Scaling Structures in Four-dimensional Simplicial Gravity *

H.S.Egawa a, T.Hotta b, T.Izubuchi b, N.Tsuda c and T.Yukawa c, d

a Department of Physics, Tokai University Hiratsuka, Kanagawa 259-12, Japan
b Department of Physics, University of Tokyo Bunkyo-ku, Tokyo 113, Japan
c National Laboratory for High Energy Physics (KEK), Tsukuba 305, Japan
d Coordination Center for Research and Education, The Graduate University for Advanced Studies, Hayama-cho, Miura-gun, Kanagawa 240-01, Japan

Four-dimensional (4D) spacetime structures are investigated using the concept of the geodesic distance in the simplicial quantum gravity. On the analogy of the loop length distribution in 2D case, the scaling relations of the boundary volume distribution in 4D are discussed in various coupling regions i.e. strong-coupling phase, critical point and weak-coupling phase. In each phase the different scaling relations are found.

1. Introduction

Simplicial gravity has witnessed a remarkable development toward quantizing the Einstein gravity. This development started with 2D simplicial gravity and has now reached the point of subjecting to simulate 4D case[1–4] about the analysis of fractal dimensions, minbu, scaling relations for the loop length distribution and the curvature distribution. The aim of this paper is to investigate 4D Euclidean spacetime structures using the concept of the geodesic distance. It is very important that the scaling relations have been obtained in 2D case. Therefore, on the analogy of the loop length distribution (LLD) in 2D[5], the scaling relations in 4D are discussed. Actually we measured the boundary volume distribution (BVD) for various geodesic distances in 4D DT-manifold, in analogy to LLD. In order to discuss the scaling relations, we assume that the scaling variable x has a form x = V/D α, where V, D and α denote the each boundary(cross section) volume, the geodesic distance and scaling parameter, respectively. Hagura et al. argue the scaling properties of the surface area distributions in 3D case by the same analysis as we employ in 4D case(in these proceedings).

2. The model

We use the lattice action of 4D model with the S^4 topology corresponding to the action as S = −κ_2 N_2 + κ_4 N_4, where N_i denotes the total number of i-simplexes. The coupling κ_2 is proportional to the inverse bare Newton’s constant and the coupling κ_4 corresponds to a lattice cosmological constant. For the dynamical triangulation model of 4D quantum gravity, we consider a partition function of the form, Z(κ_2, κ_4) = ∑_{T(S^4)} e^{-S(κ_2, κ_4, T)}. We sum up over all simplicial triangulations T(S^4). In practice, we have to add a small correction term, ∆S = δκ_4(N_4 - N_4^{(target)})^2, to the action in order to suppress the volume fluctuations from the target value of 4-simplexes N_4^{(target)} and we have used δ = 0.0005.

3. Numerical Simulations and Results

We define N_b(D) as the number of boundaries at the geodesic distance D from a reference 4-simplex in the 4D DT manifold averaged over all 4-simplexes. Fig.1 shows the distributions of N_b(D) for the typical three coupling strength with N_4 = 16K. In the strong coupling limit(κ_2 = 0), the only one boundary that is identified as the mother universe exists almost...
all the distances(see Fig.1), which means that the mother universe is a dominant structure. The branching structures are highly suppressed, which shows characteristic properties of the “crumpled manifold” similar to which observed in 2D manifold. On the other hand, in the weak coupling phase(for example, we chose $\kappa_2 = 2.0$), we observe the growth of the branches until $D \sim 60$ and can reasonably extract the relation $N_b(D) \propto D$ in the region $3 \leq D \leq 30$. Then we call this manifold as the “elongated manifold”.

3.1. The strong coupling phase

Fig.2 shows BVD, $\rho(x)$, with $x = V/D^{4.5}$ as a scaling variable in the strong coupling region, $\kappa_2 = 0$, with $N_4 = 32K$ while the fractal dimension($d_f$) reaches about $5.5$ which yet increases with the volume in our simulation size. In terms of this variable the mother universe shows scaling relation and distributes like a Gaussian distribution. We can be fairly certain that in the strong coupling phase the scaling parameter $\alpha$ of mother universe satisfies the relation $d_f = \alpha + 1$, and the manifold resembles a $d_f$-sphere ($S^{d_f}$). There seems to be a scaling property with respect to BVD with $x = V/D^{d_f-1}$ as a scaling variable.

3.2. The critical point

Next, the data near the critical point($\kappa_2 = 1.2581$) are shown in Fig.3 for various geodesic distances with $N_4 = 32K$. At this point the fractal dimension reaches about $3.5$ and yet increasing for larger values of $N_4$. We must draw attention to the double peak structure on the critical point [6]. Therefore, we measure the boundary volume distribution on both peaks, and obtain the more clear signal of the distribution of the universe on the peak which is close to the strong coupling phase. We observe the scaling properties for the mother universe $\rho(x)$ with
\[ x = V/D^{2.3} \] in Fig.3. In order to discuss the universality of the scaling relations, we assume the distribution function in terms of a scaling parameter \( x = V/D^{2.3} \) as \( \rho(x) = a_0 \frac{1}{v_0} x^{a_1} e^{-a_2 x} \), where \( a_0, a_1 \) and \( a_2 \) are some constants. Then we can calculate the fractal dimension from \( \rho(x, D) \),

\[
\lim_{x \to \infty} \int_{v_0}^{\infty} dV \rho(x, D) = V^{(4)}(D) \sim D^d, 
\]

where \( v_0 \) denotes the cut-off volume and \( V^{(4)} \) denotes the total volume of the 4D manifold. If \( a_1 > -2 \) this integration is convergent and gives a finite fractal dimension. We can extract the function of \( \rho(x, D) \) from Fig.3, and find \( a_1 \approx 0.5 \) for the distribution of mother volume and \( a_2 \approx 3.0 \). Furthermore we investigate the \( N_4 = 64K \) case and obtain the same scaling behavior as that of \( N_4 = 32K \) case except \( d_f \approx 4.0 \) and scaling parameter \( \alpha \approx 3.0 \). These results lead to the conclusion that the distributions of the baby universes show no scaling behavior. On the other hand, the distribution of the mother universe shows the scaling relation and is universal (i.e. it does not depend on the lattice cut-off \( v_0 \)).

### 3.3. The weak coupling phase

Finally, we show the data in the weak coupling phase \( (\kappa_2 = 2.0) \) within which the fractal dimension reaches about 2.0. Fig.4 shows BVD with \( x = V/D \) as a scaling variable. We can safely state \( \rho(x) \times D \propto x^{-2.3} e^{-x} \). In this phase, the dynamically triangulated manifold consists of widely expanding like branched polymers and we cannot observe the mother universe at all.

### 4. Summary and discussions

On the analogy of LLD in 2D case, the scaling relations in 4D are discussed for the three phases. BVD, \( \rho(x, D) \), at geodesic distance \( D \) gives us some basic scaling relations on the ensemble of Euclidean space-times described by the partition function \( Z(\kappa_2, \kappa_4) \).

In the strong coupling limit \( \kappa_2 = 0 \) we find that the mother part of BVD, \( \rho(x, D) \), scales trivially with \( x = V/D^{d_f-1} \) as a scaling variable. There is fairly general agreement that the 4D DT manifold seems to be a \( d_f \)-sphere(\( S^{d_f} \)). What is important is that this scaling property for the mother universe changes gradually into the scaling relation of that of the critical point. The fluctuations of the spacetime growth with \( \kappa_2 \rightarrow \kappa_2^c \). In 2D, the baby loop and the mother loop show scalings with the same parameter \( x = L/D^2 \). However, LLD of the baby loops is depend on the lattice cut-off and we think that it is not universal. An the critical coupling in 4D case we have obtained the similar BVD. However, we have a different scaling parameters \( x = V/D^{2.3} \) with \( N_4 = 32K \) and \( x = V/D^{3.0} \) with \( N_4 = 64K \) from 2D case for the mother universe. Furthermore, BVD of the baby universes seems to be non-universal.

In the weak coupling phase (see Fig.4) we have obtained the elongated manifolds, in other words, branched polymers. In this phase no mother universe exists and BVD of the baby universes shows that the scaling relation is not universal.

The results of this paper is the first step to research the universal scaling relations in 2, 3 and 4D on simplicial quantum gravity. The results of simulations can be regarded as the possibility that we may have 4D quantum gravity as the generalized DDK model.

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### REFERENCES