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DIFFERENCES IN SHIELDING VERSUS TRAPPING

BEHAVIOUR OF DYNAMICALLY STABILIZED

Nb-Ti CYLINDERS

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ABSTRACT

It was found experimentally that dynamically stabilized Nb-Ti cylinders show - in contrast to general expectation - a stability behaviour for shielding of external magnetic fields different from that for trapping. This phenomenon is discussed and a theoretical interpretation is proposed.

1. INTRODUCTION

It is generally accepted that when the magnetic field outside a body bounded by type II superconductor is varied, the field within the body remains unchanged beyond the region occupied by the persistent currents induced by the variation. Practical applications of this effect have included the preservation of essentially zero field in the bore of a superconducting tube by shielding it from an external applied field [1,2] and trapping an impressed field within a superconducting cylinder when the external field is removed [3]. The limit to the field difference which can be sustained across the superconducting wall is set by the onset of flux-jump instabilities. Several authors [4-7] have proposed methods of calculating this stability limit, but they do not differentiate between shielding and trapping - the inference being that there is no difference between the two cases. Our experiments on dynamically stabilized Nb-Ti cylinders show, however, that trapping is considerably less stable than shielding.

2. EXPERIMENTS

We have done a number of magnetization experiments on hollow superconducting cylinders made by stacking discs of Nb-Ti foil, high conductivity normal metal and wire mesh. The experimental arrangement is shown in fig. 1. The magneto-resistance probe measures the radial field distribution at the centre plane, the hall plate measures the axial distribution and the pick-up coil indicates flux changes. The cylinder is made longer than the coil to ensure that maximum penetration occurs at the centre plane.

Fig. 2 illustrates a particularly clear case of "unsymmetric" behaviour. The induction on the axis at the centre plane is plotted against the coil field. The cylinder is first immersed in liquid helium at zero coil field, then the coil field is progressively increased. The cylinder shields the coil-field up to 38 kOe when a flux jump destroys virtually all the induced currents and complete penetration occurs. The cylinder is then heated to destroy any residual current and re-immersed in liquid helium at a coil

field of 35 kOe. When the coil field is progressively reduced, the impressed field in the cylinder bore stays constant, but after an external field change of 15 kOe, partial flux jumping begins to limit the build-up of current and at zero coil field, only some 1.7 T remain in the bore.

3. DISCUSSION

In an attempt to explain this unsymmetric stability behaviour we will investigate the effect of a small temperature disturbance on the superconductor and endeavour to show that such a disturbance results in a greater temperature increase and consequently in a reduced stability for the trapping case.

Considering an infinitely long cylinder in a uniform external field, we adopt the expression of Kim [8] for the field dependence of critical current density

$$j_c = \frac{\alpha_s}{B + \beta} \quad , \quad (1)$$

where α_s and β are material constants, and the critical state model [9] for field distribution in the penetrated zone

$$\frac{dH}{dr} = j_c \lambda_s \quad , \quad (2)$$

where λ_s is the filling factor.

When these equations are combined, it is convenient to express the results in terms of the radial distribution of the induction B. Thus, for shielding

$$B = \left\{ (B_e + \beta)^2 - 2\mu_0 \alpha (r_e - r) \right\}^{\frac{1}{2}} - \beta \quad (3)$$

and for trapping

$$B = \left\{ (B_e + \beta)^2 + 2\mu_0 \alpha (r_e - r) \right\}^{\frac{1}{2}} - \beta, \quad (4)$$

where B_e = external induction,

$$\alpha = \alpha_s \lambda_s \quad \text{and}$$

r_e = external radius

For both cases the penetration depth is

$$d = \left| \frac{(B_e + \beta)^2 - (B_i + \beta)^2}{2\mu_o \alpha} \right|, \quad (5)$$

where B_i = internal induction. So far shielding and trapping are symmetric.

We will now consider a cylinder under adiabatic conditions. In this case the resultant increase in temperature of the superconductor is directly related to the energy dissipated in response to a small temperature disturbance.

If a uniform temperature increment δT is applied to the superconductor, the critical current density j_c will be reduced by

$$-\delta j_c = \frac{\delta T}{T_o} j_c. \quad (6)$$

Eq. (6) can be regarded as a definition of T_o . The resulting change in the radial field distribution is given by

$$\delta B = \mu_o \int_r^{r_e} \delta j_c dr. \quad (7)$$

Further integration yields the change of flux linked to the cylindrical shell of radius r

$$\delta \phi = \int_{r_i}^r \delta B \cdot 2\pi r dr, \quad (8)$$

where $r_i = r_e - d$ = inner radius of penetrated zone.

This flux change induces an electric field with circular field lines and an amplitude given by

$$dt \oint E ds = 2\pi r E dt = \delta\phi . \quad (9)$$

Finally, the coexistence of electric field strength E and time independent current density j_c implies a loss mechanism generating heat per unit volume of

$$q = E j_c dt. \quad (10)$$

Substituting the previous expressions in eq. (10) results in

$$q = \delta T \frac{\mu_0 \alpha^2}{r(B + \beta)} \int_{r_i}^r \left\{ \int_r^{r_e} \frac{dr}{(B + \beta) T_o} \right\} r dr , \quad (11)$$

in which B is given by eqs (3) and (4) for shielding and trapping, respectively.

Eq. (11) has been evaluated numerically using $T_o = 5 - 0.455 B$ with B in Tesla and T_o in Kelvin as an empirical fit to the data of Hampshire et al. [10] for Nb-Ti at 4.2 K and $\alpha_s = 2 \times 10^9$ AT/m², $\beta = 1$ T, $\lambda_s = 0.25$, which represent typical values for the cylinders used in the experiments. The resulting radial distributions of B , j_c , $\int E dt$ and q are given in figs 3(a) to (d). The pronounced unsymmetry of heat generation shown there is due to the fact that in the shielding case the electric field falls as the current density rises, whereas in the trapping case both rise together, thus giving a peak of heat generated per unit volume several times higher than in the shielding case.

Therefore, if the thermal condition of the superconductor is adiabatic, the higher heat generation in trapping is sufficient to explain the lower stability.

The experimental cylinders, however, were both internally cooled and dynamically stabilized. Under these conditions the temperature rise in the superconductor is also a function of the rate of heat transfer to the coolant and hence of the power generation per unit volume.

If in this case the uniform temperature increment δT is applied at time $t = 0$, the current density in the superconductor becomes instantaneously greater than the critical value by the "excess current density"

$$\delta j = \delta j_c = j_c \frac{\delta T}{T_0} \quad (\text{at } t = 0) \quad (12)$$

and the superconductor exhibits the flux flow resistivity ρ_s due to which part of the excess current is transferred to the normal metal. It can be shown that for very close magnetic coupling the transfer time becomes negligibly short. Also, under these conditions the induced voltage which in turn is balanced by the resistive voltage drop is equal in superconductor and normal conductor, i.e.

$$U = \oint E ds = \oint \rho_n j_n ds = \oint \rho_s j_s ds, \quad (13)$$

where: j_s is that part of the excess current density remaining in the superconductor, j_n is that part of the excess current density transferred to the normal conductor and ρ_n is the resistivity of the normal metal.

Further from the requirement of continuity we obtain

$$\lambda_s j_s + \lambda_n j_n = \lambda_s \delta j, \quad (14)$$

where λ_n is the filling factor for the normal metal.

If we now want to extend our considerations from uniformly applied disturbances to merely locally induced ones, we can do this by confining the temperature increment δT to a sector ψ of the cylinder (see fig. 4). Eq. (13) can then be written

$$2\pi r j_n \rho_n = \psi r j_s \rho_s \quad (15)$$

and the power dissipated per unit volume of superconductor

$$\dot{q} = j_c E = j_c (j_s \rho_s)$$

becomes

$$\dot{q}(t=0) = \frac{\lambda_s \rho_n \rho_s}{\lambda_s \rho_n + \frac{\psi}{2\pi} \lambda_n \rho_s} j_c^2 \frac{\delta T}{T_o} . \quad (16)$$

For disturbances applied to an extremely small sector one obtains

$$\lim_{\psi \rightarrow 0} \dot{q} = \rho_s j_c^2 \frac{\delta T}{T_o} , \quad (17)$$

which is the expression for a cylinder without any stabilization. Evidently, dynamic stabilization becomes progressively less effective, as the area of disturbance diminishes.

If, however, the normal metal is in intimate electrical contact with the superconductor, then the distribution of potential in superconductor and normal conductor must be the same, i.e.

$$j_n \rho_n = j_s \rho_s . \quad (18)$$

The condition expressed by eq. (18) is equivalent to putting $\psi = 2\pi$ in eqs (15) and (16) and implies that the power dissipation per unit volume becomes now independent of the extent of a disturbance. Experiments with cylinders using superconductor in good electrical contact with normal metal show increased stability in both modes, nevertheless trapping still exhibits less stability than shielding.

Fig. 3(e) shows the radial distribution of power dissipation per unit volume at $t = 0$, obtained by evaluating eq. (16) with $\psi = 2\pi$. The curves for shielding and trapping are symmetrical, but comparison with the curves for heat generation shows that in shielding the peak power

dissipation occurs where heat generation is zero whilst in trapping it occurs where heat generation is near its maximum. In addition, as r approaches r_1 the heat generation approaches zero in both cases whereas the power dissipation does not. This implies that the decay rate of excess current varies as a function of the radius and approaches infinity as r approaches r_1 , despite the dynamic stabilization.

To illustrate this point we consider the decay of a deviation δB from the critical-state field distribution. Taking the case of an infinite slab, thickness $w > 2d$ (see fig.5), to avoid the unnecessary complication of Bessel functions inherent in rotational symmetry, we can write the normal diffusion equation, with D_m as the magnetic diffusivity

$$D_m \frac{\partial^2}{\partial x^2} \delta B = \frac{\partial}{\partial t} \delta B \quad (19)$$

with $D_m = \bar{\rho} / \mu_0$.

The average resistivity $\bar{\rho}$ is defined by

$$E = \bar{\rho} \delta j = \rho_s j_s = \rho_n j_n \quad (20)$$

Combining (14) and (20) gives

$$\frac{1}{\bar{\rho}} = \frac{\lambda_n}{\lambda_s} \frac{1}{\rho_n} + \frac{1}{\rho_s} \approx \frac{\lambda_n}{\lambda_s} \frac{1}{\rho_n} \quad (21)$$

The boundary conditions for eq. (20) are

$$\delta B = 0 \quad \text{at } x = d$$

$$\mu_0 \delta j = \frac{\partial}{\partial x} \delta B = 0 \quad \text{at } x = 0 \quad .$$

The solution for (19) under these boundary conditions is the time dependent Fourier series

$$\delta B = \sum_{m=0}^{\infty} \delta B_m \cos (k_m x) e^{-\frac{t}{\tau_m}}, \quad (22)$$

which can be differentiated with respect to x

$$\delta j = \sum_{m=0}^{\infty} j_m \sin (k_m x) e^{-\frac{t}{\tau_m}}, \quad (23)$$

where $k_m = \frac{(m+1)\pi}{2d}$, $\tau_m = \frac{1}{D_m k_m^2}$, $m = 0, 1, 2, \dots$

$$\text{and } j_m = \frac{1}{d} \int_0^d \delta j_c \sin (k_m x) dx. \quad (24)$$

Analysation of eq. (23) shows that, since all space harmonics compatible with the boundary conditions are zero at $x = 0$, the decay time there must be infinitely short.

The calculated spectra of the amplitudes j_m are shown in fig. 6. One observes that more of the initial excess current occurs in the higher harmonics in shielding, so decay will be more rapid in this case.

We have also calculated numerically the distribution of power dissipation per unit volume for a typical test cylinder at times greater than $t = 0$. This was done by dividing the cylinder into concentric shells, each of which was characterized by its resistivity, self-inductance and mutual inductance to the other shells. A resistivity of zero was ascribed to the innermost shell. The resulting system of linear differential equations for the excess currents was solved numerically. The results for $t = 0$ and $t = 0.5$ sec are shown in fig. 7. As expected, these again indicate that shielding and trapping are only symmetrical at $t = 0$. Due to the very rapid decay of the excess current density near $r = r_i$ the peak power dissipation per unit volume is at all other times greater in trapping.

The higher stability in the shielding case can be attributed to the fact that here the peak power dissipation per unit volume occurs close to r_i , where the local current decay time is small compared to the thermal time constant. The local thermal conditions may then be considered as essentially adiabatic and the temperature rise due to a thermal disturbance is proportional to the density of generated heat which becomes very small close to r_i .

To test this hypothesis we have carried out experiments with cylinders cooled in coil fields of 0, 5, 10 and 20 kOe and then subjected to a negative external field change. The initial coil field is first reduced to zero and then increased with reversed polarity until a flux jump occurs. The results, shown in fig. 8, indicate that as soon as the peak density of initial power dissipation (which coincides with the point of lowest field) no longer occurs at its privileged position at r_i , the stability is reduced considerably.

CONCLUSIONS

The rearrangement of the current distribution resulting from a disturbance applied to a dynamically stabilized superconducting cylinder cannot be described by a single time constant. Deviations from the critical state relax with position dependent decay times. These tend towards zero, as one approaches the inner limit of the penetration zone. In combination with a field dependent current density this leads to a stability behaviour which favours flux shielding compared to flux trapping.

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REFERENCES

- [1] M. Firth, E.U. Haebel, L. Krempasky and F. Schmeissner, Proceedings of the International Conference on Instrumentation for High-Energy Physics, Frascati, Italy, (1973) 79.
- [2] F. Martin, S.J. St. Lorant and W.T. Toner, Nuclear Instruments and Methods 103 (1972) 503.
- [3] J.J. Hanak, RCA Review 25 (1964) 551.
- [4] H.R. Hart Jr., Proceedings of the 1968 Summer Study on Superconducting Devices and Accelerators (1968) 571.
- [5] R. Hancox, Physics Letters 16 (1965) 208.
- [6] S.L. Wipf, Physical Review 161 (1967) 404.
- [7] P.S. Swartz and C.P. Bean, Journal of Applied Physics 39 (1968) 4991.
- [8] Y.B. Kim, C.F. Hempstead and A.R. Strnad, Physical Review 129 (1962) 528
- [9] C.P. Bean, Physical Review Letters 8 (1962) 250.
- [10] R. Hampshire, J. Sutton and M.T. Taylor, Proceedings of the Conference on Low Temperatures and Electric Power, London, (1969) 69.

FIGURE CAPTIONS

- Fig. 1 Experimental Arrangement.
- Fig. 2 Magnetization curve of a test cylinder showing the unsymmetry between shielding and trapping.
- Fig. 3 a) Induction, b) critical current density, c) time integrated pulse of induced electric field strength, d) total heat generated per unit volume and e) power dissipated per unit volume at $t = 0$, all parameters plotted versus radius.
- Fig. 4 Local temperature disturbance δT in a sector ψ of the cylinder.
- Fig. 5 Cross section of a slab penetrated to depth d .
- Fig. 6 Fourier spectra of j_m for the slab.
- Fig. 7 Power density distribution at $t = 0$ and $t = 0.5$ sec, calculated for the test cylinder.
- Fig. 8 Shielding characteristics of a test cylinder with different initial coil fields.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

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