First results with non-perturbative fermion improvement

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We present initial results for light hadron masses and nucleon structure functions using a recent proposal for eliminating all $O(a)$ effects from Wilson fermion simulations in the quenched approximation. With initially limited statistics, we find a much more linear APE plot and a value of the axial coupling $g_A$ nearer to the experimental point than with comparable runs using unimproved Wilson fermions.

1. NON-PERTURBATIVE IMPROVEMENT

It is now common to seek to improve Wilson fermions by the addition of a term of the form

$$S_{SW} = \frac{i}{2} \kappa g c_{SW} a \sum_x \bar{\psi}(x) \sigma_{\mu\nu} F_{\mu\nu}(x) \psi(x)$$

(1)

in order to reduce $O(a)$ lattice cut-off effects present in the action. The original proposal \cite{1} was to use $c_{SW} = 1$, which corresponds to the tree-level perturbation theory result; this eliminates terms up to $O(a g^2)$. The elimination of tadpole contributions requires a value of $c_{SW}$ some 50\% larger depending on the coupling \cite{2}, though the leading corrections here remain formally $O(a g^2)$.

It has recently been suggested \cite{3} that $c_{SW}$ can be chosen via some suitable physical condition so as to remove all $O(a)$ effects. Here we present initial results using this action as part of our project for a non-perturbative calculation of nucleon matrix elements \cite{4}.

2. IMPLEMENTATION

Our initial runs used a $16^3 \times 32$ lattice at $\beta = 6.0$ with $c_{SW} = 1.769$ \cite{3}. Our implementation runs on a Quadrics QH2 parallel computer with an $8 \times 8 \times 4$ topology. The improvement term in equation (1) appears in the site-diagonal part of the action; the major overhead in our case is multiplication by this term during inversion of the fermion mass matrix. In our gamma matrix basis,

$$\gamma_i = i \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(2)

we can rewrite this term as

$$1 + \kappa c \sigma \cdot F \equiv \begin{pmatrix} A & B \\ B & A \end{pmatrix}$$

$$\equiv \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} A + B & 0 \\ 0 & A - B \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

(3)

(where $A, B$ are $6 \times 6$ matrices, i.e. two-spinors with colour) so that instead of a $12 \times 12$ multiplication we have two $6 \times 6$ multiplications and two inexpensive co-ordinate transformations. This reduces the overhead for the improvement in the inverter from 45\% to 30\%. Also, the inverse of the term in equation (3) is required on half the
lattice due to the red-black preconditioning; we now have to invert two $6 \times 6$ instead of a $12 \times 12$ matrix. However, this is only required once for each propagator inversion, so is a less significant factor.

At $\beta = 5.7$ we see large numbers of configurations where the fermion matrix inversion fails to converge even after 2000 minimal residual sweeps. For $c_{SW} > 1$ these amount to more than 5% of the total. For $\beta < 6.0$, no non-perturbatively calculated $c_{SW}$ is available [3] for essentially this reason.

3. HADRON MASSES

![Figure 1. The APE plot for the unimproved (unfilled symbols) and $c_{SW} = 1.769$ improved (filled) data. The physical and heavy quark points are shown as squares.](image)

We have calculated pion, rho and nucleon masses for $\kappa = 0.1324, 0.1333$ and 0.1342 from 125 configurations, and on a $24^3 \times 32$ lattice at $0.1342, 0.1346$ and 0.1348 with similar statistics. The APE plot is shown in figure 1, where we compare the results with previous runs using $c_{SW} = 0$ at $\kappa = 0.1515, 0.1530$ and 0.1550 with large statistics (some 5000 data points) on the smaller lattice and 0.1550, 0.1558 and 0.1563 again with around 100 configurations on the larger lattice (the high statistics and light quark results are both previously unpublished). The $\kappa$’s have been chosen to lead to similar pion masses in the improved and unimproved data sets; we also find that the nucleon masses are similar. However, the rho mass with improved fermions is some 20% larger for the higher mass points, which are thus shifted towards the origin in comparison with the Wilson case.

As may be seen, we find that the unimproved Wilson hadron masses eventually converge with the improved values, and indeed the extrapolated values are comparable, although the extrapolation of the ratios deviates considerably more from the linear in the former case. We have also found extrapolation of the individual masses to the chiral limit to be more linear for the improved fermions.

4. STRUCTURE FUNCTIONS

For a full non-perturbative calculation of the structure functions we need two additional ingredients.

First, we require operator improvement which needs to be calculated separately for each operator. The perturbative case has

$$V \equiv \overline{\psi} \gamma_\mu \psi \rightarrow \overline{\psi} \gamma_\mu \psi + b_V A(\overline{\psi} \gamma_\mu \psi - \partial_\mu \overline{\psi} \sigma_{\mu\nu} \psi),$$

$$A \equiv \overline{\psi} \gamma_\mu \gamma_5 \psi \rightarrow \overline{\psi} \gamma_\mu \gamma_5 \psi + b_A \overline{\psi} \gamma_\mu \gamma_5 \psi + b_A A(\partial_\mu \overline{\psi} \gamma_5 \psi + \overline{\psi} \sigma_{\mu\nu} D_\nu \gamma_5 \psi);$$

in this preliminary calculation we have set $b_V = b_A = 1/2$, corresponding to the ‘rotation’ valid in the $c_{SW} = 1$ case. Non-perturbatively, we need to determine the coefficients of each term separately. Secondly, we also need to determine the renormalisation constants $Z_O$ non-perturbatively. At present we use a perturbative calculation assuming the $b_V = b_A = 1/2$ rotation but with $c_{SW} = 1.769$. A point to note is that definitions of $Z_O$ vary, depending on whether the residual effect of the rotation in the chiral limit is absorbed into $Z_O$ or not: we follow Martinelli et al. [5]. This is dif-
different to the prescription being used by the ALPHA collaboration for these calculations, hence their results presented at this conference [6] are not immediately applicable to us. The calculation of the $Z_O$ is then similar to that presented in reference [7].

A fully non-perturbative calculation of the renormalisation constants is currently in progress.

4.1. Local vector current
We have calculated the local vector current and extrapolated to the chiral limit, where the perturbative value for $Z_V$ was calculated. As this current is conserved in the continuum we require only to recover the values 2 for the up quark and 1 for the down quark content of the proton. This acts as a test of the consistency of our procedure. The results are $2.0(3)$ and $1.0(2)$, which are entirely satisfactory.

4.2. Axial vector current
The axial vector current $\Delta q$ is shown in figure 2. This was formerly (in the naive quark model) connected with the spin contribution of the quarks, though emphasis is now on the violation of the OZI rule implied by the low values for the flavour singlet operator [8].

Our main interest is in the combination $g_A = \Delta u - \Delta d$ (the axial coupling) where disconnected fermion loops which we have not calculated cancel out. The experimental value is 1.26. With the improvement, our calculated quantity has changed from $1.07(9)$ to $1.22(14)$. This is better but suffers from large errors.

5. CONCLUSIONS
We have performed a first QCD calculation with Wilson fermions improved up to $O(a^2)$. Hadron masses with improved fermions at $\beta = 6.0$ extrapolate more linearly to the chiral limit than with ordinary Wilson fermions.

Our preliminary analysis of the local vector current in the nucleon shows the procedure to be consistent, and our results for the axial vector indicate a promising trend in our value for the axial coupling despite low statistics. A fully non-perturbative analysis remains to be done.

The work was done on the APE Quadrics computers at DESY-IfH, Zeuthen, Germany. We thank the computer center for technical help.

REFERENCES
6. ALPHA collaboration, Lat96 talk by H. Wittig.
8. See e.g. Proceedings of DIS 96, Rome, Italy, 15-19 Apr 1996.