HADRO-PRODUCTION OF QUARKONIA
IN FIXED TARGET EXPERIMENTS * 

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In this talk I review the recent progress made in the calculations of quarkonia production in fixed target experiments. NRQCD organizes the calculations in a systematic expansion in $\alpha_s$ and $v$, the relative velocity between the heavy quarks. Within this formalism there are octet contributions which are not included in the color singlet model. These contributions depend upon unknown matrix elements of local operators which are fit to the data. Using these fits, there are several predictions which do indeed improve agreement with the data. However, the prediction for the polarization of the produced states as well as the ratio of the $\chi_1$ to $\chi_2$ cross sections differ substantially from the data for the case of pion beams. Possible large corrections from higher twist effects are discussed as is the issue of the proper choice of masses.

1. Introduction

The use of non-relativistic QCD (NRQCD) allows us to calculate both production and annihilation rates of heavy quark bound states in a systematic expansion in $\alpha_s$ and $v$, the relative velocity of the heavy quarks. Moreover, the inclusion of higher Fock states, which emerge naturally in the formalism, allows for a consistent factorization of long and short distant effects, thus validating the use perturbative QCD to calculate the Wilson coefficients. Furthermore, the long distance effects are now written in terms of well defined operators, which can be calculated on the lattice, instead of potential model wave functions.

In light of this progress, it is interesting to revisit the issue of hadroproduction in fixed target experiments. Previous calculations within the confines of the color singlet model were found to be inconsistent with the data. The prediction for the overall normalization of the cross section is too small as is the ratio of the production cross sections for $\chi_1$ and $\chi_2$. The direct production rate for $J/\psi$ is too small, and the $J/\psi$ are predicted to be partially transversely polarized, which they are not. This disparity between theory and data is, at least for the first two observables discussed, crying out for a new production channel. NRQCD supplies just such a channel, namely the state in which the two heavy quarks are in a relative

octet configuration. This state will have a finite overlap after soft gluon emission acts as a color sink. The overlap will be suppressed by powers of $v$ as dictated by the velocity scaling rules $^7$.

Before proceeding to the results I would like to briefly discuss the levels of rigor which go into the various approximations in the calculations. First, there is no operator product expansion in these calculations. Thus, the factorization is performed via a diagrammatic analysis, as is done for Drell-Yan and other such processes. Factorization in such cases is known to be violated by higher twist effects which are suppressed by powers of the large invariant mass scale involved in the process. For the case discussed here, this scale would be quarkonium mass. In this regard, the proofs of factorization in the case of small $p_T$ production is no less rigorous than at large $p_T$. The only difference in the two cases is that the higher twist corrections at large $p_T$ are suppressed by $1/p_T^2$ as opposed to $1/m_Q^2$. Thus, we expect larger errors to be incurred at small $p_T$.

In showing factorization it is imperative that one sum over all relative color states of the quarkonium, otherwise, as was discussed in the case of the decay of P wave states $^8$, there will be an infrared divergent Wilson coefficient which obviously destroys any hope of calculating in a model independent fashion. Once the higher Fock states are included, the factorization is restored and the final result of our calculation depends upon unknown non-perturbative matrix elements which are enumerated by the velocity scaling rules. Thus, the production cross section for the reactions

$$A + B \rightarrow H + X,$$

(1)

can be written as

$$\sigma_H = \sum_{i,j} \int_0^1 dx_1 dx_2 f_{i/A}(x_1) f_{j/B}(x_2) \hat{\sigma}(ij \rightarrow H),$$

(2)

$$\hat{\sigma}(ij \rightarrow H) = \sum_n C_{Q\bar{Q}[n]}^{ij} \langle O_{H}^n \rangle.$$

(3)

Here the first sum extends over all partons in the colliding hadrons and $f_{i/A}$ etc. denote the corresponding distribution functions. The short-distance ($x \sim 1/m_Q \gg 1/(m_Q v)$) coefficients $C_{Q\bar{Q}[n]}^{ij}$ describe the production of a quark-antiquark pair in a state $n$ and have expansions in $\alpha_s(2m_Q)$. The parameters $\langle O_{H}^n \rangle$ describe the subsequent hadronization of the $Q\bar{Q}$ pair into a jet containing the quarkonium $H$ and light hadrons.

The velocity scaling rules are derived via the multipole expansion, which tells us that soft gluon couplings to a heavy quark bound state are suppressed by the ratio of the size of the bound state to the wavelength of the gluon. This expansion was first used within the confines of the strong interaction by Gottfried $^9$ a while back, and it seems to work quite well. Furthermore, present extractions of matrix

$^9$Their precise definition is given in Sect. VI of $^1$.
elements seem to agree with their predicted scalings with $v$. As such, we will continue under the assumption that these scaling rules are valid. Furthermore, we will ignore effects due to the finite size of the target. Presumably, the soft gluons which will be exchanged between the target and the quark-antiquark pair during the hadronization process should lead to higher order effects in $v^2$ for the same reasons as stated above. We will assume this is true for now, and will keep this, perhaps dubious assumption, in the back of our minds when we confront the data.

2. $\psi'$ production

The case of $\psi'$ production is simplest to analyze since there are no states below open charm threshold which contribute to its indirect production rate. Many of the arguments discussed in this simple case will apply for the other states as well.

The production of $\psi'$ in the singlet channel begins at $O((\frac{4}{\pi})^3 v^3)$ due to charge conjugation, while the octet channel production is $O((\frac{4}{\pi})^2 v^7)$. Given that numerically, $\alpha(4m_E^2) \propto v^2$, it would seem that octet production should be of the same order as the singlet channel. However, the singlet cross section vanishes at threshold where there is small $x$ enhancement due to the gluon distribution functions and, as such, the octet actually dominates the singlet.

Before going on to quantitative issues however, we must address the issue of the proper choice of the hadron mass. It is clear that the short distance coefficients should be calculated using the quark mass, since binding effects are neglected by definition in this part of the calculation. However, at face value it seems that as far as the phase space boundary is concerned we should be using the hadron mass instead of twice the quark mass. This issue was vehemently debated during the workshop. As I emphasized then, the proper choice of mass in the phase space boundary is indeed the $2m_c$. This is the choice which is consistent with the $v^2$ expansion, as the binding effects are always higher order in this expansion parameter. This is best illustrated in the case of heavy-light meson decay where the expansion parameter is $\Lambda_{QCD}/m_Q$. In this case, we may perform an operator product expansion, with no question as to the proper choice of mass. At leading order in the OPE, we find that the phase space is dictated unambiguously by the quark mass as a consequence of unitarity. If we consider the lepton spectrum, then there is an explicit factor of $\theta(1 - 2E_l/m_Q)$ in the differential rate. The fact that the true phase space boundary is determined by the meson mass is seen when one goes to higher order in the OPE where the expansion looks like

$$\frac{d\Gamma}{dE_l} \propto \theta(1 - 2E_l/m_Q) + \delta(1 - 2E_l/m_Q) + \delta'(1 - 2E_l/m_Q) + \ldots \ldots \ldots (4)$$

We see that the expansion breaks down near the partonic endpoint $E_l = 2m_Q$ and is signaling the need for a resummation of the non-perturbative effects. Such a resummation leads to the construction of a structure function which has support all the way to the hadronic endpoint. In our case a resummation of higher order $v^2$ effects will lead to shifting the space limits from being partonic to hadronic. Whether
or not the non-perturbative corrections are large depends upon the observable of interest. In the example above we may safely use the partonic mass if we are not interested in the endpoint region. With that said, let us confront the theory with the data.

In previous analyses, performed using the color singlet model, it was found that the theoretical predictions needed a $K$ factor of $25^{9}$. However, this discrepancy is greatly reduced once we use twice the charm quark mass instead of the hadron mass. Indeed, given the uncertainty in the charmed quark mass, varying $m_c$ between $1.3-1.7 \text{ GeV}$, changes the total cross section by a factor of 8 at $\sqrt{s} = 30 \text{ GeV}$. This large variation is a consequence of the steep rise in the gluon distribution at small $x$. Nonetheless, let us press on assuming that the expansion is well behaved and consider the consequences. In ref $^3$, it was found that, using $m_H = 2m_c = 3 \text{ GeV}$ the color singlet contribution fell a factor of 3 below the data. Including the color octet contribution leads to a fit of the data with the choice $\Delta_8 = 5.2 \cdot 10^{-3} \text{ GeV}$. At large $p_T$, the authors of $^{11}$ found $\langle \mathcal{O}_8^{H}(1S_0) \rangle + \frac{3}{m_c^2} \langle \mathcal{O}_8^{H}(3P_0) \rangle = 1.8 \cdot 10^{-2} \text{ GeV}^3$.

If we assume $\langle \mathcal{O}_8^{H}(1S_0) \rangle \simeq \langle \mathcal{O}_8^{H}(3P_0) \rangle / m_c^2$, the fixed target value is a factor of four smaller than those found in $^{11}$. This discrepancy should not concern us, nor should we consider this particular observable a good test of the color octet mechanism for the reasons discussed above.

3. $J/\psi$ production

Using the data from proton beam fixed target experiments we may again fit the data using the value

$$\Delta_8(J/\psi) = 3.0 \cdot 10^{-2}. \tag{5}$$

As in the previous case this observable is very sensitive to the choice of the quark mass and, as such, the fact that the color singlet contribution is below the data is not strong evidence for the existence of the octet channel. However, in this case we may also look at the ratio for the direct to total cross section which is not sensitive to the quark mass. Indeed, we find that the pure singlet contribution gives a ratio of 0.21 whereas inclusion of the octet with the matrix element extracted above gives $^{3} 0.63$ which is in much better agreement with the experimentally extracted value of $0.62 \pm 0.04$ for a proton beam at $\sqrt{s} = 23.7 \text{ GeV}$. Note that this is not a trivial consequence of fitting the color octet matrix element since the indirect contribution is dominated by color singlet gluon fusion and the singlet matrix elements are fixed in terms of wavefunctions $^{12}$.

Another very interesting observable is $\sigma_{\chi_1}/\sigma_{\chi_2}$ which has been measured in proton as well as pion beam experiments. The singlet cross section for $\chi_1$ production is suppressed by a factor of $\alpha$ relative to $\chi_2$ cross section, while the leading octet contribution is $O(\alpha^2 v^3)$ but is suppressed because it is a quark initiated process. $\chi_2$ production on the other hand, is dominated by color singlet gluon fusion at $O(\alpha^2 v^5)$. The E771 experiment measured a value $^{20} 0.34 \pm 0.16$ for a proton beam
Fig. 1. Total (solid) and singlet only (dotted) \( \psi' \) production cross section in pion-nucleon collisions \((x_F > 0)\) only. The solid line is obtained with \( \Delta S(\psi') = 5.2 \cdot 10^{-3} \text{ GeV}^3 \) determined using the data from proton beam experiments.

at \( \sqrt{s} = 38.7 \text{ GeV} \), and NRQCD predicts a value of 0.07\(^{1b}\). However, the relativistic corrections can be substantial\(^{14}\) given that the leading singlet contribution scales like \( \alpha^3 v^3 \) and numerically \( \alpha(2m_c)/\pi \propto v^3 \). Indeed at \( O(\alpha^2 v^9) \) there will contributions coming from intermediate \( ^1S_0 \) and \( ^3P_J \) octet as well as singlet states, as well a octet \( ^3D_J \) states. Though these states are suppressed in \( v^2 \) given the large number of channels which contribute, the net contribution could be substantial. A naive use of the velocity scaling rules leads to \( \simeq 0.3 \).\(^c\)

For pion beams theory does not seem to do as well. The latest reported value for this ratio for pion beams is given by 0.57 \( \pm 0.18 \), whereas the leading NRQCD prediction is given by 0.07 even including the next order corrections in \( v^2 \), it is clear that the theory falls short. Furthermore, the overall normalization for the total \( J/\Psi \) and \( \psi' \) production cross sections, found using the fitted values of the octet matrix elements using the data from proton beam experiments, also falls short as is shown in figures 1 and 2\(^d\).

4. Polarization in fixed target experiments

There are some interesting theoretical issues involving polarized cross sections

\(^{a}\)In ref. \(^3\) the ratio was weighted by the branching ratios to \( J/\psi \).

\(^{b}\)The value quoted in the first reference in \(^{14}\) was not weighted by branching fractions of \( \chi_J \) into \( J/\psi \) contrary to what is stated in the text.

\(^{c}\)data points in the plot for figure 2 in ref. \(^3\) are off set in \( \sqrt{s} \) due to an error in the plotting routine.
Fig. 2. $J/\psi$ production cross sections in pion-nucleon collisions for $x_F > 0$. Direct $J/\psi$ production in the CSM (dashed line) and after inclusion of color-octet processes (dotted line). The total cross section (solid line) includes radiative feed-down from the $\chi_{cJ}$ and $\psi'$ states. The solid line is obtained with $\Delta_8(1S) = 3.0 \cdot 10^{-2} \text{GeV}^3$.

within the NRQCD formalism. However, due to space limitations, they will not be discussed here, and I refer the reader to refs. 18,3,19 for discussions. As we will see polarized production is a useful tool for investigating the octet mechanism.

Polarization measurements have been performed for both $\psi$ and $\psi'$ production in pion scattering fixed target experiments. Both experiments observe an essentially flat angular distribution in the decay $\psi \rightarrow \mu^+\mu^- \ (\psi = J/\psi, \psi')$,

$$\frac{d\sigma}{d\cos \theta} \propto 1 + \alpha \cos^2 \theta,$$

where the angle $\theta$ is defined as the angle between the three-momentum vector of the positively charged muon and the beam axis in the rest frame of the quarkonium. The observed values for $\alpha$ are $0.02 \pm 0.14$ for $\psi'$, measured at $\sqrt{s} = 21.8 \text{ GeV}$ in the region $x_F > 0.25$ and $0.028 \pm 0.004$ for $J/\psi$ measured at $\sqrt{s} = 15.3 \text{ GeV}$ in the region $x_F > 0$. In the CSM, the $J/\psi'$s are predicted to be significantly transversely polarized 6, in conflict with experiment.

The polarization yield of color octet processes can be calculated along the lines of the previous subsection. We first concentrate on $\psi'$ production and define $\xi$ as the fraction of longitudinally polarized $\psi'$. It is related to $\alpha$ by

$$\alpha = \frac{1 - 3\xi}{1 + \xi}.$$
For the different intermediate quark-antiquark states we find the following ratios of longitudinal to transverse quarkonia:

\[
\begin{align*}
3S_1^{(1)} & : 3.35 & \xi = 0.23 \\
1S_0^{(8)} & : 2 & \xi = 1/3 \\
3P_J^{(8)} & : 6 & \xi = 1/7 \\
3S_1^{(8)} & : 1 & \xi = 0
\end{align*}
\] (8)

where the number for the singlet process (first line) has been taken from \(^6\). Let us add the following remarks:

(i) The \(3S_1^{(8)}\)-subprocess yields pure transverse polarization. Its contribution to the total polarization is not large, because gluon-gluon fusion dominates the total rate.

(ii) For the \(3P_J^{(8)}\)-subprocess \(J\) is not specified, because interference between intermediate states with different \(J\) could occur as discussed in the previous subsection. As it turns out, interference does in fact not occur at leading order in \(\alpha_s\), because the only non-vanishing short-distance amplitudes in the \(JJ_z\) basis are \(00, 22\) and \(2(-2)\), which do not interfere.

(iii) The \(1S_0^{(8)}\)-subprocess yields unpolarized quarkonia. This follows from the fact that the NRQCD matrix element is

\[
\langle 0 | \chi^T \psi a_{\psi}^{(\lambda)} a_{\psi}^{(\lambda)} \chi^T | 0 \rangle = \frac{1}{3} \langle O_8^{\psi'} (1S_0) \rangle,
\] (9)

independent of the helicity state \(\lambda\). At this point, we differ from \(^7\), who assume that this channel results in pure transverse polarization, because the gluon in the chromomagnetic dipole transition \(1S_0^{(8)} \rightarrow 3S_1^{(8)} + g\) is assumed to be transverse. However, one should keep in mind that the soft gluon is off-shell and interacts with other partons with unit probability prior to hadronization. The NRQCD formalism applies only to inclusive quarkonium production. Eq. (9) then follows from rotational invariance.

(iv) Since the \(3P_J^{(8)}\) and \(1S_0^{(8)}\)-subprocesses give different longitudinal polarization fractions, the \(\psi'\) polarization depends on a combination of the matrix elements \(\langle O_8^{\psi'} (1S_0) \rangle\) and \(\langle O_8^{\psi'} (3P_0) \rangle\) which is different from \(\Delta_8(\psi')\).

To obtain the total polarization the various subprocesses have to be weighted by their partial cross sections. We define

\[
\delta_8(H) = \frac{\langle O_8^{H} (1S_0) \rangle}{\Delta_8(H)}
\] (10)

and obtain

\(^c\)This number is \(x_F\)-dependent and we have approximated it by a constant at low \(x_F\), where the bulk data is obtained from. The polarization fractions for the octet \(2 \rightarrow 2\) parton processes are \(x_F\)-independent.
\[ \xi = 0.23 \frac{\sigma_{\psi'}(3S_1^{(1)})}{\sigma_{\psi'}} + \left[ \frac{1}{3} \delta_8(\psi') + \frac{1}{7} (1 - \delta_8(\psi')) \right] \frac{\sigma_{\psi'}(1S_0^{(8)} + 3P_J^{(8)})}{\sigma_{\psi'}} \]

\[ = 0.16 + 0.11 \delta_8(\psi'), \]  

(11)

where the last line holds at \( \sqrt{s} = 21.8 \text{ GeV} \) (The energy dependence is mild and the above formula can be used with little error even at \( \sqrt{s} = 40 \text{ GeV} \)). Since \( 0 < \delta_8(H) < 1 \), we have \( 0.16 < \xi < 0.27 \) and therefore

\[ 0.15 < \alpha < 0.44. \]  

(12)

In quoting this range we do not attempt an estimate of \( \delta_8(\psi') \). Note that taking the Tevatron and fixed target extractions of certain (and different) combinations of \( \langle O_{\psi'}(1S_0) \rangle \) and \( \langle O_{\psi'}(3P_0) \rangle \) seriously (see Sect. 5.1), a large value of \( \delta_8(\psi') \) and therefore low \( \alpha \) would be favored. Within large errors, such a scenario could be considered consistent with the measurement quoted earlier. From a theoretical point of view, however, the numerical violation of velocity counting rules implied by this scenario would be rather disturbing.

In contrast, the more accurate measurement of polarization for \( J/\psi \) leads to a clear discrepancy with theory. In this case, we have to incorporate the polarization inherited from decays of the higher charmonium states \( \chi_{cJ} \) and \( \psi' \). This task is simplified by observing that the contribution from \( \chi_{c0} \) and \( \chi_{c1} \) feed-down is (theoretically) small as is the octet contribution to the \( \chi_{c2} \) production cross section. On the other hand, the gluon-gluon fusion process produces \( \chi_{c2} \) states only in a helicity \( \pm 2 \) level, so that the \( J/\psi \) in the subsequent radiative decay is completely transversely polarized. Weighting all subprocesses by their partial cross section and neglecting the small \( \psi' \) feed-down, we arrive at

\[ \xi = 0.10 + 0.11 \delta_8(J/\psi) \]  

(13)

at \( \sqrt{s} = 15.3 \text{ GeV} \), again with mild energy dependence. This translates into sizeable transverse polarization

\[ 0.31 < \alpha < 0.63. \]  

(14)

The discrepancy with data could be ameliorated if the observed number of \( \chi_{c1} \) from feed-down were used instead of the theoretical value. However, we do not know the polarization yield of whatever mechanism is responsible for copious \( \chi_{c1} \) production.

Thus, color octet mechanisms do not help to solve the polarization problem and one has to invoke a significant higher-twist contribution as discussed in \(^6\). To our knowledge, no specific mechanism has yet been proposed that would yield predominantly longitudinally polarized \( \psi' \) and \( J/\psi \) in the low \( x_F \) region which dominates the total production cross section. One might speculate that both the low \( \chi_{c1}/\chi_{c2} \) ratio and the large transverse polarization follow from the assumption
of transverse gluons in the gluon-gluon fusion process, as inherent to the leading-twist approximation. If gluons in the proton and pion have large intrinsic transverse momentum, as suggested by the $p_t$-spectrum in open charm production, one would be naturally led to higher-twist effects that obviate the helicity constraint on on-shell gluons.

5. What Needs to Be Done

The uncertainties in the theoretical prediction at fixed target energies are substantial and preclude a straightforward test of universality of color octet matrix elements by comparison with quarkonium production at large transverse momentum. Small-$x$, as well as kinematic effects, could bias the extraction of these matrix elements in different directions at fixed target and collider energies. The large uncertainties involved, especially due to the charm quark mass, could hardly be eliminated by a laborious calculation of $\alpha_s$-corrections to the production processes considered here. To more firmly establish existence of the octet mechanism there are several experimental measurements which need to be performed. Data on polarization is presently only available for charmonium production in pion-induced collisions. A polarization measurement for a proton beam would be very interesting given that we seem to have a better handle on the theory in this case, as is demonstrated by the observed value of the $\chi_1/\chi_2$ ratio. A measurement of polarization at large transverse momentum or for bottomonium is of crucial importance, because higher twist effects should be suppressed. Furthermore, a measurement of direct and indirect production fractions in the bottom system would provide further confirmation of the color octet picture and constrain the color octet matrix elements for bottomonium.

From a theoretical standpoint there are still several issues that warrant further investigation. To begin with, factorization in hadro-production of quarkonia is presently just a working hypothesis. This is true at large $p_T$ as well as at small $p_T$. Of course while formally this puts both calculation on the same theoretical footing; practically, there is still an important difference between the two cases. Namely, the size of the higher twist effects will be suppressed by $1/p_T^2$ at large $p_T$, as opposed to $1/(4m_c^2)$, for our fixed target cross section. It is furthermore possible, and this again applies to the case of large $p_T$ as well, that the higher twist effects could be enhanced by powers of $1/v$. Indeed, understanding higher twist effects in quarkonium production is complicated by the presence of the scales $mv$ and $mv^2$. Needless to say, there is still much work to be done on this subject before we can get a good handle on the errors due to higher twist effects.

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References

2. See G. Bodwin's talk in these proceedings.
14. See note added in ref. 3.
    NORDITA report NORDITA-96/18 P [hep-ph/9603266].
20. See K. Hagans' talk in these proceedings for the most recent data from E771. Also see,