LONGITUDINAL BEAM ECHO IN THE CERN SPS

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Abstract
The presented work summarises the results of two experiments in the CERN SPS. Longitudinal echo signals can be produced in the CERN-SPS by exciting a coasting proton beam at 120 GeV/c with two short RF pulses at different harmonics of the revolution frequency, separated by a suitable time-delay. The aim of the experiments was to confirm the analytical predictions for a beam echo in the SPS and to probe the applicability of the beam echo for a measurement of the energy distribution and diffusion coefficients in the accelerator.

1 INTRODUCTION
Echo phenomena have been well known in plasma physics for many years [1]. However, the effect has only been recently introduced to accelerator physics. The first measurements of the effect were made one year ago in the 8 GeV anti-proton accumulator ring at FNAL [2]. These first results were very promising and encouraged further studies. For example, echo techniques might offer a new approach for investigating diffusion and collective effects in hadron storage rings [2]. The aim of the presented work was to measure echo signals in the CERN SPS and to investigate the applicability of the beam echo for a measurement of the energy distribution and diffusion coefficients in the accelerator.

The echo signal is an interference pattern of two consecutive short RF-pulses. A theoretical analysis of the echo effect shows that the time and the amplitude of the echo response depends on the time delay between the two RF-kicks. The basic idea of the analysis is to treat the energy gain during the two RF-kicks as small perturbations and to expand the initial distribution function in a Taylor series around the unperturbed variables [3].

Without diffusion, the equations of motion for a coasting beam can be easily solved and one can show that the beam echo appears at a time

\[ t^* = \frac{h_1}{h_2 - h_1} \cdot \Delta t \]  \hspace{1cm} (1)

after the second RF-kick, where \( \Delta t \) is the time separation of the two RF-kicks. Adding diffusion to the particle dynamics and using similar approximations as in the case without diffusion, one gets for the echo response in the beam current [3][4]

\[ I_{\text{echo}}(t) \approx \epsilon \omega_0 2 \epsilon_1 \cdot \{ J_1(x) + \tau \epsilon_2 [J_0(x) - J_1(x)] \} \]  \hspace{1cm} (2)

where \( \epsilon_1 \) and \( \epsilon_2 \) are the perturbation parameters in the Taylor expansion

\[ \epsilon_i = \frac{\omega_0 T_i}{2\pi} \cdot \frac{e V_i}{E_0} \]  \hspace{1cm} (5)

and

\[ \tau = \int \rho(\hat{p}) \cdot \cos(p \tau) \, dp \]

\[ \times \, e^{-D k_0^2 [h_1^2 (\Delta t)^3 + (h_2 - h_1)^2 (t - t^*)^3]/3}, \]

where \( D \) is the diffusion coefficient, \( \rho(\hat{p}) \) the initial energy distribution, \( \tau \) the time measured relative to the centre of the echo response \((\tau = k_0(h_2 - h_1) \cdot (t - t^*))\) and \( t \) the time measured from the second RF kick. \( J_n(x) \) are Bessel functions of the first kind with

\[ x = 2h_1 \epsilon_2 \sigma_p k_0 \Delta t \]  \hspace{1cm} (3)

Figure 1: The analytical estimate for the echo shape for a Gaussian energy distribution: \( \Delta t = 45 \text{ ms}, \sigma_p = 6 \cdot 10^{-4} \) and parameters of Table 1.
2 EXPERIMENTAL DATA

Fig. 3 shows the experimental setup for the echo measurement. All the data discussed in this section was measured for two RF kicks with harmonic numbers \( h_1 = 4600 \) and \( h_2 = 4620 \) and a kick amplitude of 500 kV per kick. Each kick lasted for approximately 4 turns (\( T_i \approx 92.221 \mu s \)). The beam intensity was approximately \( 1.2 \times 10^{12} \) particles. Table 1 lists the relevant parameters for the CERN SPS storage ring. Fig. 4 shows a longitudinal Schottky spectrum towards the beginning of a new proton fill and Fig. 5 shows the corresponding echo signal for a time separation of the two RF kicks of \( \Delta t = 45 \text{ ms} \). One clearly recognises the multiple peaks from a non-Gaussian energy distribution.

Both examples illustrate nicely the sensitivity of the echo response on the energy distribution in the beam. In principle, the distribution function can be recovered by an inverse Fourier transform of the echo response (see Equation (2)). However, a detailed study of this potential application for beam echos is still in progress and a reconstruction of the energy distribution from the beam echo has not yet been done. The energy spread in the beam distribution is given by the time separation \( \Delta_{\text{peak}} \) of the two maxima in the echo response. For example, for a Gaussian energy distribution,

\[
\begin{array}{|c|c|c|c|}
\hline
f_{\text{rec}} \text{ [kHz]} & \eta & E_0 \text{ [GeV]} & f_{\text{RF}} \text{ [MHz]} \\
\hline
43.23 & 1.8 \times 10^{-9} & 120 & 200 \\
\hline
\end{array}
\]

Table 1: Machine parameters of the SPS.
one obtains from Equation (2)

\[ \sigma_p^2 = \frac{\beta^2}{(h_2 - h_1) \cdot \pi \cdot f_{\text{res}} \cdot |\eta| \cdot \Delta_{\text{peak}}} \]  

Using Equation (6), we get from Fig. 7 \( \sigma_p = 4.1 \cdot 10^{-4} \). Measuring the energy spread directly from the longitudinal Schottky signal in Fig. 6, we get \( \sigma_p = 5.2 \cdot 10^{-4} \). Both values agree rather well. However, reading the frequency spread from the Schottky signal is more cumbersome than reading the time separation of the two maxima in the echo signal.

Fig. 8 shows a series of 22 echo measurements for different time separations between the two RF kicks. The vertical axis shows the amplitude of the echo signal, and the horizontal axis the time measured after the second RF kick. The appearance of the echo signal is proportional to the time delay between the two RF kicks (see Equation (1)) and the beam echo appears the later in time, for a larger time separation between the two RF kicks. The time separation in Fig 8 varied between 5 ms and 220 ms and horizontal scale varies from 0 s to 50 s. The solid envelope shows the analytical estimate for the maximum echo response from Equation 2 with \( D = 0 \).

of diffusion on the beam echo is still in progress. In a recent experiment in the CERN SPS storage ring an additional diffusion was generated by applying a noise signal to one of the RF cavities. The noise signal was calibrated by measuring the growth of the energy spread in the beam for different noise amplitudes with the longitudinal Schottky signal. Over a time interval of 100 s, the Schottky signal was sensitive to noise signals which correspond to \( D > 10^{-9} s^{-1} \). Once the noise signal was calibrated the noise amplitude was reduced to values where we could no longer observe a growth of the energy spread in the longitudinal Schottky signal \( (D < 10^{-9} s^{-1}) \). The beam echo could be successfully used to measure noise amplitudes which were at least two orders of magnitude smaller \( (D \approx 10^{-11} s^{-1}) \).

3 CONCLUSIONS

The beam echo measurements in the CERN SPS agree well with the analytical expectations and encourage attempts to use the beam echo for measurements of beam distributions and diffusion coefficients in the beam. The measurement of intensity dependent effects might be another potential application of beam echoes. The ongoing study at the CERN SPS foresees to analyse the effect of wake fields on the beam echo in the upcoming experiments. So far, the studies in the CERN SPS were limited to longitudinal beam echos for different separation times \( \Delta t \) as a function of time after the second RF kick. The time separation of the two RF kicks varies between 5 ms and 220 ms. The solid envelope shows the analytical estimate for the maximum echo response from Equation 2 with \( D = 0 \).

4 REFERENCES