Aharonov–Bohm Effect in 3D Abelian Higgs Theory

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We study a field-theoretical analogue of the Aharonov–Bohm effect in the 3D Abelian Higgs Model: the corresponding topological interaction is proportional to the linking number of the vortex and the particle world trajectories. We show that the Aharonov–Bohm effect gives rise to a nontrivial interaction of tested charged particles.

1. Introduction

It is well known that the Abelian Higgs Model in three and four dimensions has the classical solutions called Abrikosov–Nielsen–Olesen vortices [1]. These vortices carry quantized magnetic flux (vorticity) and the wave function of the charged particle which is scattered on the vortex acquires an additional phase. The shift in the phase is the physical effect which is the field-theoretical analog [2] of the quantum-mechanical Aharonov–Bohm effect [3]: vortices play the role of solenoids, which scatter the charged particles.

In Section 2 we show that in the 3D lattice Abelian Higgs model with the non-compact gauge field, the Aharonov–Bohm effect gives rise to the long-range Coulomb-like interaction between test particles. In the three dimensional case the induced potential is confining, since the Coulomb interaction grows logarithmically.

In Section 3 we present the results of the numerical calculations of the induced potential. Our numerical results show the existence of the Aharonov–Bohm effect in this model.

2. Potential Induced by the Aharonov–Bohm Effect

The partition function of 3D non-compact lattice Abelian Higgs Model is:

\[ Z = \int_{-\infty}^{+\infty} dA \int_{-\pi}^{+\pi} d\varphi \sum_{l(c_1) \in \mathbb{Z}} e^{-S(A, \varphi, l)}, \]

where

\[ S(A, \varphi, l) = \beta \| dA \|^2 + \gamma \| d\varphi + 2\pi l - NA \|^2, \]

\( A \) is the non-compact gauge field, \( \varphi \) is the phase of the Higgs field and \( l \) is the integer-valued one-form. For simplicity we consider the limit of the infinite Higgs boson mass, then the radial part of the Higgs field is frozen and we use the Villain form of the action.

One can rewrite the integral (1) as the sum over the closed vortex trajectories using the analogue of Berezinski–Kosterlitz–Thaless (BKT) transformation [4]:

\[ Z \propto Z^{BKT} = \sum_{\ast j(\ast c_1) \in \mathbb{Z}} \delta_{\ast j = 0} e^{-4\pi^2 \gamma (\ast j, (\Delta + m^2)^{-1} \ast j)}, \]

where \( m^2 = N^2 \gamma / \beta \) is the classical mass of the vector boson \( A \). The closed currents \( \ast j \) which are defined on the dual lattice represent the vortex trajectory. It can be easily seen from eq.(3) that the currents \( \ast j \) interact with each other through the Yukawa forces.

In the limit \( N^2 \gamma \gg 1 \gg \beta \) the partition function (3) becomes:

\[ Z^{BKT}_0 = \sum_{\ast j(\ast c_1) \in \mathbb{Z}} \exp \left\{ -\frac{4\pi^2 \beta}{N^2} \| \ast j \|^2 \right\}. \]

In the corresponding continuum theory the term \( \| \ast j \|^2 \) is proportional to the length of the trajectory \( \ast j \), therefore the vortices are free. The

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quantum average of the Wilson loop $W_M(C) = \exp\{iM(A,jc)\}$ in the discussed region of the parameters is:

$$< W_M(C) >_N = \frac{1}{Z_{BKT}} \sum_{j' \in \mathbb{Z}} \exp\left\{ \frac{4\pi^2\beta}{N^2} ||*j||^2 + 2\pi i \frac{M}{N} \mathcal{L}(jc, j') \right\}.$$  (5)

The last long–range term has the topological origin: $\mathcal{L}(jc, j')$ is the linking number between the world trajectories of the defects $*j$ and the Wilson loop $jc$:

$$\mathcal{L}(jc, j') = \langle jc, \Delta^{-1}d*j \rangle.$$  (6)

The trajectory of the vortex $*j$ is a closed loop and the linking number $\mathcal{L}$ is equal to the number of points at which the loop $jc$ intersects the two dimensional surface bounded by the loop $*j$. The equation (6) is the lattice analogue of the Gauss formula for the linking number. This topological interaction corresponds to the Aharonov–Bohm effect in the field theory [2,5]. Thus, eq.(5) describes the Aharonov–Bohm interaction of the free vortices carrying the flux $\frac{2\pi}{N}$ with the test particle of the charge $M$. Therefore the interaction between the charged particles is due to Aharonov–Bohm effect only.

The estimation of (5) in the saddle–point approximation ($N^2/\beta \gg 1$) gives in the leading order [5]:

$$< W_M(C) >_N = \text{const.} e^{-\kappa_{(M,N)}^{(0)}(jc, \Delta^{-1}jc)},$$  (7)

where

$$\kappa_{(M,N)}^{(0)} = \frac{q^2 N^2}{4\beta}, \quad q = \min_{K \in \mathbb{Z}} \frac{M}{N} - K,$$  (8)

$q$ is the distance between the ratio $M/N$ and a nearest integer number. Expressions (7–8) depend on the fractional part of $M/N$, this is the consequence of the Aharonov–Bohm effect. Interaction of the testing charges is absent if $q = 0$ ($M/N$ is integer), this corresponds to the complete screening of the test charge $M$ by the Higgs bosons of the charge $N$.

Consider the product of two Polyakov lines: $W_M(C) = L^+_M(0) \cdot L_M(R)$. Then $(jc, \Delta^{-1}jc) = 2T \Delta^{-1}_{(2)}(R)$, where $\Delta^{-1}_{(2)}(R)$ is the two–dimensional massless lattice propagator. At large $R$ the propagator $\Delta^{-1}_{(2)}(R)$ grows logarithmically, $\Delta^{-1}_{(2)}(R) = \frac{C_0}{R} \ln R + \ldots$, where $C_0$ is some numerical constant. Then eq.(7) is reduced to:

$$< L^+_M(0)L_M(R) >_N = \text{const.} e^{-V_{(M,N)}(R)},$$  (9)

where

$$V_{(M,N)}(R) = C_0 \kappa_{(M,N)}^{(0)} \cdot \ln R + \ldots$$  (10)

is the long–range potential induced by the Aharonov–Bohm effect.

One can prove the following general statement: if the potential $V_{(M,N)}$ is induced by the Aharonov–Bohm effect, then it must satisfy the following relations:

$$V_{(M,N)} = V_{(N,M,N)}, \quad V_{(N,N)} = 0,$$  (11)

see e.g. the definition of $q$ (8).

3. Numerical Calculations

We calculated numerically the potential between the tested particles with the charge $M$ in the three dimensional Abelian Higgs Model, the charge of the Higgs boson is $N = 6$. The action of the model is chosen in the Wilson form: $S[A, \varphi] = \beta||dA||^2 - \gamma \cos(d\varphi + NA)$. In our calculations the standard Monte–Carlo method is used. The simulations are performed on the lattice of the size $16^3$ for the charges $M = 1, \ldots, N$.

We fit the numerical data for the potential, (9), by the formula:

$$V_{(M,N)}(R) = 2 \kappa_{num}^{(M,N)} \cdot T \cdot \Delta^{-1}_{(2)}(R) + C_{num},$$  (12)

where $\kappa_{num}$ and $C_{num}$ are numerical fitting parameters. It turns out that the numerical data for $\kappa_{num}$ are well described by the formula (12).
We present on the Figure 1 the plot of the dependence of the coefficients $\kappa_{num}^M$ on $\beta$ for $M = 1, 2, 4, 5$ and $\gamma = 15$. It’s seen, that $\kappa_{num}^M = \kappa_{num}^N - M$ within the numerical errors. The potential for the charge $M = N$ is equal to zero within the errors. These relations are in agreement with the Aharonov–Bohm nature of the potential $V_{(M,N)}(R)$, cf. eq.(11).

The Abelian Higgs model is in the Coulomb phase at the considered values of the parameters $\beta$ and $\gamma$. The usual Coulomb interaction of the test particles with the charge $M$ is proportional to $M^2$, the fact that the potential $V_{(M)}$ satisfies the relations (11) also means that the Coulomb interaction of the tested particles is small.

Although both theoretical and measured numerical potentials satisfy eq.(11) the measured coefficients $\kappa_{num}^M(\beta)$ are not described by semiclassical formula (8). This deviation is due to the renormalization of $\kappa^{(0)}$ by quantum corrections. The same effect exists in the considered model at the finite temperature [7].

Conclusion and Acknowledgments

In this talk we show both analytically and numerically that the Aharonov–Bohm interaction between the vortices and the charged particles induce the long–range Coulomb–type interaction between the particles. Due to the long–ranged nature of the induced potential the Aharonov–Bohm effect may play a role in the dynamics of colour confinement in nonabelian gauge theories [8].

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REFERENCES