Evaluation of the mean intensity of the P–odd mixing of nuclear compound states

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Abstract

A temperature version of the shell-optical-model approach for describing the low-energy compound-to-compound transitions induced by external single-particle fields is given. The approach is applied to evaluate the mean intensity of the P-odd mixing of nuclear compound states. Unified description for the mixing and electromagnetic transitions allows one to evaluate the mean intensity without the use of free parameters. The valence-mechanism contribution to the mentioned intensity is also evaluated. Calculation results are compared with the data deduced from cross sections of relevant neutron-induced reactions.

1. The weak nucleon-nucleon interaction gives rise to the parity-violating part $V_w$ of the nuclear mean field. In the case of intermediate and heavy mass nuclei, this part is the main source of the P-odd mixing of compound states (CS) corresponding, as a rule, to neutron resonances. In view of complexity of CS wave functions only the mean intensity of the mixing seems to be a subject for theoretical studies. This intensity is usually characterized either by root-mean-squared matrix elements $M_w$ of the field $\hat{V}_w = \sum_a V_w(a)$ between the CS wave functions or by the quantity $\Gamma_{\downarrow}^w = 2\pi M_w^2/d_c$ ($d_c^{-1}$ is the density of CS). The experimental values of the weak matrix elements $M_{w}^{\exp}$ are deduced from the cross sections of the scattering of the low-energy polarized neutrons from nuclei. These matrix elements are known now for the $^{233}\text{Th}$, $^{239}\text{U}$ [1] and for $^{114}\text{Cd}$, $^{116}\text{In}$ [2] compound nuclei. The weak matrix elements found for some other nuclei are usually assigned to the mixing of certain $p_{1/2}$ and $s_{1/2}$ CS (see, e.g., ref.[3]).

Some experimental data on the intensities of electromagnetic low-energy compound-to-compound ($c - c'$) transitions are also available. These intensities are characterized by the photon strength functions $s_\gamma(\omega) = \Gamma_{\gamma}(\omega)/d_c$ ($\omega$ is the transition energy, $\Gamma_{\gamma}(\omega)$ is the mean partial radiative width ). The mean total radiative width $\Gamma_{\gamma}^{\text{tot}}$ of neutron resonances is the integral characteristic of the mentioned transitions. The experimental data on $\Gamma_{\gamma}^{\text{tot}}$ are available for many nuclei [4].

Being analyzed in terms of the mean quantities $\Gamma_{\downarrow}^w$ the $c - c'$ P-odd mixing can be considered as the weak $c - c'$ transitions (with $\omega = 0$) by analogy with the low-energy electromagnetic $c - c'$ transitions. Really, both of the transitions are induced by the single-particle fields ($\hat{V}_w$ and $\hat{V}_{E1,M1}$, respectively) and, therefore, take place only due to coupling of particle-hole and many-quasiparticle configurations. For these reasons, the transitions can be described in terms of the particle-hole (or giant-resonance) strength functions taken in the low-energy limit, i.e. for the energy $\omega$, which is less than the energy $D$ of the corresponding giant resonance ($0^{-}$...
and $1^-, 1^+$, respectively). Thus, the unified description for the weak and electromagnetic low-energy $c - c'$ transitions seems to be profitable because the same adjustable parameter is used for calculations of the corresponding strength functions (see below).

Since the first qualitative estimations of mean intensities of $c - c'$ transitions [5], two alternative mechanisms (valence and statistical) are widely discussed [6]–[12]. According to the valence mechanism, the $c - c'$ transitions are due to single-quasiparticle components of the CS wave functions, whereas according to the statistical mechanism, they are due to many-quasiparticle components. Most of the authors agree in that the statistical (or temperature) mechanism is dominant (see, however, ref.[6]). The valence mechanism can provide a special interest [9].

Note in this connection that the method for conversion from the product of the mean neutron widths of the $s_{1/2}$ and $p_{1/2}$ compound resonances to the valence contribution to the $M_w^2$ value is given in refs.[8, 9]. The other considerations of this contribution [6, 12] seem to be only estimative.

It is known, that the photon strength function $s_\gamma(\omega, U')$ is proportional to the energy-averaged total cross section of photoabsorption by the nucleus being in the final state. For this reason, it is convenient to use the following definition of strength function $S_V(\omega, U')$ for the $c - c'$ transitions induced by an external field $\hat{V} : S_V(\omega, U') = < \sum_c | \hat{V}_{cc'} |^2 \delta(U_c - U_{c'} - \omega) >$, where $U_c$ and $U_{c'}$ are excitation energies of initial ($c$) and final ($c'$) nuclear states, respectively; brackets $< \ldots >$ are taken to mean averaging over energy interval $I \gg d_c$. Bearing in mind averaging over final CS we can consider the strength function $S_V(\omega, U') = M_w^2 / d_c$ as the quantity, which is calculated for the nucleus heated up to the temperature $t(U')$ corresponding to the mean energy $U'$ of final CS ($U' = U - \omega, U$ is the mean energy of initial CS). The idea to evaluate $S_V(\omega, U')$ by this way seems to be adequate to the considered problem, but the method for practical realization of this idea is another matter. The method for description of the particle-hole excitations corresponding to giant resonances in heated nuclei has long been used and consists in the temperature modification of nucleon occupation numbers in the RPA equations (see, e.g., ref.[13]). But there are no well-grounded method for the description of giant-resonance spreading due to the coupling of the particle-hole states to many-quasiparticle configurations in heated nuclei. As mentioned above the low-energy $c - c'$ transitions induced by a single-particle field take place only due to this coupling. In other words, in heated nuclei the low-energy (up to $\omega = 0$) "tail" of giant resonances arises due to the mentioned coupling.

This statement has been widely used in attempts to evaluate the ($c-c'$)-transition intensities by means of an extrapolation of the Lorentz–like parametrization of the giant-resonance strength functions to small $\omega$ [7, 10]. The approach used in refs.[8] (called by the authors as semiempirical one) and based on both microscopical (shell-model) description of particle-hole configurations and a phenomenological consideration of quasiparticle damping (in terms of the imaginary part of an optical potential) seems to be the most advanced description for the "tail" of giant resonances in heated nuclei. The $M_w$ evaluation based on a consideration of the so-called "principal" components of the CS wave functions [12] and on the microscopical description of many-particle states [11] were realized only for $^{233}Th$ and $^{239}U$ compound nuclei, respectively.

In this work the approach used in refs.[8] for evaluation of $M_w$ is extended in the following lines. (i) We take into consideration the complementary term in the expression for the strength function $S_V(\omega, U')$. This term (omitted in ref.[8]) ensures the $\omega \rightarrow -\omega$ invariance of this strength function and gives essential contribution to $S_V$ for small $\omega$. (ii) For each considered nucleus we find the value of single adjustable parameter (describing the strength of particle-hole coupling to many-quasiparticle configurations) by fitting the calculated value of $\Gamma_\gamma^{tot}$ to the experimental one. It allows us to calculate the root-mean-squared matrix element $M_w$ without the use of free parameters because other model parameters can be chosen from independent data.(iii) We evaluate $M_w$ for many nuclei (with $A > 50$) and compare the calculation results
with available experimental data. (iv) For all considered nuclei we evaluate also the valence-mechanism contribution to $M_\nu$ by the convenient method proposed in ref. [9].

2. The semiempirical approach for calculating the low-energy part of the strength function $S_V(\omega, U')$ corresponding to a single-particle external field $V$ is formulated in two steps. The first one is formulated for cold nuclei ($U' = 0$). The method for calculating the giant-resonance strength function $S_V(\omega \sim D, 0)$ has long been formulated in following approximations [14]: (i) the shell-model basis of appropriate particle-hole states has been limited by discrete (and quasidiscrete) spectrum; (ii) spreading of single-particle and single-hole states has been described in terms of the imaginary part of an optical-model potential. Thus, the method can be called as the shell-optical-model approach. For small excitation energies ($\omega \ll D$) the approach has been generalized to the case of exact consideration for the single-particle continuum [15] by the methods of the finite Fermi-system theory [16]. The shortcoming of the approaches developed in refs. [14, 15] is neglect of possible interference of particle and hole damping at the description of the giant-resonance spreading. For this reason the imaginary part of an optical potential used in these approaches should be considered as effective one.

We give the basic equations of the shell-optical-model approach in rather schematic form:

$$S_V(\omega, 0) = -\frac{1}{\pi} Im \int \tilde{V}^+(x, \omega)A(x, x'; \omega)\tilde{V}(x', \omega)dxdx'. \quad (1)$$

$$\tilde{V}(x, \omega) = V(x) + \int F(x, x_1)A(x_1, x_2; \omega)\tilde{V}(x_2, \omega)dxdx_1. \quad (2)$$

Here $x$ is the set of nucleon coordinates including spin and isospin variables, $\tilde{V}(x, \omega)$ is the effective field that differs from the external field $V(x)$ due to the polarization effect caused by the particle-hole interaction $F(x, x')$; $A(x, x'; \omega)$ is the energy-averaged particle-hole propagator, which is the main quantity in the considered problem. Within the shell-model approach, when the coupling of particle-hole and many-quasiparticle configurations is neglected, eqs. (1),(2) correspond to the continuum-RPA, widely used in the theory of giant resonances [17, 18]. In this case $A \rightarrow A^{(RPA)}$, where

$$A^{(RPA)}(x, x'; \omega) = \sum_\nu n_\nu \phi^{*\nu}(x)\phi_\nu(x')(g^{(0)}(x, x'; \nu + \omega) + g^{(0)}(x, x'; \nu - \omega)), \quad (3)$$

Here $\nu$ and $\phi_\nu$ are the single-particle energies and bound-state wave functions, respectively; $n_\nu$ are occupation numbers; $g^{(0)}(x, x'; \nu)$ is the Green function of the single-particle Schrödinger equation with the shell-model potential $u(x)$. In the low-energy limit $\omega \ll D$ the function $\int A^{(RPA)}(x, x'; \omega)\tilde{V}(x', \omega)dxd' \tilde{V}(x, \omega)$ has no poles in the complex $\omega$-plane, so that $S_V^{(RPA)}(\omega \ll D, 0) = 0$.

A non-zero value of $S_V(\omega \ll D, 0)$ results from the coupling of particle-hole states to many-quasiparticle configurations. The consideration for this coupling is beyond of the RPA and has been undertaken within the shell-optical-model approach. The expression for $A(x, x'; \omega)$ has been obtained in ref. [15] with the use of the spectral expansion for single-particle Green function in the coordinate representation:

$$A \simeq A^{(RPA)} + i \text{Im}A; \quad \text{Im}A(x, x'; \omega) = \text{Im}(\delta A(x, x'; \omega) + \delta A(x, x'; -\omega)), \quad (4)$$

$$\text{Im}\delta A(x, x'; \omega) \simeq \text{Im} \sum_\nu \phi^{*\nu}(x)\phi_\nu(x')(n_\nu g^{(p)}(x, x'; \nu + \omega) - (1 - n_\nu)g^{(h)}(x, x'; \nu - \omega)). \quad (5)$$

Here $g^{(p,h)}(x, x'; \nu)$ is the Green function of the single-particle Schrödinger equation with the optical-model potential $u^{opt}(x) = u(x) - iw^{(p,h)}(x)f(r)$, where the intensity of the imaginary part of the optical potential for particles ($p$) and holes ($h$) is defined as follows ($f(r)$ is the formfactor of this part):
Here $\mu$ is the the chemical potential determined by the equation $\sum \nu n_\nu = N$, where $N$ is the number of particles for the given subsystem; $\theta(y) = 1$, when $y < 0$, $\theta(y) = 0$, when $y > 0$. In the low-energy limit $| \int A^{(RPA)}(|x, x'; \omega) \tilde{V}(x', \omega) dx' | \gg \int | mA(x, x'; \omega) \tilde{V}(x', \omega) dx' |$ (as it follows from eqs.(3)-(6) when $\omega \ll D$ and $|w| \ll D$), so that the quantity $A^{(RPA)}$ can be used in eq.(2) for the effective (quasistatic) field. To clarify the meaning of eqs.(4)-(6) we notice, that (i) the first and the second terms in eq.(5) correspond to damping (or spreading) single-particle and single-hole states, respectively; (ii) $Im\delta A(\omega \rightarrow 0) \rightarrow 0$, because the quasiparticle damping vanishes provided that the quasiparticle energy $|\epsilon - \mu|$ tends to zero (for instance, $w^{(p,h)}(\epsilon) = \alpha(\epsilon - \mu)^2$ [16]); (iii) $Im\delta A(\omega)$ is identically equal to zero ($\omega \geq 0$), because in cold nuclei there are no both single-particle excitations with $\epsilon < \mu$ and single-hole excitations with $\epsilon > \mu$.

At first sight the above expressions can be directly applied for calculating the particle-hole strength function $S_V(\omega, U')$ in accordance with the known Brink’s hypothesis. That is correct provided that $\omega \gg t(U')$. The last statement has been checked by calculations of the giant-resonance strength function for heated nuclei ($\omega \sim D, \omega \gg t$) [13]. However, it is obvious, that the Brink’s hypothesis should be modified for the case $\omega \sim t$ because of temperature smearing out the Fermi surface.

A semiempirical way to take this effect into consideration has been proposed in ref.[15] and partially realized in refs.[8]. The basic point of the semiempirical approach (and the second step in its formulation) consists in the reasonable assumption that one can come to the case of heated nuclei by the a priori use of an appropriate modification of the occupation numbers and intensity $w(\parallel \epsilon - \mu \parallel)$ in eqs.(4)-(6):

$$\begin{align*}
\theta(\epsilon - \mu) &\longrightarrow \theta_t(\epsilon - \mu) \equiv (1 + e^{\frac{(\epsilon - \mu)}{t}})^{-1}, \\
n_\nu &\longrightarrow \theta_t(\epsilon_\nu - \mu), \\
\sum \nu \theta_t(\epsilon_\nu - \mu) &\equiv N, \\
w(\parallel \epsilon - \mu \parallel) &\longrightarrow w(\parallel \epsilon - \mu \parallel, t) = \alpha((\epsilon - \mu)^2 + (\pi t)^2).
\end{align*}$$

These equations are similar to those that are used to describe the single-quasiparticle damping in infinite Fermi-systems at the finite temperature [20]. The first two equations describe temperature smearing out the Fermi surface in heated nuclei. Due to this effect: (i) both single-particle excitations with $\epsilon < \mu$ and single-hole excitations with $\epsilon > \mu$ are possible; (ii) the intensity $w(\parallel \epsilon - \mu \parallel)$ is modified and $w(\epsilon \rightarrow \mu, t) \neq 0$ as opposite to the case of cold nuclei. As consequences of the above statements two results of the substitution of eqs.(7) into eqs.(1),(4)-(6) should be noted. The quantity $Im\delta A(\omega)$ in eq.(4) is comparable with $Im\delta A(\omega)$ up to $\omega \simeq t$ (taking this quantity into account ensures the $\omega \rightarrow -\omega$ invariance of strength functions $S_V(\omega, U')$). In the static limit ($\omega = 0$) the value of the strength function $S_V(0, U)$ is nonzero, as it follows from eqs.(1)-(7), and can be estimated as $\alpha(\pi D^{-1} M^{sp}_v)^2 A^{1/3}$, where $M^{sp}_V$ is the so-called single-particle matrix element. The last quantity is not well-defined one when the low-energy transitions ($\omega \ll D$) are studied. Nevertheless, it is considered in most attempts to estimate the $M_V$ value [6, 7, 12]. The use of the coordinate representation for the particle-hole propagator (4),(5) allows one to avoid the use of $M^{sp}_V$ because the whole single-particle spectrum is exactly taken into account. Similar statements about the use of $M^{sp}_V$ are valid in consideration of the valence-mechanism contribution to the $M_V$ value (see Sect.4 and compare refs.[6, 12] with refs.[8, 9]).

The temperature $t$ in eqs.(7) is naturally determined by the mean excitation energy of the final compound states: $t = \sqrt{U'/a}$, where $a$ is the parameter in the well-known semiempirical
formula for the level density (see e.g. ref.[21]). One can suppose that possible errors in the
calculation of the strength function $S_V(\omega, U')$, which are connected with the a priori use of
eqs.(7) in eqs.(1),(4)-(6), can be compensated, to a certain extent, by an appropriate choice of
single adjustable parameter $\alpha$ in the expression for $w(|\epsilon - \mu|, t)$. For instance, if one finds this
parameter by fitting the calculated value of $\Gamma_\gamma^\text{tot}$ to the experimental one for the given nucleus,
then one can calculate the $\Gamma_w^\dagger$ and $M_w^2$ values for the same nucleus without the use of free
parameters. Such a procedure is realized below.

3. The mean total radiative width of neutron resonances can be calculated by means of the
well-known expression (see,e.g. ref.[23]):

$$\Gamma_\gamma^\text{tot} = \Gamma_\gamma^\text{tot}(E1) + \Gamma_\gamma^\text{tot}(M1),$$
$$\Gamma_\gamma^\text{tot}(E1, M1) = 3\rho_0^{-1}(U_n) \int_0^{U_n} s_{E1,M1}(\omega, U_n - \omega)\rho_0(U_n - \omega)d\omega,$$  \(8\)

where $s_{E1,M1}$ are the photon strength functions mentioned above,$U_n = B_n - 2\Delta$, $B_n$ is the
neutron binding energy, $\Delta$ is the adjustable parameter in the level-density formula [21] (for
double-even nuclei $\Delta$ is approximately equal to the pairing energy per nucleon), $\rho_0(U) \sim (aU^2)^{-1} \exp(2\sqrt{aU})$ is the density of levels with the zero angular momentum. The external
fields corresponding to E1 and M1 transitions can be chosen in the form:

$$V_{E1}(x) = -\tau^{(3)}z, \quad V_{M1}(x) = \frac{1}{2}\{\mu_n(1 + \tau^{(3)}) + (\mu_p - 1/2)(1 - \tau^{(3)})\}\sigma^{(3)},$$  \(9\)

where $\tau^{(3)}$ and $\sigma^{(3)}$ are the isospin and spin Pauli matrixes respectively; $z$ is the nucleon
coordinate ; $\mu_n = -1.91, \mu_p = 2.79$. According to the well-known expressions for the dipole
partial widths $\Gamma_\gamma(\omega)$, the photon strength functions $s_{E1,M1}$ are determined by the strength
functions $S_{E1,M1}$ corresponding to fields (9) as follows:

$$s_{E1,M1}(\omega, U) = \frac{\omega^3}{3k_{E1,M1}}S_{E1,M1}(\omega, U),$$  \(10\)

where $k_{E1} = 137 \cdot (hc)^2, k_{M1} = 137 \cdot (mc^2)^2, m$ is the nucleon mass.

The weak nuclear mean field is chosen in the form [22]:

$$V_w = \frac{\Lambda_N^3c}{2} \cdot \{\xi_n(1 + \tau^{(3)}) + \xi_p(1 - \tau^{(3)})\}[\vec{\sigma}\vec{p}, \varrho(r)]_+,$$  \(11\)

where $\Lambda_N = h/mc, \vec{p}$ is the nucleon momentum, $\varrho$ is the nuclear density normalized to the total
number of nucleons $A$. Calculations were performed using $\varrho(r) = \frac{3}{4\pi}r_0^{-3}f_{WS}(r, R, a_0)$, where
$f_{WS}$ is the Woods–Saxon function, $r_0 = 1.24$ fm, $R = r_0A^{1/3}$, $a_0 = 0.65$ fm, and two sets of the
parameters of the weak nucleon-nucleus interaction [22]: $\xi_n = -0.7, \xi_p = 3.3$ (set (I)) and
$\xi_n = \xi_p = 3.3$ (set (II)). Then the values of $M_w^2$ and $\Gamma_w^\dagger$ are determined by the strength function
corresponding to field (11):

$$M_w^2 = S_w(\omega = 0, U = U_n)d_c; \quad \Gamma_w^\dagger = 2\pi S_w(\omega = 0, U = U_n)$$  \(12\)

Using the above-given estimation of $S_V(0, U)$ with $D \simeq \varepsilon_F A^{-1/3}, \alpha \simeq 0.1 MeV^{-1}, t \simeq 0.5 MeV$
($\varepsilon_F \simeq 40 MeV$ is the Fermi energy) and the amplitude of field (11) with $\xi = 1, p \simeq p_F = \sqrt{2m\varepsilon_F}$,
we obtain $S_w \sim 10^{-8} eV = 10 neV$.

Except for the parameter $\alpha$ in eqs.(7), two types of input data are used for evaluating $M_V^2$
within the framework of the temperature version of the shell-optical-model approach. The first
one is the nuclear mean field and the particle-hole interaction. In the following we use the
Landau-Migdal particle-hole interaction [16]:

5
\[ F(x_1, x_2) = C\{f + f'(\vec{r}_1 \vec{r}_2) + (g + g'(\vec{r}_1 \vec{r}_2))(\vec{\sigma}_1 \vec{\sigma}_2)\} \delta(\vec{r}_1 - \vec{r}_2), \]  

where the dimensionless phenomenological parameters \( f', g, g' \) are involved into the calculation of the strength functions \( S_V \) corresponding to the external fields (9) (the strength parameter \( C = 300 \text{ MeV} \cdot \text{fm}^3 \)). The mentioned parameters are chosen as follows: \( f' = 0.85 \cdot (1 + 2.55 \cdot A^{-2/3}) \), \( g = g' = 0.7 \). As for the nuclear mean field, we use the shell-model potential of the Saxon-Woods type from ref.[24]. The set of parameters in the semiempirical formula for the level density is the second type of input data for the calculations. These parameters are taken from ref.[21].

For practical calculations of strength functions \( S_V \) it is necessary to separate the spin-angular and isospin variables in basic eqs.(1),(2), in which the external fields (9),(11) are used. This procedure is straightforward and leads to the rather cumbersome final formulae. For brevity sake, we give here the corresponding formulae only for the case of weak \( c - c' \) transitions in the form, which is rather different from that given in the second reference [8]. In the case of E1 transitions the corresponding formulae are just the same except for taking into account term \( \delta A(x, x' ; -\omega) \) in eq.(4). According to eqs. (1),(4)-(7),(11) we get the following expressions for \( S_w \):

\[
S_w = \sum_{\beta=\alpha,\rho} s_w^{\beta, \xi_s^2} = \frac{2}{\pi} \epsilon_w^2 I m \int d r d r' \sum_{\lambda, \lambda'} t(\lambda(\lambda')) \chi_{\lambda}^{\beta}(r) \hat{L}_{(\lambda)(\lambda')}(r) \chi_{\lambda'}^{\beta}(r') \theta(\epsilon_{\lambda} - \mu_{\lambda}) g^{(\lambda; \lambda')}_{\lambda}(r, r'; \epsilon_{\lambda}) \\
- (1 - \theta(\epsilon_{\lambda} - \mu_{\lambda})) g^{(\lambda; \lambda')}_{\lambda}(r, r'; \epsilon_{\lambda}) \]  

Here \( \epsilon_w = 0.75 \cdot 10^{-6}(\Lambda_N/r_0)^3 \cdot hc; \) \( \chi^{n(p)}_{\lambda} \) and \( \chi^{n(p)}_{\lambda}(r) \) are, respectively, the energy and the radial wave functions for the single-neutron(proton) bound states determined by the (real) shell-model potential; \( \lambda = \epsilon_{\lambda}, j_{\lambda}, l_{\lambda} \equiv \epsilon_{\lambda}, (\lambda) \) is the set of the bound-state quantum numbers; \( g_{\lambda}(r, r'; \epsilon_{\lambda}) \) is the Green function of the Schrödinger radial equation with the optical-model potential whose real part is coincident with the shell-model potential and the imaginary part is defined for particles and holes according to eqs.(6),(7); \( t(\lambda(\lambda')) = (2j_{\lambda} + 1)\delta_{j_{\lambda} j_{\lambda'}} \); the operator \( \hat{L}_{(\lambda)(\lambda')} \) is defined as follows:

\[
\hat{L}_{(\lambda)(\lambda')}^r = 2f_{WS}(r, R, a_0) \frac{\partial}{\partial r} + \frac{B_{(\lambda)(\lambda')}}{r} + \frac{\partial f_{WS}(r, R, a_0)}{\partial r}, 
\]

where \( B_{(\lambda)(\lambda')} = (l_{\lambda}(l_{\lambda} + 1) - l_{\lambda'}(l_{\lambda'} + 1))/2 \). In the case of the static (\( \omega = 0 \)) weak transitions there is no core-polarization effect caused by the momentum-independent interaction (13). Really, because of time-reversal invariance of the field \( V_w \) (11) only the additional term proportional to \( \omega \bar{\sigma} \tilde{F}/r \) can appear in the expression for the effective field \( \tilde{V}_w \) satisfying eq.(2). The static core-polarization effect takes place only due to the momentum-dependent part of the particle-hole interaction. One can expect that the intensity of this part is small [16].

As mentioned above, the single adjustable parameter \( \alpha \) determining the strength of the optical-potential imaginary part according to eqs.(7) can be found for each nucleus by fitting the calculated value of \( \Gamma_{\gamma}^{\text{tot}} \) to the experimental one. The experimental values of \( \Gamma_{\gamma}^{\text{tot}} \) are taken from ref.[4]. The calculations are performed according to eqs.(1)-(10). The radial Green functions are calculated by means of the regular and nonregular solutions of the corresponding Schrödinger equation. The formfactor \( f(r) \) of \( w(r, \epsilon) \) is taken in the volume form \( (f(r) = f_{WS}(r, R, a_0)) \). The parameters needed for the calculation of both the level density and temperature are taken from ref.[21]. The results of \( \alpha \) matching for several nuclei are given in the Table I. The calculated values of relative contribution of M1 transitions to \( \Gamma_{\gamma}^{\text{tot}} \) fall into the range \((0.05 - 0.35)\). In some
cases this contribution is relatively large due to the existence of the low-energy single-particle M1 transitions, which has been taken into account in the $S_{M1}$ calculation by the method given in ref.[14]. The accuracy of the $\Gamma_{\gamma}^{tot}$ calculation and, therefore, of $\alpha$ matching is not high mainly due to: (i) the uncertainties in the level-density parameters $a$ and $\Delta$ (see e.g. ref.[21]); (ii) neglect of the contribution of the E1 transitions to the ground and low-lying states of simple nature to $\Gamma_{\gamma}^{tot}$; (iii) neglect of the nucleon pairing at finite temperature. Notice in this connection that it is not necessary to match $\alpha$ with a high accuracy because the matrix elements $M_w$ are proportional to $\alpha^{1/2}$.

The $\alpha$ parameters found by the presented way are further used for calculating $S_w$ according to eq.(14). The results of the $s_w^{np}$ and $M_w$ calculations are also given in the Table I. Notice, that the calculations are performed for the $^{233}Th$ and $^{239}U$ compound nuclei as for spherical ones. The mean energy interval $d_s$ between $s_{1/2}$ compound resonances at the neutron binding energy for the considered nuclei is taken from ref.[4]. Bearing in mind that in some cases the calculated root-mean-squared matrix elements $M_w$ are compared with the "individual" experimental matrix elements, the agreement with the known $M_{w}^{exp}$ values seems to be satisfactory (see the Table I). Notice also, that in the case of even–$A$ compound nuclei the $M_{w}^{exp}$ values given in the Table I are actually only the low limit of $M_{w}^{exp}$ [25].

4. Let us turn to calculation of the valence part of the $M_w^2$ value following refs.[8, 9]. As shown in ref.[8] the quantity $(M_{w}^{2})_{val}$ can be calculated by means of an optical model. The more convenient way has been proposed in ref.[9] where the method of conversion of the reduced (to 1eV) mean neutron widths $\Gamma_{s_{1/2}}^{0}$ and $\Gamma_{p_{1/2}}^{0}$ of $s_{1/2}$ and $p_{1/2}$ compound resonances to the $(M_{w}^{2})_{val}$ value was given. After separation of spin–angular variables in the corresponding expression obtained in ref.[9] we get

$$ (M_{w}^{2})_{val} = \Gamma_{s_{1/2}}^{0} \Gamma_{p_{1/2}}^{0} q^2 $$

(15)

$$ q^2 = -\left(\frac{\epsilon}{\lambda_1 \pi}\right)^2 \frac{2 \xi_2 \xi_w}{S_{s_{1/2}} S_{p_{1/2}}} \int dr dr' \hat{L}_{r,r'}(s_{1/2}) \hat{L}_{r',r}(s_{1/2}) \left[ |\text{Im} g_{s_{1/2}}(r,r';\epsilon)|^2 \right] $$

Here $S_{s_{1/2}}$, $S_{p_{1/2}}$ are the $s_{1/2}$ and $p_{1/2}$ neutron strength functions, $\epsilon$ is the neutron kinetic energy.

The following comments to exprs.(15) should be given. (i) Because the neutron strength functions can be calculated by means of the optical model ($S_{s_{1/2}} = \frac{2}{\pi} \eta_{s_{1/2}}$, where $\eta_{s,p}$ are the imaginary part of the neutron scattering phases), the quantity $q^2$ can be also calculated within the optical model. (ii) In contrast to its physical meaning, the quantity $(M_{w}^{2})_{val}$ determined by eqs.(15) is not equal to zero when $w = 0$ due to the contribution of single-particle continuum to $\text{Im} g(r,r';\epsilon > 0)$. To exclude this contribution the following substitution should be used in eqs.(15) (compare with consideration of the valence part of the E1-radiative strength function of neutron resonances [19]):

$$ \text{Im} g(r,r';\epsilon) \to \text{Im} g(r,r';\epsilon) + \pi \chi_{\epsilon}^{opt}(r) \chi_{\epsilon}^{opt*}(r') $$

(16)

After this substitution $(M_{w}^{2})_{val} \to 0$, and $q^2$ tends to a finite limit when $w \to 0$. (iii) The substitution (16) has no practical importance when the realistic optical–model parameters are used. In this case inequality $\Gamma_{sp}^{↓} \gg \Gamma_{sp}^{↑}$ is fulfilled ($\Gamma_{sp}^{↑}$ and $\Gamma_{sp}^{↓}$ are the single–particle escape and spreading widths, respectively), and as a result, the last term in eq.(16) can be neglected.

The calculations of $q^2$ show that this quantity is practically independent (with an accuracy of 20%) of $w$ and mass number $A$, but it is markedly dependent on the choice of the formfactor $f(r)$. When the surface–type absorption is chosen ($f(r) = 4a_0 df_{WS}(r,R,a_0)/dr$) $q^2 \approx 0.7$. In the case of volume–type absorption the value of $q^2$ is less by the factor of 3.0. The difference
is explained by the fact that single-pole approximation for both the single particle Green
function and the potential-scattering matrix is valid when the considered nucleus is close to
the relevant shape resonance, but none of the nuclei can belong simultaneously to the $s_{1/2}$ and
$p_{1/2}$ shape resonances. Therefore, the single–particle weak matrix element in the representation
$q^2 = (M^2_{sp})_{sp}/(\Gamma^{0}_{s_{1/2}})_{sp}(\Gamma^{0}_{p_{1/2}})_{sp}$ [9] is not well-defined quantity.

For estimation of the upper limit of the valence-mechanism contribution to $M_w$ according to
eqs.(15) we use: (i) the surface-type formfactor of the imaginary part of the optical potential;
(ii) the strength of the neutron-nucleus weak interaction $\xi_n = 3.3$; (iii) the experimental values
of neutron widths $\Gamma^{0}_{s_{1/2}p_{1/2}} = S^{0}_{s_{1/2}p_{1/2}} d_{s,p}$. The calculated ratios $r = (M_w)_{val}/M_w$ are given in
the last column of the Table I. As a rule these ratios are about $10^{-3}$.

5. In the present work we extend the temperature version of the shell-optical-model ap-
proach for describing the low-energy compound-to-compound transitions induced by external
single-particle fields. On this base we evaluate the root-mean-squared matrix elements $M_w$
for the weak mixing of $s_{1/2}$ and $p_{1/2}$ neutron resonances. The unified description for the ele-
tromagnetic $c - c'$ transitions and the P-odd mixing of nuclear compound states allows us to
evaluate the intensity of this mixing without the use of free parameters. It is also quantitatively
shown that the valence-mechanism contribution to $M_w$ is small. The satisfactory description of
all known experimental data on $M_w$ for intermediate and heavy mass nuclei has been obtained
within the framework of the given approach.

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TABLE I. Calculated matrix elements $M_w$ in comparison with experimental values of $M_w^{exp}$ taken from refs. [1, 2, 3]. Calculated ratios $r = (M_w)^{val}/M_w$ are also given.

<table>
<thead>
<tr>
<th>compound nucleus</th>
<th>$d_s$, eV</th>
<th>$\alpha$, MeV$^{-1}$</th>
<th>$s^0_w$, neV</th>
<th>$s^0_w$, neV</th>
<th>$M_w^{(I)}$, meV</th>
<th>$M_w^{(II)}$, meV</th>
<th>$M_w^{exp}$, meV</th>
<th>$r$, $10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{57}Fe$</td>
<td>1.7 · 10^4</td>
<td>0.10</td>
<td>31</td>
<td>14</td>
<td>54</td>
<td>92</td>
<td>46+15 −13 [3]</td>
<td>19</td>
</tr>
<tr>
<td>$^{82}Br$</td>
<td>94</td>
<td>0.18</td>
<td>17</td>
<td>43</td>
<td>6.7</td>
<td>7.8</td>
<td>3.0 ± 0.5 [3]</td>
<td>2.6 ± 0.1 [3]</td>
</tr>
<tr>
<td>$^{108}Ag$</td>
<td>16</td>
<td>0.07</td>
<td>25</td>
<td>6.8</td>
<td>1.2</td>
<td>2.3</td>
<td>*</td>
<td>1.0</td>
</tr>
<tr>
<td>$^{110}Ag$</td>
<td>14</td>
<td>0.10</td>
<td>31</td>
<td>8.4</td>
<td>1.2</td>
<td>2.5</td>
<td>*</td>
<td>0.9</td>
</tr>
<tr>
<td>$^{112}Cd$</td>
<td>20</td>
<td>0.05</td>
<td>18</td>
<td>4.3</td>
<td>1.1</td>
<td>2.2</td>
<td>1.6+8 −6 [3]</td>
<td>1.6</td>
</tr>
<tr>
<td>$^{114}Cd$</td>
<td>21</td>
<td>0.11</td>
<td>33</td>
<td>8.0</td>
<td>1.5</td>
<td>3.1</td>
<td>2.0+1.6 −0.9 [2]</td>
<td>0.7</td>
</tr>
<tr>
<td>$^{116}In$</td>
<td>9</td>
<td>0.05</td>
<td>15</td>
<td>3.0</td>
<td>0.6</td>
<td>1.4</td>
<td>0.84 ± 0.23 [3]</td>
<td>0.7</td>
</tr>
<tr>
<td>$^{118}Sn$</td>
<td>48</td>
<td>0.03</td>
<td>10</td>
<td>2.2</td>
<td>1.2</td>
<td>2.5</td>
<td>0.7 ± 0.1 [3]</td>
<td>1.7</td>
</tr>
<tr>
<td>$^{140}La$</td>
<td>208</td>
<td>0.04</td>
<td>5.5</td>
<td>11</td>
<td>5.0</td>
<td>6.1</td>
<td>1.7 ± 0.1 [3]</td>
<td>2.3</td>
</tr>
<tr>
<td>$^{233}Th$</td>
<td>17</td>
<td>0.08</td>
<td>18</td>
<td>9.0</td>
<td>1.4</td>
<td>2.2</td>
<td>1.39+0.55 −0.38 [1]</td>
<td>1.1</td>
</tr>
<tr>
<td>$^{239}U$</td>
<td>21</td>
<td>0.08</td>
<td>19</td>
<td>8.0</td>
<td>1.5</td>
<td>2.5</td>
<td>0.56+0.41 −0.20 [1]</td>
<td>1.6</td>
</tr>
</tbody>
</table>

*) The data are expected [2]

References


