The $OSp(32|1)$ versus $OSp(8|2)$ supersymmetric M-brane action from self-dual (2,2) strings

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Abstract

Taking the (2,2) strings as a starting point, we discuss the equivalent integrable field theories and analyze their symmetry structure in $2+2$ dimensions from the viewpoint of string/membrane unification. Requiring the ‘Lorentz’ invariance and supersymmetry in the (2,2) string target space leads to an extension of the (2,2) string theory to a theory of $2+2$ dimensional supermembranes ($M$-branes) propagating in a higher dimensional target space. The origin of the hidden target space dimensions of the M-brane is related to the maximally extended supersymmetry implied by the ‘Lorentz’ covariance and dimensional reasons. The Kähler-Chern-Simons-type action describing the self-dual gravity in $2+2$ dimensions is proposed. Its maximal supersymmetric extension (of the Green-Schwarz-type) naturally leads to the $2+10$ (or higher) dimensions for the M-brane target space. The proposed $OSp(32|1)$ supersymmetric action gives the pre-geometrical description of M-branes, which may be useful for a fundamental formulation of F&M theory.

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1 Introduction. Since a discovery of string dualities, much evidence was collected for the idea that ‘different’ string theories can be understood as particular limits of a unique underlying theory whose basic formulation is yet to be found. The fundamental theory does not seem to be a theory of strings but it describes fields, strings and membranes in a democratic way. A candidate for the unified theory was also proposed under the name of \textit{M-theory} [1, 2] or its refined \textit{F-theory} formulation [3], which can be reduced to all known 10-dimensional superstrings and 11-dimensional supergravity as well. There should be also room for strings with \textit{extended} world-sheet supersymmetry in the unified theory. The anticipated relation between the $N = (2, 1)$ heterotic strings and M-theory in some particular low-dimensional backgrounds was recently used [4] to propose the definition of the underlying M-theory as a theory of $2 + 2$ dimensional membranes (called \textit{M-branes} [5]) embedded in higher dimensions. The origin of M-branes should therefore be understood from the basic properties of $N=2$ strings. It is the purpose of this Letter to argue that the hidden membrane (both world-volume and target space) dimensions are in fact \textit{required} by natural symmetries which are broken in the known $N=2$ string formulations. By the \textit{natural} symmetries I mean ‘Lorentz’ invariance and supersymmetry which should be made explicit and linearly realized. That symmetries uniquely determine the dynamics of M-branes.

The basic idea for describing M-branes naturally arises from the known \textit{world-sheet/target space duality} of $N=2$ strings. In the early days of $N=2$ string theory, when only two-dimensional target spaces were considered, Green [6] suggested to use the $N=2$ string world-sheet as the target space, which implies a duality between the world-sheet moduli and their target space counterparts. The four-dimensional nature of the $(2, 2)$ string target space as a (hyper) Kähler manifold was understood later by Ooguri and Vafa [7], who suggested to associate with the $N=2$ string world-sheet (Riemann surface) a four-dimensional symplectic space – the so-called ‘cotangent bundle of the Riemann surface’. The latter has an equal number of moduli to be associated with non-trivial deformations of a complex structure \textit{and} of a Kähler class. The duality (in fact, triality) symmetries then appear between the world-sheet moduli, the target space complex structure moduli, and the target space Kähler-class moduli. I am going to use the world-sheet/target space duality of $N=2$ strings as the (first) working principle of string/membrane unification, namely, as a route for constructing the self-dual theory of M-branes out of the target space field theory of $(2,2)$ strings, along the lines of ref. [4]. However, unlike the way of reasoning in ref. [4], which puts forward the $(2,1)$ heterotic strings, I consider closed and open $(2,2)$ strings as a starting point. The critical $(2,2)$ strings naturally live in $2+2$ dimensions, which are crucial for self-duality, whereas the $(2,0)$ or $(2,1)$ heterotic strings require the $1+1$
or 1 + 2 dimensional target space [7]. The duality principle is however not enough to deliver the M-brane action, since it does not say enough about the symmetries of the M-brane. Hence, I postulate the second working principle by requiring all the natural symmetries to explicitly appear in the target space action. Despite its innocent content, the ‘Lorentz’ invariance in 2 + 2 dimensions appears to be non-trivial for N=2 strings. By gauging the ‘Lorentz’ group $SO(2,2)$, I formulate a Kähler-Chern-Simons-type gauge-invariant action in five dimensions, whose dynamics describes the self-dual gravity in four dimensions. It gives the relevant part of the M-brane action, according to the (first) duality principle above. The rest of the M-brane action is fixed by requiring the maximal supersymmetry (the second working principle) in the M-brane target space, whose dimension is 2 + 10, or it can be even higher. The supersymmetric action is proposed to describe the M-branes, which may be the fundamental constituents of the putative F&M theory.

2 Summary of (2,2) strings. The N=2 strings are strings with two world-sheet (local) supersymmetries. The gauge-invariant $N = 2$ string world-sheet actions in the NSR-type formulation are given by couplings of a two-dimensional N=2 supergravity to a complex N=2 scalar matter [10], and they possess global $U(1,1) \times \mathbb{Z}_2$ target space symmetry. A covariant gauge-fixing introduces conformal ghosts $(b,c)$, complex superconformal ghosts $(\beta^\pm, \gamma^\mp) = (\partial \xi^\pm e^{-\phi^\mp}, \eta^\mp e^{\phi^\mp})$, and real abelian ghosts $(\tilde{b}, \tilde{c})$, as usual. The chiral N=2 (superconformal) current algebra comprises a stress-tensor $T(z)$, two supercurrents $G^\pm(z)$, and an abelian current $J(z)$. The critical closed and open (2,2) strings live in four dimensions with a signature $2 + 2$. The current algebras of the N=2 heterotic strings have the additional abelian null current. It is needed for a nilpotency of the BRST charge, and implies a reduction of the N=2 string target spacetime dynamics down to 1 + 2 or 1 + 1 dimensions [7].

The BRST cohomology and on-shell amplitudes of N=2 strings were investigated by several groups [7, 11, 12, 13]. There exists only a single massless physical state in the open or closed (2,2) string spectrum. This particle can be identified with the Yang scalar of self-dual Yang-Mills theory for open strings, or the Kähler scalar of self-dual supergravity for closed strings, while infinitely many massive string modes are all unphysical. The (2,2) strings thus lack ‘space-time’ supersymmetry. Though twisting the N=2 superconformal algebra yields some additional twisted physical states which would-be the target space ‘fermions’, they actually decouple. It is consistent with another observation that the ‘space-time fermionic’ vertex operators constructed in

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3See refs. [8, 9] for a review.
4The signature is dictated by the (2,2) world-sheet supersymmetry. The euclidean signature is excluded by trivial kinematics for massless particles.

An \( n \)-point function of closed N=2 strings is given by a topological sum indexed by the genus \( g \) and the instanton number (Chern class) \( c \), with each term in the sum being an integral over metric, N=2 fermionic and Maxwell moduli. The Maxwell moduli parameterize the space of flat connections or harmonic 1-forms \( H \) on the \( n \)-punctured world-sheet \( \Sigma \), and they enter the gauge-fixed action as \( \int_{\Sigma} H \wedge \ast J \). Making a shift \( H \rightarrow H + h \) changes the action as

\[
\int_{\Sigma} h \wedge \ast J = \sum_{i=1}^{g} \left( \int_{a_i} h \int_{b_i} \ast J - \int_{b_i} h \int_{a_i} \ast J \right) + \sum_{l=1}^{n} \int_{c_l} h \int_{p_l}^{p_0} \ast J,
\]

where a canonical homology basis \((a_i, b_j)\) on \( \Sigma \), the contours \( c_l \) encircling punctures \( p_l \), and a reference point \( p_0 \) have been introduced. Therefore, the shift gives rise to twists \( \text{SFO}(\theta) \equiv \exp \{ 2\pi i \theta \int \ast J \} \) around the homology cycles as well as around the punctures, with \( \theta \in [0, 1] \). This phenomenon is known as spectral flow. A twist around a puncture at \( z \) can be absorbed into a redefined (twisted) vertex operator [13]

\[
V(z) \rightarrow V^{(\theta)}(z) = \exp \left\{ 2\pi i \theta \int_{z_0}^z \ast J \right\} V(z) .
\]

The spectral flow operator \( \text{SFO} \) is BRST-closed but only its zero mode is not BRST-exact. Hence, the position of \( \text{SFO} \) in an amplitude is irrelevant, and all the \( n \)-point functions are invariant,

\[
\langle V_1^{(\theta_1)} \cdots V_n^{(\theta_n)} \rangle = \langle V_1 \cdots V_n \rangle ,
\]

as long as the total twist vanishes, \( \sum_i \theta_i = 0 \). The bosonized spectral flow operator reads

\[
\text{SFO}(\theta) = e^{-2\pi i \phi(z_0)} \exp \{ 2\pi i \theta \phi(z) \} ,
\]

where the \( U(1) \) current has been bosonized as \( \ast J = d\phi \). The two factors in eq. (3) are separately neutral under the local \( U(1) \), but carry opposite charges under the global \( U(1) \) symmetry. Eq. (3) relates the spectral flow to Maxwell instantons on the world-sheet. Indeed, choosing \( \theta = 1 \) yields an instanton-creation operator, \( \text{ICO} \equiv \lambda \text{SFO}(\theta = 1) \), which changes the world-sheet instanton number \( c \) by one. Amplitudes with different instanton backgrounds are therefore related as

\[
\langle V_1 \cdots V_n \rangle_c = \langle V_1 \cdots V_n (\text{ICO}) \rangle_{c=0} = \lambda^c \langle V_1^{(\theta_1)} \cdots V_n^{(\theta_n)} \rangle_{c=0} ,
\]

with a total twist of \( \sum_i \theta_i = c \). Hiding the reference point ambiguity by declaring the Maxwell coupling constant to be \( \lambda = \exp \{ 2\pi i \phi(z_0) \} \) implies that both \( \text{ICO} \) and \( \lambda \)
have a non-vanishing charge with respect to the $U(1)$ subgroup of the actual global symmetry group $U(1,1) \subset SO(2,2)$ [13].

The only non-vanishing N=2 string scattering amplitudes are 3-point trees (and, maybe, 3-point loops as well), while all the other tree and loop amplitudes vanish due to kinematical reasons.\footnote{Similar results are valid for open (2,2) strings too.} As a result, a (2,2) string theory appears to be equivalent to an integrable field theory. In particular, the open (2,2) string amplitudes are reproduced by either the Yang non-linear sigma-model action [14] or the Leznov-Parkes cubic action [15], each following from a field integration of the self-dual Yang-Mills (SDYM) equations in a particular gauge, and related to each other by a duality transformation. As far as the closed (2,2) strings in the zero-instanton sector are concerned, the equivalent non-covariant field theory action is known as the Plebański action [16] for the self-dual gravity (SDG). The world-sheet instanton effects lead to a deformation of self-duality: the Ricci-tensor does not vanish, while the integrability implies the self-dual Weyl tensor instead.

The natural (global) ‘Lorentz’ symmetry of a flat 2 + 2 dimensional ‘space-time’ is $SO(2,2) \cong SU(1,1) \otimes SU(1,1)'$. The NSR-type N=2 string actions used to calculate the amplitudes have only a part of it, namely, $SU(1,1)$, so is the symmetry of the N=2 string amplitudes. The full ‘Lorentz’ symmetry $SO(2,2)$ can be formally restored in the twistor space, which adds the (harmonic) space $SU(1,1)/U(1)$ of all complex structures in 2 + 2 dimensions [11]. The ladder generators of the second $SU(1,1)'$ factor can be explicitly constructed as follows [11]:

$$J_− = \int \xi^−\eta^−(1 - \tilde{c}\tilde{b})ICO, \quad J_+ = \int \xi^+\eta^+(1 + \tilde{c}\tilde{b})ICO^{-1}.$$ \hspace{1cm} (5)

Closing the underlying N=2 superconformal algebra to be appended by the additional currents $J_\pm$ results in the so-called ‘small’ twisted N=4 superconformal algebra. This remarkable property allows one to treat the N=2 string theory as an N=4 topological field theory [11, 17]. The embeddings of the N=2 algebra into the N=4 algebra are just parameterized by twistors: a choice of a complex structure selects a $U(1,1)$ subgroup of the ‘Lorentz’ group, while world-sheet Maxwell instantons rotate that complex structure.

The (real) coupling constant $g$ of the (2,2) string interaction and the Maxwell coupling constant (phase) $\lambda$ can be naturally unified into a single complex coordinate parameterizing the moduli space of complex structures. The complex N=2 string coupling can also be interpreted as the vacuum expectation value of a complex dilaton.
field. Hence, the N=2 string dilaton is not inert under the full ‘Lorentz’ transformations! The dilaton thus takes its values in $SU(1,1)'/U(1)'$, and it can therefore be represented by an anti-self-dual (closed) two-form $\omega$ satisfying a nilpotency condition $\omega \wedge \omega = 0$ (see sects. 4 and 5 also).

3 Adding supersymmetry in $2+2$ dimensions. Because of the isomorphisms $SU(1,1) \cong SL(2,\mathbb{R})$ and $SO(2,2) \cong SL(2,\mathbb{R}) \otimes SL(2,\mathbb{R})'$, it is natural to represent the $2+2$ ‘space-time’ coordinates as $x^{\alpha,\alpha'}$, where $\alpha = (+,-)$ and $\alpha' = (+',-')$ refer to $SL(2)$ and $SL(2)'$, respectively. The $N$-extended supersymmetrization of self-duality amounts to extending the $SL(2)$ factor to $OSp(N|2)$, while keeping the $SL(2)'$ one to be intact. One has $\delta^{AB} = (\delta^{ab}, C^{\alpha\beta})$, where $\delta^{ab}$ is the $SO(N)$ metric and $C^{\alpha\beta}$ is the (part of) charge conjugation matrix, $A = (a,\alpha)$. In superspace $Z = (x^{\alpha,\alpha'}, \theta^{A'})$, the $N$-extended (gauged) self-dual supergravity (SDSG) is defined by the constraints in the light cone-gauge can be reduced to a single equation for the pre-potential $V_{\alpha\alpha'} = \partial_{\alpha'} M_{\alpha'\mu'} \partial_{\mu'} + \frac{i}{2} \delta_{AA'} BC M^{CB}$, as [18]:

$$\{\nabla^{\alpha a}, \nabla^{b\beta} \} = C^{\alpha\beta} M^{ab} + \delta^{ab} M^{\alpha\beta},$$  \hspace{1cm} (6a)$$
$$\{\nabla^{aa}, \nabla_{bb} \} = \delta^{ab} C^{\alpha\beta} \nabla_{\beta\beta'}, \hspace{1cm} [\nabla^{\alpha a}, \nabla_{\beta\beta'}] = \delta^{\alpha\beta} \delta^{ab} \nabla_{b\beta'},$$  \hspace{1cm} (6b)$$
where $M^{AB} = (M^{ab}, M^{\alpha\beta}, \nabla^{aa})$ are the generators of $OSp(N|2)$. Eqs. (6) have the $OSp(N|2) \otimes SL(2)'$ symmetry, and they can be 'solved' in the lightcone gauge in terms of a SDSG pre-potential. It is well-known that, as far as the SDG is concerned, one has

$$R_{a_1 a_2 a_3 a_4} \sim \partial_{\alpha_1+} \partial_{\alpha_2+} \partial_{\alpha_3-} \partial_{\alpha_4-} \sim \partial_{\alpha_1+} \partial_{\alpha_2+} \partial_{\alpha_3+} \partial_{\alpha_4-} V_{\alpha\alpha'},$$  \hspace{1cm} (7)$$
where the prepotential $V_{\alpha\alpha'}$ has a single component representing the helicity $(+2)$. Eq. (7) can be generalized in superspace, $R_{A_1 A_2 A_3 A_4} (Z) \sim \partial_{A_1+} \cdots \partial_{A_4+} V_{\alpha\alpha'} (Z)$, where $V_{\alpha\alpha'}$ is a SDSG pre-potential of dimension $(−1)$. The free field equation for the SDSG pre-potential, $\partial_{\alpha'} A \partial_{\alpha'} V_{\alpha\alpha'} (Z) = 0$, can be solved for all $\theta^{\alpha\alpha'}$ dependence. It reduces $V_{\alpha\alpha'} (Z)$ to a self-dual superfield $V_{\alpha\alpha'} (x^{\alpha,\alpha'}, \theta^{\alpha\alpha'})$, which merely depends on a half of $\theta's$. Of course, it breaks the ‘Lorentz’ symmetry. As a result, the SDSG constraints in the light cone-gauge can be reduced to a single equation for the pre-potential, which is obtained from the $N$-extended super-Plebański action [18],

$$S_{SDSG} = \int d^{2+2} xdN \theta \left[ 1 V_{\alpha\alpha'} \Box V_{\alpha\alpha'} + \frac{i}{\beta} V_{\alpha\alpha'} \partial_{\alpha+} \partial_{\alpha+} V_{\alpha\alpha'} \right] \delta^{BA} (\partial_{B+} \partial_{\alpha+} V_{\alpha\alpha'}) \hspace{1cm} (8)$$

As was noticed by Siegel [18], the action (8) implies the maximal supersymmetry! Indeed, dimensional analysis immediately yields $N = 8$, and the same follows from counting the total $GL(1)'$ charge of the action (8), where $GL(1)'$ is the unbroken part.
of the ‘Lorentz’ factor $SL(2)'$. Similarly, the $N$-extended super-Leznov-Parkes action for the self-dual supersymmetric Yang-Mills (SDSYM) theory implies $N = 4$ [18]:

$$S_{SDSYM} = \int d^{2+2}x d^4\theta \left[ \frac{i}{2} V_{\alpha'} \Box V_{\alpha'} + \frac{i}{3} V_{\alpha'}(\partial^{\alpha'} V_{\alpha'}) (\partial_{\alpha'} V_{\alpha'}) \right].$$  \hspace{1cm} (9)

The SDSG and SDSYM theories in eqs. (8) and (9) are similar to the non-self-dual supersymmetric gauge theories in the light-cone gauge [19].

The natural appearance of the maximal $N = 8$ supersymmetry and the (gauged) $SO(8)$ internal symmetry in the supersymmetrized (2,2) string effective action in $2 + 2$ dimensions is very remarkable, since that effective action is supposed to be a (dual) part of an M-brane action. We may now proceed in the usual way known in supergravity, and ‘explain’ the maximally extended local supersymmetry as a simple local supersymmetry in higher dimensions. For example, one may use the embedding

$$SO(2,2) \otimes SO(8) \subset SO(2,10),$$  \hspace{1cm} (10)

which implies going up to $2 + 10$ dimensions. Indeed, the $2 + 10$ dimensions are the nearest ones in which Majorana-Weyl spinors and self-dual tensors also appear, like in $2 + 2$ dimensions. It should be noticed that twelve dimensions for string theory were originally motivated in a very different way, namely, by a desire to explain the S-duality of type IIB string in ten dimensions as the T-duality of a 12-dimensional F-theory dimensionally reduced on a two-torus. The type IIB string is then supposed to arise upon double dimensional reduction from the F-theory.

There is, however, a problem with that naive approach. One has to double the on-shell number (8) of the anticommuting coordinates in a covariant M-brane action while maintaining the number of their degrees of freedom. One then gets $2 \times 16 = 32$ off-shell components, which is just needed for a single Majorana-Weyl spinor in $2 + 10$ dimensions. As is well known in superstring theory, it is the $\kappa$-symmetry of the Green-Schwarz superstring action that makes the doubling to be possible, while the Green-Schwarz action itself can be understood as the particular Wess-Zumino-Novikov-Witten (WZNW) model with superspace as the target supermanifold [20]. Therefore, one should look for a Green-Schwarz-type reformulation of self-duality in $2 + 2$ dimensions, and then maximally supersymmetrize the target space, instead of (or, maybe, in addition to) the world-volume (or NSR-type) supersymmetrization.

4. **Kähler-Chern-Simons actions for SDYM and SDG.** A more geometrical (dual) description of the SDYM theory is provided by the five-dimensional hyper Kähler-Chern-Simons action [21]:

$$S_{hKCS} = -\frac{1}{4\pi} \int_Y \text{tr} \left( \tilde{A} \wedge d\tilde{A} + \frac{2}{3} \tilde{A} \wedge \tilde{A} \wedge \tilde{A} \right) \wedge \omega^i e_i,$$  \hspace{1cm} (11)
where \( Y = M_4 \otimes R \), with \( M_4 \) being the 2 + 2 dimensional hyper Kähler world-volume and \( R \) being the auxiliary dimension called extra ‘time’ \( t \). Here \( \hat{A} \) is the Lie algebra valued 1-form on \( Y \), \( \omega^i \) is the hyper Kähler structure on \( M_4 \), and \( e_\mu = (1, e_i) \) is a basis of quaternions. Since \( \omega^i \) are closed, the action (11) is invariant under the gauge transformations \( \hat{A}^h = h\hat{A} h^{-1} - dh h^{-1} \) which should be trivial on the boundary \( \partial Y \). I assume that the boundary conditions for the gauge field \( \hat{A} \) are chosen in such a way that no boundary terms appear in the equations of motion. It is convenient to decompose both the gauge field and the exterior derivative into the ‘time’ and ‘rest’ components, \( \hat{A} = A_t + \hat{A} \) and \( \hat{d} = dt \frac{\partial}{\partial t} + \hat{d} \). One finds that \( A_t \) and \( \omega^i \) appear in eq. (11) as Lagrange multipliers, which implement the self-duality equations

\[
F \wedge \omega^i = 0 , \quad i = 1, 2, 3 ,
\]

where the YM field strength \( F = dA + A \wedge A \) has been introduced. Varying with respect to \( A \) implies (in the gauge \( A_t = 0 \)) that the A-field is \( t \)-independent. In the gauge \( A_t = 0 \), the gauge symmetry is represented by the \( t \)-independent gauge transformations. Therefore, the action (11) describes on-shell the SDYM in 2 + 2 dimensions. Eq. (12) for \( i = 1, 2 \) can be solved in complex coordinates \((z^a, \bar{z}^{\bar{a}})\) on \( M_4 \) as \( A_a = (U)^{-1} \partial_a U \) and \( A_{\bar{a}} = -\partial_{\bar{a}} U^i (U^i)^{-1} \), where \( U \) is locally defined. In terms of the gauge-invariant potential \( J = UU^\dagger \), the remaining eq. (12) at \( i = 3 \) is just the Yang equation,

\[
\omega \wedge \bar{\partial} \left( J^{-1} \partial J \right) = 0 .
\]

Eq. (13) can be obtained from the Donaldson-Nair-Schiff (DNS) action \[21\]

\[
S_{\text{DNS}}[J; \omega] = -\frac{1}{4\pi} \int_{M_4} \omega \wedge \text{tr}(J^{-1} \partial J \wedge J^{-1} \bar{\partial} J) + \frac{i}{12\pi} \int_{M_4 \times [0,1]} \omega \wedge \text{tr}(J^{-1} dJ)^3 .
\]

The action similar to eq. (11) can also be constructed for SDG. Let us simply replace the YM Chern-Simons form by the ‘Lorentz’ Chern-Simons form \( C_{3L} \),

\[
C_{3L} = \text{tr} \left( \bar{\Omega} \wedge \bar{\partial} \bar{\Omega} + \frac{2}{3} \bar{\Omega} \wedge \bar{\Omega} \wedge \bar{\Omega} \right) = \text{tr} \left( \bar{\Omega} \wedge \bar{\Omega} - \frac{1}{3} \bar{\Omega} \wedge \bar{\Omega} \wedge \bar{\Omega} \right) ,
\]

where \( \bar{R} = \bar{\partial} \bar{\Omega} + \bar{\Omega} \wedge \bar{\Omega} \), and the 1-form \( \bar{\Omega} \) takes values in the Lie algebra of \( SO(2,2) \). The SDG action for a hyper Kähler manifold \( M_4 \) equipped with the anti-self-dual two-form \( \omega \) is given by

\[
S_{\text{SDG}}[\bar{\Omega}; \omega] = -\frac{1}{4\pi} \int_Y C_{3L} \wedge \omega .
\]

\[6\]There exist the Kähler (1,1) form \( \omega \) and a closed (2,0) form \( \omega^+ \) on any hyper Kähler manifold \( M_4 \). The hyper Kähler structure is defined by \( \omega^1 = \text{Re} \omega^+ \), \( \omega^2 = \text{Im} \omega^+ \) and \( \omega^3 = \omega \).

\[7\]If one first solves eq. (12) for \( i = 2, 3 \), the remaining equation for \( i = 1 \) follows from the dual Leznov-Parkes action (sect. 2).
The self-duality condition $R \wedge \omega = 0$ appears from varying eq. (16) with respect to $\Omega_t$ (in the gauge $\Omega_t = 0$). One can check that the vanishing variation of the action (16) with respect to $\Omega$ is consistent with the self-dual geometry, and the physics associated with eq. (16) is four-dimensional indeed. The anti-self-dual two-form $\omega$ is interpreted as the $(2, 2)$ string dilaton field (sect. 2), rather than the world-volume gravity.

5. Higher dimensions versus extended supersymmetry. The maximally supersymmetric SDSG and SDSYM actions (8) and (9) have manifest $OSp(8|2)$ or $OSp(4|2)$ supersymmetry, respectively. In fact, they possess an even larger superconformal symmetry $SL(8|4)$ or $SL(4|4)$, respectively [18], which may be the fundamental world-volume symmetries of (closed or open) M-branes. Indeed, the conformal extension of $SO(2, 2)$ is given by $SO(3, 3) \cong SL(4)$, whereas its $N$-supersymmetric extension is just $SL(N|4)$.

Since the internal symmetry of the supergroup $SL(4|4)$ is also $SL(4) \cong SO(3, 3)$, combining it with the ‘space-time’ conformal group $SO(3, 3)$ implies ‘hidden’ twelve dimensions in yet another way: $SO(3, 3) \otimes SO(3, 3) \subset SO(6, 6)$. The 6+6 dimensions is the only alternative to 2 + 10 dimensions where Majorana-Weyl spinors also exist. I do not consider this possibility.

The gauge actions (11) and (16) for SDYM and SDG, or the equivalent DNS action (14), can be naturally supersymmetrized à la Green-Schwarz. The simple supersymmetry with one spinor generator (minimal grading) in the maximal dimensions (twelve) amounts to the simple superalgebra $OSp(32|1)$. The choice of $OSp(32|1)$ is unique since it simultaneously represents the minimal supersymmetric extension of (i) the (self-dual) ‘Lorentz’ algebra in 2+10 dimensions, (ii) de Sitter algebra in 1+10 dimensions and (iii) the conformal algebra in 1+9 dimensions [22]. A supersymmetry part of $OSp(32|1)$ reads (cf. eq. (6a)):

$$\{Q_\alpha, Q_\beta\} = \gamma^{\mu\nu}_{\alpha\beta} M_{\mu\nu} + \gamma^{\mu_1\ldots\mu_6}_{\alpha\beta} Z^+_{\mu_1\ldots\mu_6} , \quad (17)$$

where $Q_\alpha$ is a 32-component Majorana-Weyl spinor, the Dirac $\gamma$-matrices are chirally projected, $M_{\mu\nu}$ are 66 ‘Lorentz’ generators, $8$ and $462$ generators $Z^+_{\mu_1\ldots\mu_6}$ comprise a self-dual six-form (all in 2+10 dimensions). The pre-geometrical action I propose for M-branes is given by

$$S_{M}[\tilde{\Omega}; \omega] = -\frac{1}{4\pi} \int_Y \text{str} \left( \tilde{\Omega} \wedge d\tilde{\Omega} + \frac{2}{3} \tilde{\Omega} \wedge \tilde{\Omega} \wedge \tilde{\Omega} \right) \wedge \omega , \quad (18)$$

$^8$At this point my approach differs from that of Bars [23].
where $\omega$ is an anti-self-dual two-form ($N=2$ string dilaton) in the world-volume, and $\tilde{\Omega}$ is the $OSp(32|1)$ Lie superalgebra valued 1-form gauge potential. The action (18) can be further (doubly) supersymmetrized with respect to the world-volume, as in sect. 3. Currently, it is unclear to me whether it should be done or not.

The action in eq. (18) is called pre-geometrical because of the apparent absence of the translation generators (momenta) in the gauged superalgebra $OSp(32|1)$. However, the momenta can be easily recovered after a Wigner-Inönü-type contraction of $OSp(32|1)$ to lower dimensions. For instance, the 66 Lorentz generators $M_{\mu\nu}$ are decomposed into 55 Lorentz generators and 11 translations in eleven dimensions. The additional generators $Z^{\pm}_{\mu_1,\ldots,\mu_6}$ can be interpreted either as the off-shell charges that do not transform the physical states [22], or as the active charges which are related to boundaries of extended objects (6-branes) [23]. The most degenerate contraction of $OSp(32|1)$ yields the flat target space in $66 + 462 = 528$ (!) dimensions (cf. ref. [23]).

6. Conclusion. My arguments in this Letter support the idea [4] that the fundamental framework for describing the secret F(or M, S, ...) theory is provided by the 2 + 2 dimensional supermembranes (M-branes) living in 2 + 10 dimensions. The integrability (or self-duality) of M-branes naturally substitutes and generalizes the conformal symmetry of the string world-sheet. The basic assumptions were merely the $N=2$ string (world-sheet/target space) duality, and the manifest 'Lorentz' invariance and supersymmetry in 'space-time'. The hidden superconformal symmetries of M-branes are to be responsible for their full integrability and the absence of loop divergences in 2 + 2 world-volume dimensions despite of the fact that the DNS action is non-linear and, hence, is formally non-renormalizable in four dimensions. It fact, the DNS action is known to be one-loop finite, at least [24, 25]. Its maximally supersymmetric extension may have no divergences at all, presumably because of having a chiral current symmetry algebra similar to that in the two-dimensional supersymmetric WZNW models [24, 25]. Unlike the $N=2$ strings having severe infrared divergences in loops [26], no such problems are expected for M-branes due to the higher world-volume dimension. The theory of M-branes should therefore exist as a quantum theory, in which strings would appear as asymptotic states of M-branes.

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9A super-Poincaré algebra does not exist in 2 + 10 dimensions.
References


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