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Calculation of Flow in Transition Duct Using Second-Order Closure and Wall Functions

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Introduction

Transition ducts are relevant to a number of industries but in particular they are important in the aeronautical field for intake and exhausts of high performance aircraft. The real-life application of this geometry is the motivation behind the extensive research conducted in this area over the last decade. Detailed experimental measurements of a circular-to-rectangular transition duct have been carried out by Davis¹ and he reported the results for the mean flow as well as for the turbulence quantities, including all six components of the Reynolds-stress tensor.

Sotiropoulos and Patel² conducted an extensive numerical study of transition ducts and obtained excellent agreement between their simulation and the experimental data of Davis¹. They used the near wall, full Reynolds-stress transport model of Launder and Shima³ for closure. Furthermore, they analyzed in detail previous numerical simulations^{4,5,6} for this particular geometry and argued that they were deficient in many aspects of the flow, including turbulence characteristics. One of the main reasons why the studies prior to the one of Sotiropoulos and Patel² failed to capture the main phenomena present in the transition duct flows was attributed to the usage of inappropriate turbulence models not capable of accurately capturing the turbulence anisotropy. However, why their results were substantially better than an earlier simulation⁵ utilizing the second-order closure could not be answered as numerical uncertainties were present in the earlier paper. To clear up these uncertainties and to provide more solid conclusions, they followed up this initial work with simulations using the standard $k-\epsilon$ model and the Reynolds-stress model of Gibson and Launder⁷ for closure. In both of these methods they used wall functions. These results were compared with both the experimental as well as with the predicted results from their earlier simulation. The results obtained⁸ showed the same deficiencies as the earlier attempts by other

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researchers. The overall conclusions from this work were that a second-moment closure is a minimum level of modeling required and, more importantly, that explicit modeling of the near wall flow is needed to correctly predict secondary motion and vorticity strength. It is also worth mentioning here that recently Craft and Lien⁹ also calculated the present type of flow utilizing the full Reynolds-stress transport model of Craft and Launder¹⁰ but again in conjunction with the low-Reynolds-number approach.

The purpose of this Technical Note is to determine whether the deficiencies reported in previous study⁸ could be overcome by using an alternative high-Reynolds-number second order closure which will be described below.

Governing Equations

The equations for conservation of mass and momentum for constant density flow are:

$$\frac{\partial U_i}{\partial x_i} = 0. \quad (1)$$

$$U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial U_i}{\partial x_j} - \overline{u_i u_j} \right) \quad (2)$$

where U_i is the mean-velocity vector, p the pressure, ν the kinematic viscosity and $\overline{u_i u_j}$ the Reynolds-stress tensor.

The unknown Reynolds-stresses in equation (2) are obtained here from the solution of six separate differential transport equations that are derivable exactly from the momentum equations. Unfortunately, the Reynolds-stress equations contain a number of terms that are themselves unknown and they require therefore an adequate level of approximation. Since the ultimate goal of the present study is to develop a generally applicable numerical procedure, it is essential to employ a turbulence model which would be able to offer an acceptable accuracy for the various types of flow geometries. However, if the Reynolds-stress model is to be part of a generally applicable numerical procedure, there is obviously a need for a model formulation which will be free of all parameters which could limit its generality (e.g. wall distance). The need to drop the wall-distance parameters from second-order closure has been recognized by many scientists as a first step towards the goal of applying this type of closure to complex geometries. Since the wall distance appears as part of the pressure-strain model, a number of alternative models for this term have been reported in the last couple of years (e.g. Speziale et al.¹¹ and Launder and Li¹²). Here, we adopt the proposal of Speziale et. al¹¹ (SSG) and, with the use of Cartesian-tensor notation, the complete modeled Reynolds-stress equations may be written as:

$$\overbrace{U_k \frac{\partial \overline{u_i u_j}}{\partial x_k}}^{\text{convection}} = - \overbrace{\left(\overline{u_i u_k} \frac{\partial U_j}{\partial x_k} + \overline{u_j u_k} \frac{\partial U_i}{\partial x_k} \right)}^{\text{production}} + \overbrace{\frac{\partial}{\partial x_k} \left[C_s \frac{k}{\epsilon} \overline{u_k u_l} \frac{\partial \overline{u_i u_j}}{\partial x_l} + \nu \frac{\partial \overline{u_i u_j}}{\partial x_k} \right]}^{\text{diffusion}} - \overbrace{\frac{2}{3} \delta_{ij} \epsilon}^{\text{dissipation}}$$

$$\begin{aligned}
& \overbrace{-(C_1 \epsilon + C_1^* P_k) b_{ij} + C_2 \epsilon (b_{ik} b_{kj} - \frac{1}{3} b_{mn} b_{mn} \delta_{ij}) + [C_3 - C_3^* (b_{mn} b_{mn})^{\frac{1}{2}}] k S_{ij}}^{\text{pressure-strain}} \\
& + \overbrace{C_4 k (b_{ik} S_{jk} + b_{jk} S_{ik} - \frac{2}{3} b_{mn} S_{mn} \delta_{ij}) + C_5 k (b_{ik} W_{jk} + b_{jk} W_{ik})}^{\text{pressure-strain}} \quad (3)
\end{aligned}$$

where b_{ij} , S_{ij} and W_{ij} stand for Reynolds-stress anisotropy, the mean rate of strain and the mean vorticity tensor respectively. The rate of dissipation of turbulence kinetic energy (ϵ) is determined from the solution of the standard transport equation:

$$U_j \frac{\partial \epsilon}{\partial x_j} = \frac{\partial}{\partial x_k} \left(C_\epsilon \frac{k}{\epsilon} \overline{u_k u_l} \frac{\partial \epsilon}{\partial x_l} \right) + \frac{\epsilon}{k} (C_{\epsilon 1} P_k - C_{\epsilon 2} \epsilon) \quad (4)$$

where the P_k stands for production of turbulence kinetic energy. The model coefficients were assigned the original values proposed by originators¹¹.

Numerical Method

The numerical procedure used follows that of Basara et. al¹³ but, for completeness, the main points regarding the numerical procedure will be included here. The governing equations together with turbulence model equations are solved using the finite-volume method which utilizes a general non-orthogonal, body-fitted coordinate system. The velocity and Reynolds-stress tensor components retain a Cartesian orientation within the co-located grid arrangement whereby all dependent variables are stored at the center of the control volume. In order to avoid the numerical oscillations associated with the use of this grid arrangement, a pressure-smoothing technique of Rhie and Chow¹⁴ was used to determine the velocity at the cell face. All the results reported here are obtained using the second-order accurate Linear-Upwind Differencing Scheme. The boundary conditions used are as follows. The universal log.-law of the wall is used for the cells adjacent to the rigid boundary which enabled the usage of the standard wall functions for the calculation of the wall shear stresses. The rate of the turbulence energy dissipation was set to the rates of turbulence-energy production in the near-wall region. The zero-gradient boundary condition was employed for all six components of the Reynolds-stress tensor but it should be noted that the stress equations are solved even for the cells adjacent to the wall whereby all velocity gradients entering those equations are calculated through the log.-law. The zero gradient boundary conditions are also employed at the symmetry and the outlet planes while the inlet values for all dependent variables are prescribed in accordance with the information provided by Davis¹.

Once the algebraic equations have been assembled through the discretization procedure, they are solved using an appropriate conjugate gradient algorithm. The criterion for convergence was that the absolute sum of the normalized residuals for all dependent variables fell below 6×10^{-5} . For a typical run performed on a grid of $81 \times 41 \times 41$, this level of convergence was attained after 3000 iterations and required 360 CPU minutes to complete on 8 processors of an IBM-SP2 machine. The parallel implementation employs a spatial decomposition of the global domain into sub-domains which are assigned to separate processes. The processors communicate and synchronize their actions with other processors by exchanging messages. To accommodate the finite volume computational stencils a buffer of halo cells is added to the sub-domains.

Results

The duct topology with cross sectional grids shown at various stations is shown in Fig. 1. This type of geometry is particularly complex as the change in shape produces, simultaneously, stretching and squeezing which induces the generation of streamwise vorticity which distorts the flow. Full details of the geometry can be found in Davis¹. The measurements were taken at a Reynolds number of 390,000 based on the bulk velocity and duct inlet diameter. Due to the symmetry of the geometry and of the inlet conditions only a quadrant of the geometry is modeled. In order to investigate the grid sensitivity of the present results, the predictions were carried out on a coarse ($61 \times 21 \times 21$), medium ($71 \times 31 \times 31$) and fine grid ($81 \times 41 \times 41$). The results were monitored along the major axis at Station 5. For example, the differences in the computed maximum values for turbulent intensity u' and Reynolds-stress $\overline{v^2}$ and $\overline{uv}$ between the coarse and medium grids were 14%, 21% and 23% respectively. Comparing the medium and fine grid cases these differences were reduced to 2.7%, 0.4% and 3.1% respectively and therefore the fine grid ($81 \times 41 \times 41$) was regarded adequate for a grid independent solution. All results reported here are for Station 5, shown in Fig. 1, where the change of shape from circular to rectangular cross-section ends.

The calculated and measured secondary-velocity vectors are compared in Fig. 2. Both predictions and measurements show the existence of the secondary flow vortex confined to the side wall region of the channel. The effects of the secondary flow on the primary mean velocity can be seen from Fig. 3 where the predicted and measured contours of this quantity, averaged by bulk velocity (U/U_b), are presented. The secondary velocity vortex acts to transport slow-moving fluid from the vicinity of the side walls into the central portion of the duct along the horizontal symmetry plane. This distorts the primary velocity field quite significantly, and it can be seen that the prediction follows the measured behavior quite well. It is important to state here that all previous calculations^{5,8} which used wall functions failed to capture, almost completely, the level of distortion of the mean velocity field presented in

this study.

Consideration is now turned to the turbulence field and the axial turbulence intensity averaged by the local axial velocity component as shown in Fig. 4. The position of the maximum turbulence intensity is in good qualitative agreement with the measurements with a very similar shape of contours, including also their distortions toward the duct corner. The shear-stress component $\overline{uw}$ is shown in Fig. 5. Predicted quantities is in a good qualitative agreement with the experimental data. This is particularly true for the shape of the $\overline{uw}$ contours where predictions capture even small local maximum confined in the corner between the symmetry plane and the side wall. However, it is evident that the peak value for this quantity is somewhat overestimated when compared to the measured values. The levels of the normal-stress anisotropy is particularly important for this type of flow since this quantity represents the primary driving mechanism for the secondary motion. The contours of the transverse normal-stress anisotropy $(\overline{v^2} - \overline{w^2})$ reported by Davis¹ are compared here with the present predictions in Fig. 6. It can be seen from this figure that the contour lines are very close in the region close to the side wall which means that gradients of the normal-stress anisotropy are high and hence strong secondary velocities are generated. Agreement, overall, is fair although some overprediction is observed for the peak value of the contours which suggests that the model has exaggerated the reduction wrought by the top wall on the vertical fluctuating-velocity component $(\overline{w^2})$.

Conclusions

The results presented in this paper are part of an ongoing program of research aimed at the development and verification of a generally-applicable portable parallel code for the prediction of turbulent flows which utilize advanced turbulence models. The focus here has been on the flow in a circular-to-rectangular transition duct which has well-documented experimental data¹ for model validation. The Reynolds-stress model used here utilizes the complete pressure strain model of Speziale et al.¹¹, which is free of any wall-distance parameters. It was found that the model can reproduce, fairly satisfactorily, the principal features of the flow, including the distortions of the mean velocity and turbulent fields caused by the cross-sectional secondary-velocity field. However, the most important conclusion coming out of this short technical note is that, contrary to some previous studies^{5,8}, it is possible to capture all the main features of this type of flow with second-order closure when used in conjunction with wall functions.

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Fig. 1 Computational grid and geometry. Data of Davis¹.

Fig. 2 Cross-sectional secondary flow. Data of Davis¹.

Fig. 3 Axial mean velocity contours U/U_b . Data of Davis¹.

Fig. 4 Axial turbulence intensity contours $u'/Ux10^2$. Data of Davis¹.

Fig. 5 Reynolds-stress contours $\overline{uw}/U_b \times 10^4$. Data of Davis¹.

Fig. 6 Transverse normal stress anisotropy $(\overline{v^2} - \overline{w^2})/U_b \times 10^4$. Data of Davis¹.