The scatter fraction in a DELAC of 41 cm ring radius based on such a detector is comparable to that of the ECAT-953B in 3D mode.

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