Lattice Chiral Gauge Theories in a Renormalizable Gauge

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The lattice formulation of gauge theories in a renormalizable gauge is discussed. The formulation invokes a new phase diagram, and it may allow for a lattice definition of Chiral Gauge Theories.

1. INTRODUCTION

We report here on a new approach [1] to the long-standing problem of constructing chiral gauge theories using the lattice regularization. A renormalizable gauge fixed action plays a central role in the lattice formulation. It was first suggested in ref. [2] that gauge fixing is vital for the construction of Lattice Chiral Gauge Theories (LCGTs). Here we make an important step forward, and explicitly construct a lattice model where the gauge fixing paradigm is realized.

Most of the modern attempts to define LCGTs invoke a fermion action where gauge invariance is explicitly broken, thus avoiding a direct conflict with well-known No-Go theorems [3]. In a gauge invariant theory like QCD, the fermion spectrum can be read off from the lattice action by going to the free field limit. But in the absence of exact gauge invariance, the degrees of freedom along the lattice gauge orbit couple to the fermions, and one has to understand the consequences of this coupling.

Under these circumstances, the fermion spectrum is determined by a reduced model, in which only the longitudinal modes of the lattice gauge field are kept. The action of the reduced model is obtained by substituting $U_{x,\mu} = \phi_x \phi_{x+\mu}^\dagger$ into the original action, where $\phi_x \in G$ is a group valued scalar field. The original measure $\int DU$ is replaced by $\int D\phi$. Formally, the reduced model corresponds to setting $g_0 = 0$ in the action. Thus, finding the fermion spectrum by going to the reduced model is a natural generalization of what one does in the gauge invariant case. In both cases, the procedure is justified because the continuum limit corresponds to $g_0 \to 0$.

One can go back from the reduced model to the original model in two steps. In the first step one gauges the reduced model. This leads to a manifestly gauge invariant, generalized Higgs model, where both $\phi_x$ and $U_{x,\mu}$ appear as independent fields (“generalized” means that the higgs action is not the conventional one). In the second step, gauge invariance of the generalized Higgs model is used to completely eliminate the $\phi_x$ field, while leaving the partition function invariant [4]. Any observable defined from the vector picture of the theory (where the partition function contains only the $U_{x,\mu}$ field), coincides with a corresponding gauge invariant observable defined from the partition function in the Higgs picture. The group valued field is seen to play the dual role of a Higgs or a St"{u}ckelberg field. Which interpretation captures the physics better depends on the dynamics.

The original gauge group reappears as an exact global symmetry of the reduced model, that acts on the Higgs–St"{u}ckelberg field $\phi_x$ by right multiplication. This global symmetry serves to assign the fermions (or any other matter fields) to representations of the gauge group. Thus, one can map out the phase diagram, and study the fermion spectrum in the various phases.

In most models that have been investigated so far, the fluctuations of the Higgs–St"{u}ckelberg field are not controlled by any small parameter. This has the consequence that one cannot rely on perturbation theory for finding the fermion spectrum. Where non-perturbative meth-
ods are available, one finds that either the fermion spectrum is vector-like (symmetric phase) or the gauge bosons have acquired cutoff masses and decoupled (broken phase). The fact that the fermion spectrum is vector-like in symmetric phases can be understood in terms of a generalized No-Go theorem [5]. For a more detailed discussion including references to the original literature see ref. [6].

2. A NEW PHASE DIAGRAM

The attempts to construct LCGTs in a symmetric phase lead to an impasse. This raises the question, can we do better in the broken phase? In the broken phase the doublers can acquire a mass \( m_d \sim yv \), where \( v \) is the Higgs VEV and \( y \) is a Yukawa (or Yukawa-Wilson) coupling. (In the non-abelian case we assume a matrix valued VEV \( v_{AB} = \delta v_{AB} \).) Keeping the primary fermions massless is not a problem [1]. Now, if we want to decouple the doublers, we have to send \( v \) to infinity, in physical units. (The formula \( m_d \sim yv \) holds only for small \( y \). There is conclusive evidence that the fermion spectrum is different in the limit \( y \to \infty \), and that one is back to the same problem as in the weak coupling symmetric phase). However, for a conventional Higgs theory, gauge invariance implies the relation \( m \sim gy \) for the gauge bosons mass. Sending \( v \) to infinity will then decouple the gauge bosons too, and we seem to have ended in a different impasse.

What happens if we allow for a Higher Derivative (HD) Higgs action? Now the formula \( m \sim gy \) is no longer valid. In the classical approximation, a HD Higgs action gives rise to no gauge boson mass in the broken phase. As a result, one can send \( v \) to infinity and decouple the doublers, while retaining the gauge bosons in the low energy spectrum. We use this loophole to escape from the previous impasses.

A HD action will eventually give rise to some gauge boson mass via quantum corrections. Therefore, we must be able to tune the renormalized vector boson mass \( m_v \) to zero, while staying in the broken phase. If we have a parameter that allows us to adjust the curvature of the potential of a vector field, and tune it to zero at the origin, then there should be a parameter range where the curvature at the origin is negative. In this range condensation of a vector field will take place.

The phase structure that we need thus consists of two broken phases: an ordinary broken phase and a new one, denoted FMD, which has a preferred direction defined by a vectorial order parameter. In the reduced model, the vectorial order parameter corresponds to a non-zero momentum of the ferromagnetic ground state, and the phase transition separates the FMD phase from an FM phase. For \( g_0 \neq 0 \), a vector field develops a non-zero VEV in the FMD phase. (There are no physical Goldstone modes for \( g_0 \neq 0 \), because the lattice rotation group is discrete.) With “ordinary broken phase,” we refer to the rotationally invariant region on the other side of the phase boundary, which belongs to a Higgs or Higgs-confinement phase.

The continuum limit is defined by approaching a gaussian critical point on the FMD phase boundary (see below). This new continuum limit is qualitatively different from the conventional Higgs transition. The cutoff masses acquired by the fermion doublers are attributed to the scalar VEV in the reduced model. In this sense, \( \phi_x \) plays the usual role of a Higgs field from the fermion’s point of view. (The primary fermions remain massless because their Yukawa couplings are set to zero.) From the point of view of the gauge sector, however, \( \phi_x \) plays the role of a St"{u}ckelberg field. The reason is that the continuum limit is taken far from any symmetric phase to keep \( v \sim 1 \) in lattice units. As a result, radial fluctuations of the \( \phi_x \) field (which do not exist classically) are suppressed also at the quantum level.

A non-trivial continuum limit should be described by some renormalizable continuum lagrangian. In the absence of a Higgs resonance, the new critical region is controlled by a renormalizable vector lagrangian, which can be read off from the lattice action by going to the vector picture of the full model. This vector lagrangian contains kinetic terms for all polarizations. A transversal kinetic term is provided as usual by the plaquette term, whereas a longitudinal kinetic term \( (\partial \phi_x)^2 \) arises naturally if one chooses the simplest HD Higgs action. We believe that the longitudinal
kinetic term is in fact indispensable, and that an attempt to get rid of it by some tuning is bound to end up in uncontrolled IR divergences.

A renormalizable, but otherwise arbitrary, vector theory is not unitary. In view of the presence of a longitudinal kinetic term, the idea is to bring the marginal gauge symmetry breaking terms in the vector lagrangian to the form

$$\frac{1}{2\alpha_0} (\text{gauge condition})^2. \quad (1)$$

This allows the interpretation of a gauge fixing action. The gauge fixing term can be either $(\partial \cdot A)^2$ [7] or, alternatively, $(\partial \cdot A + gA^2)^2$ [1], which corresponds to a non-linear gauge. An appropriate Faddeev-Popov (FP) ghost action has to be included. Provided the fermion spectrum is anomaly free, perturbative unitarity is now recovered by adding appropriate counter-terms to enforce the Slavnov-Taylor (BRST) identities in the continuum limit, as first proposed in ref. [2]. The issue of exact unitarity is left open. Notice that one of the BRST identities is $m_r = 0$. This BRST identity is at the heart of our approach, as it amounts to approaching the FMD phase boundary in the continuum limit.

All the new features of our approach have to do with the treatment of the gauge sector of the theory. A success in achieving the critical behaviour described above, will allow us to couple the gauge field to chiral fermions using one of several existing fermion actions. Thus, what we have done boils down to proposing an answer to the question of how to define lattice gauge theories in a renormalizable gauge. It is interesting that the qualitative features of the desired phase diagram can be deduced by requiring consistency of the gauge fixing procedure, while making no reference to the presence or absence of fermions. This alternative point of view is discussed in ref. [7].

3. THE FORMULATION

We first present a simple non-linear model that provides the basic phase diagram that we need. The vector theory that gives rise to this reduced model can be reconstructed as described in the introduction. The action, which borrows from previous work on higher derivative models [8], is given by

$$S_H = \sum_{xy} (-\kappa \phi_x^* \Box_{xy} \phi_y + \bar{\kappa} \phi_x^* \Box_{xy}^2 \phi_y), \quad (2)$$

where $\Box_{xy}$ is the standard nearest-neighbor laplacian. The lattice spacing $a$ is set equal to one. The $\phi_x$ field takes values in some compact Lie group. For simplicity we will consider the case $\phi_x \in U(1)$. The generalization to non-abelian theories is given in ref. [1,7].

As mentioned earlier, the vectorial order parameter of the non-linear model takes the form of a non-zero momentum for the ground state. Assuming

$$\langle \phi_x \rangle = ve^{iqx}, \quad (3)$$

one can study the phase diagram using standard mean-field techniques. For large $\bar{\kappa}$ one is in a broken phase and $v \neq 0$. Minimizing the mean-field hamiltonian with respect to $q_\mu$, one finds a second order transition between an FM phase and an FMD phase at $\kappa = 0$, with $q_\mu \neq 0$ for $\kappa < 0$. (In the vector picture of the full model, the classical vector VEV is $g_0 \langle A_\mu \rangle = q_\mu$.)

A central question is whether the continuity of the FMD transition is not too much spoiled by quantum effects. In the reduced model, quantum corrections can be studied in a systematic expansion around $\bar{\kappa} = \infty$, which is a zero temperature limit. The critical region near the FMD phase boundary is governed by a Goldstone Boson lagrangian whose coupling constant is $1/\bar{\kappa}$. The GB lagrangian is derived by substituting $\phi_x = \exp(i\theta_x/\sqrt{2\bar{\kappa}})$ into the action. $\theta_x$ is the Goldstone field. The classical continuum limit of the lattice action leads to

$$L_{GB} = \frac{\kappa}{2\bar{\kappa}} \partial_\mu \theta \partial_\mu \theta + \frac{1}{2} (\Box \theta)^2 + \frac{1}{4\bar{\kappa}} (\partial_\mu \theta \partial_\mu \theta)^2. \quad (4)$$

This lagrangian exhibits the unusual feature of containing a $p^4$ kinetic term. (The analytic continuation of the GB lagrangian to Minkowski space is not unitary, but this is of no direct concern to us, since unitarity in the continuum limit is needed only after the gauge field is introduced). The GB lagrangian is renormalizable, and leads to correlation functions of the $\partial_\mu \theta$ field obeying
the standard power counting both in the UV and the IR limits. The situation with respect to correlation functions of the \(\phi\)-field is more subtle.

Gauging the reduced model amounts to replacing the ordinary laplacian with a covariant one in eq. (2), and adding the usual plaquette action for the gauge field. After going to the vector picture (where \(\phi_x = 1\)), we find that the covariant HD action leads to the following new terms

\[
\tilde{\kappa} \phi^4 \square^2 (U) \phi |_{\phi_x=1} = \frac{1}{2\alpha_0} \left( \sum_{\mu} \Delta^\mu_{\mu} A_{A\mu} \right)^2 + \frac{g_0^2}{2\alpha_0} \left( \sum_{\mu} A_{A\mu}^2 \right)^2 + \cdots \quad (5)
\]

Here we have defined \(1/2\alpha_0 = \tilde{\kappa} g_0^2\), and \(\Delta^\mu_{\mu}\) is the backward lattice derivative. We made use of the standard weak coupling expansion \(U_{\mu} = \exp(ig_0 A_{\mu})\). The dots stand for irrelevant terms. This result is still not acceptable, however. What we need is an additional term in the HD Higgs action that does not spoil the phase diagram, whose effect is to bring the vector action to the form of some gauge fixing action, cf. eq. (1). For the linear gauge one has to cancel the \((\sum_{\mu} A_{A\mu}^2)^2\) term, whereas for the non-linear gauge one has to add a mixed term proportional to \((\sum_{\mu} \Delta A_{A\mu})(\sum_{\mu} A_{A\mu})\).

A detailed construction of the complete HD action is given in ref. [7]. In both cases, the marginal terms in the lattice gauge fixing action are lattice transcriptions of the corresponding continuum expressions. The irrelevant terms are chosen such that the unique minimum of the gauge fixing action is \(U_{x,\mu} = 1\). Consequently, lattice artefact Gribov copies are suppressed, and perturbation theory is a valid starting point for the investigation of the model.

As can be seen from eq. (5), it is natural to take \(1/\tilde{\kappa}\) to scale like \(g_0^2\). In the continuum limit one therefore approaches the gaussian critical point \(g_0 = 1/\tilde{\kappa} = 0\). For \(g_0 = 0\), we expect that the FMD transition (as a function of the scaling variable \(\kappa/\tilde{\kappa}\)) will remain continuous in the limit \(\tilde{\kappa} \to \infty\). Off the critical point, in particular for \(g_0 \neq 0\), the transition may become weakly first order. In view of the classical stability of the FMD transition, and the consistency of the weak coupling expansion in \(g_0\) and \(1/\tilde{\kappa}\), any dynamically generated IR scale should be a non-perturbative function of the coupling constants. This, in turn, should imply the existence of a scaling region.

In conclusion, the coupling between the fermions and the gauge degrees of freedom entails the need for a good control over the longitudinal dynamics. This can be achieved by formulating lattice gauge theories in a renormalizable gauge. The generic phase diagram needed for this new formulation was discussed above. Promoting the standard perturbative gauge fixing procedure to a non-perturbative one requires us to study many new questions. Work on a number of issues is in progress.

REFERENCES