DAMPING RATE OF QUASIPARTICLES IN DEGENERATE ULTRARELATIVISTIC PLASMAS

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Abstract

We compute the damping rate of a fermion in a dense relativistic plasma at zero temperature. Just above the Fermi sea, the damping rate is dominated by the exchange of soft magnetic photons (or gluons in QCD) and is proportional to \((E - \mu)\), where \(E\) is the fermion energy and \(\mu\) the chemical potential. We also compute the contribution of soft electric photons and of hard photons. As in the nonrelativistic case, the contribution of longitudinal photons is proportional to \((E - \mu)^2\), and is thus non leading in the relativistic case.

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The properties of quasiparticles in ultrarelativistic (UR) plasmas have attracted much attention in a recent past [1]. A crucial property of a quasiparticle is its decay (or damping) rate: a quasiparticle which propagates in a plasma is not stable, as it undergoes collisions with the other particles of the plasma, and the very concept of a quasiparticle makes sense only if its damping rate is small enough.

The damping rate of an electron propagating in a nonrelativistic (NR) plasma was computed almost forty years ago by Quinn and Ferrell [2], [3]. At first sight, this damping rate is infinite, due to the singular behavior of the Rutherford cross-section at small angles. However Quinn and Ferrell were able to obtain a finite result because the Coulomb interaction is screened in a plasma (Debye screening), and in the case of a degenerate plasma, they showed that the damping rate is proportional to \((\varepsilon_p - \varepsilon_F)^2\), where \(\varepsilon_p = p^2/2m\) is the NR kinetic energy and \(\varepsilon_F\) the Fermi energy, when \(\varepsilon_p\) is slightly larger than \(\varepsilon_F\). The damping rate remains finite for a non zero temperature \(T\): only the value of the Debye screening length is modified.

It is interesting to extend the calculation of the damping rate to the case of relativistic plasmas: one may have in mind either electromagnetic (QED) plasmas, such as found in white dwarves or in the core of nascent neutron stars, or chromodynamic (QCD) plasmas such as the quark-gluon plasma which is believed to be formed for large enough values of the temperature \(T\) and/or the chemical potential \(\mu\). The NR results are not easily transposed to the relativistic case, because the exchange of magnetic (or transverse) photons in QED or of magnetic gluons in QCD becomes important, while in the NR case it is suppressed by powers of \((v/c)^2\) with respect to the exchange of electric (or longitudinal) gauge bosons, and is usually neglected. These magnetic photons, or gluons, give rise to severe infrared (IR) divergences which are not easily cured, because there is no static magnetic screening analogous to Debye screening in the electric case, but only a weaker dynamical screening [4]. In many cases, this dynamical screening is sufficient to remove the IR divergences [5], [6] but it has been known for some time that it cannot solve easily the IR problem of the damping rate [7], at least for non zero \(T\). In a recent paper [8], Blaizot and Iancu were nevertheless able to derive a finite result in the \(T \neq 0, \mu = 0\) case, by using a non perturbative approach to resum the leading divergencies. However they also discovered that the decay law is no longer exponential.

In this Letter, we address the problem of computing the damping rate of quasiparticles in degenerate UR plasmas. For the sake of definiteness, we treat the case of a QED plasma,
but our results may be trivially extended to the QCD case by substituting to the QED coupling \( e \) the QCD coupling \( g \), and by taking into account some color group factors. In this computation, the basic physical idea is that the collisions of the charged quasiparticle with the particles in the plasma are governed by photon exchange, and that one must take into account the fact that the photon propagator is dressed by the interactions. Actually, this approach is a particular case of the resummation method proposed by Braaten and Pisarski [9], which relies on the properties of the so-called “hard thermal loops” [9], [10] or “hard dense loops” in the degenerate case [11]. Braaten and Pisarski pointed out the importance of a hierarchy of scales, based on the existence of a “hard scale” of order \( T \) (or \( \mu \)), and a “soft scale” of order \( eT \) (or \( e\mu \)), with \( e \ll 1 \). When soft scales are involved, one must use dressed (or resummed) propagators and vertices instead of the bare ones in a perturbative expansion. An important feature of the resummation method is that it leads to gauge independent results, due to the gauge independence of the hard thermal (or dense) loops.

Our main result is that in the case \( T = 0, \mu \neq 0 \), dynamical screening is able to cure the IR divergences of the damping rate due to magnetic photon exchange in UR plasmas; however, in contrast to the NR case, the damping rate is dominated by magnetic exchange and is proportional to \( (E - \mu) \), where \( E \) is the relativistic energy of the quasiparticle, while electric photon exchange gives a contribution proportional to \( (E - \mu)^2 \), as in the NR case, which may be in fact obtained as a low velocity limit of the relativistic one [12]. Note, however, that by convention energies and chemical potentials differ by the rest mass of the particle in the NR and relativistic cases. Note also that we use a system of units where \( \hbar = c = k_B = 1 \), and that we follow closely the notations of [1].

Let us now proceed to the derivation of our result. We assume that the quasiparticle energy \( E \) is hard (this is automatically ensured in the case of a degenerate plasma). The damping rate \( \gamma(E) \) is given by the imaginary part of the quasiparticle self-energy \( \Sigma \) [14]; more precisely

\[
\gamma(E) = -\frac{1}{4E} \text{Tr} \left[ \text{Im} \left( p_0 + i\eta \mathbf{p} \right) \mathbf{P} + m \right] \bigg|_{p_0 = E},
\]

where \( m \) is the electron mass, \( E = (\mathbf{p}^2 + m^2)^{1/2} \), \( \eta \to 0^+ \), and we have used the by now standard notation: \( P_\mu = (p_0, \mathbf{p}) \); the lowest order graph for \( \Sigma \) is drawn in Fig. 1. We are mainly interested in the contribution of soft photons, so that the electron-photon vertex and the electron propagator may be replaced by the bare ones [1], [9]: only the photon
propagator need be dressed.

We perform the calculation of $\Sigma$ in the imaginary time formalism; then the (free) electron propagator is given by

$$S_f(i\omega_n, k) = \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \frac{(K + m)\rho_f(K)}{k_0 - i\omega_n - \mu}, \quad (2)$$

with

$$\rho_f(K) = 2\pi \varepsilon(k_0) \delta(k_0^2 - E_k^2) . \quad (3)$$

In (2), $\omega_n = \pi(2n + 1)T$ is a fermionic Matsubara frequency and $\varepsilon(k_0) = k_0/|k_0|$. The (resummed) photon propagator $\Delta_{\mu\nu}(Q)$ is written in the Coulomb gauge

$$\Delta_{\mu\nu}(Q) = \delta_{\mu0}\delta_{\nu0} \Delta_L(Q) + (\delta_{ij} - \hat{q}_i\hat{q}_j) \Delta_T(Q) , \quad (4)$$

where the spectral representations of $\Delta_T$ and $\Delta_L$ read

$$\Delta_L(i\omega_s, q) = \int_{-\infty}^{\infty} \frac{dq_0}{2\pi} \rho_L(q_0, q) - \frac{1}{q^2} , \quad (5a)$$

$$\Delta_T(i\omega_s, q) = \int_{-\infty}^{\infty} \frac{dq_0}{2\pi} \rho_T(q_0, q) . \quad (5b)$$

In (4) and (5), $\hat{q}_i = q_i/|q|$ and $\omega_s = 2\pi sT$ is a bosonic Matsubara frequency. The explicit expressions of the longitudinal and transverse spectral functions $\rho_L$ and $\rho_T$ are found by taking the imaginary parts of $\Delta_L$ and $\Delta_T$

$$\rho_L(q_0, q) = 2\text{Im}\Delta_L(q_0 + i\eta, q) = 2\text{Im} \frac{-1}{q^2 + 3\omega_P^2 \left(1 - \frac{x}{2} \ln \frac{x+1}{x-1}\right)} , \quad (6a)$$

$$\rho_T(q_0, q) = 2\text{Im}\Delta_T(q_0 + i\eta, q) = 2\text{Im} \frac{-1}{(q_0 + i\eta)^2 - q^2 - \frac{3}{2} \omega_P^2 \left(x^2 + \frac{x(1-x^2)}{2} \ln \frac{x+1}{x-1}\right)} , \quad (6b)$$

where $x = (q_0 + i\eta)/q$, $\omega_P = M/\sqrt{3}$ is the plasma frequency which is related to the Debye mass $M$ given by

$$M^2 = \frac{e^2}{\pi^2} \left(\mu^2 + \frac{\pi^2 T^2}{3}\right) . \quad (7)$$

The diagram in Fig. 1 is now evaluated in the imaginary time formalism $(P = (i\omega_n, p))$

$$\Sigma(P) = e^2 T \sum_s \int \frac{d^3q}{(2\pi)^3} \gamma_\mu S_f(i(\omega_n - \omega_s), p - q)\gamma_\nu \Delta_{\mu\nu}(i\omega_s, q) . \quad (8)$$

The sum over Matsubara frequencies is easily performed when one plugs in (8) the spectral representations (2) and (6) of the propagators. Taking the imaginary part of $\Sigma$
after the analytical continuation $i \omega_n + \mu \to p_0 + i \eta$ to Minkowski space, and taking the trace in (1), one finds for the damping rate, with $k = p - q,$

\[
\gamma(E) = \frac{\pi e^2}{E} \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \rho_f(k_0) \int_{-\infty}^{\infty} \frac{dq_0}{2\pi} \times (1 + n(q_0) - \tilde{n}(k_0)) \delta(E - k_0 - q_0) \left\{ [p_0k_0 + p \cdot k + m^2] \right.
\]

\[
\times \rho_L(q_0, q) + 2[p_0k_0 - (p \cdot \hat{q})(k \cdot \hat{q}) - m^2] \rho_T(q_0, q) \right\} .
\]

In (9), $n$ and $\tilde{n}$ are Bose-Einstein and Fermi-Dirac distribution functions ($\beta = 1/T$)

\[
n(q_0) = \frac{1}{e^{\beta q_0} - 1}, \quad \tilde{n}(k_0) = \frac{1}{e^{\beta(k_0 - \mu)} + 1} .
\]

Eq. (9) could have also been derived from kinetic theory [15], using standard identities between the distribution functions (10). The case $k_0 < 0$ (see (3)) corresponds in kinetic theory to $e^+ - e^-$ annihilation, which is not IR singular and is even absent in the $T = 0$ case. We thus concentrate on the $k_0 > 0$ case, which corresponds in kinetic theory to $e^- - e^-$ scattering. From now on, we shall also restrict ourselves to the $m = 0$ and $T = 0$ case, leaving the general case to a forthcoming publication by one of the authors [12]. In the $T = 0$ limit, $(1 + n(q_0)) = \Theta(q_0)$ and $\tilde{n}(k_0) = \Theta(\mu - E + q_0)$, where $\Theta$ is the step function. The $q_0$ integration is then limited by

\[
0 \leq q_0 \leq E - \mu .
\]

This is also easily seen in kinetic theory, since, due to Pauli blocking, the quasiparticle can only scatter into states with energy $E_{|p-q|}$ such that $E_{|p-q|} \leq E$ and furthermore the particle on which it scatters must leave the Fermi sea, so that $E_{|p-q|} \geq \mu$. Note also that the exchanged photon must be space-like: $Q^2 < 0,$ so that the pole part of $\rho_{L,T}$ [1] does not contribute.

Now, the IR singular contribution comes from small values of the photon momentum $q$; in order to isolate this kinematical region, we follow Braaten ans Yuan [13] and introduce an intermediate cut-off $q^*$ such that $e\mu \ll q^* \ll \mu$. The “soft” region is defined by $q < q^*$, the “hard” one by $q > q^*$: in this latter region we may take the $M^2 = 0$ limit in the denominators of the spectral functions $\rho_{L,T}$ in (9) [1]. Let us concentrate on the soft region, where we can make the approximation

\[
E_{|p-q|} = E - q_0 \approx E - \hat{p} \cdot q .
\]
Keeping only the leading terms in (9), we find the contribution from the soft region to \( \gamma(E) \)

\[
\gamma_{\text{soft}}(E) \simeq \frac{e^2}{2} \int \frac{d^3q}{(2\pi)^3} \left( \Theta(q_0) - \Theta(\mu - E + q_0) \right) \Theta(q^* - q) \\
\times \left\{ \rho_L(q_0, q) + (1 - \cos^2 \theta)\rho_T(q_0, q) \right\},
\]

(13)

with \( q_0 = \hat{p} \cdot \mathbf{q} = q \cos \theta \). It is convenient to use as integration variables \( q_0 \) and \( q \), the integration domain \( D \) being

\[
D: \{0 \leq q_0 \leq E - \mu; \ q_0 \leq q \leq q^* \}.
\]

(14)

Then (13) becomes \((x = q_0/q)\)

\[
\gamma_{\text{soft}}(E) \simeq \frac{e^2 M^2}{4\pi} \int_D dq_0 \frac{q_0}{2 \left[ q^2 + M^2 Q_1(x) \right]^2 + \frac{M^4 \pi^2 x^2}{2}} \\
+ \frac{q_0}{\left[ 2 q^2 + M^2 Q_2(x) \right]^2 + \frac{M^4 \pi^2 x^2}{4}},
\]

(15)

where

\[
Q_1(x) = 1 - \frac{x}{2} \ln \left( \frac{1 + x}{1 - x} \right), \quad Q_2(x) = -Q_1(x) + \frac{1}{1 - x^2}.
\]

(16)

Note that in the absence of screening (namely, by setting \( M = 0 \) in the denominators of (15)), one would get IR divergent integrals. In general, the integrals in (15) must be computed numerically. Fortunately, it is possible to derive an accurate analytical result in the physically interesting case \((E - \mu) \ll M\). Indeed, it is easy to check that in this region one may expand the denominators in (15) in powers of \( q_0 \). Keeping only the leading terms, the first denominator in (15), corresponding to longitudinal photon exchange, may be replaced by \((q^2 + M^2)\), which leads to Debye screening. The second denominator in (15), corresponding to transverse photon exchange, may be replaced by

\[
4 q^4 + \frac{\pi^2 M^4 x^2}{4} + 8 M^2 q^2 x^2.
\]

(17)

It can be shown that the last term in (17) gives a subdominant contribution [12], while the second term leads to the usual form of dynamical screening [4] - [6]. Computing separately the longitudinal and transverse contributions, we find, with \( u^* = (q^*/M)^2 \),

\[
\gamma_{\text{soft}}^L(E) \simeq \frac{e^2 (E - \mu)^2}{32\pi M} \int_0^{u^*} \frac{du}{\sqrt{u}(u + 1)^2} \simeq \frac{e^2 M^2}{16\pi} (E - \mu)^2 \left( -\frac{\pi}{4M^3} - \frac{1}{3q^*^3} \right),
\]

(18)

\[
\gamma_{\text{soft}}^T(E) \simeq \frac{e^2 M}{16\pi^3} \int_0^{u^*} du \sqrt{u} \ln \left( 1 + \frac{\pi^2 (E - \mu)^2}{16 M^2 u^3} \right) \\
\simeq \frac{e^2}{24\pi} (E - \mu) + \frac{e^2 M^2}{32\pi} (E - \mu)^2 \left( -\frac{1}{3q^*^3} \right),
\]

(19)
where we have only kept the leading terms in \((E - \mu)\) and \(1/q^*\).

The total contribution of the soft region to the decay rate is obtained by adding the longitudinal and transverse contributions to get

\[
\gamma_{\text{soft}}(E) \simeq \frac{e^2}{24\pi} (E - \mu) + \frac{e^2 M^2}{32\pi} (E - \mu)^2 \left( \frac{\pi}{2M^3} - \frac{1}{q^*^3} \right). \tag{20}
\]

The transverse contribution dominates over the longitudinal one for small values of \((E - \mu)\).

We finally evaluate the contribution from the hard region. Since we are only interested in extracting the leading dependence in the fermionic energy of the decay rate we will use a simple approach to compute the hard contribution. It is possible to recover bare or unresummed perturbation theory to order \(e^4\) by using the spectral densities (6) neglecting \(M^2\) in the denominators [1]. This is only valid in the momentum transfer region \(q > q^*\). Therefore one finds for the hard contribution to the decay rate

\[
\gamma_{\text{hard}}(E) \simeq \frac{e^2 M^2}{8\pi} \int_{q^*}^{q_{\text{max}}} dq \int_{0}^{E-\mu} dq_0 \left\{ \frac{q_0}{q^4} + \frac{q_0}{2q^4} \right\}. \tag{21}
\]

After a straightforward computation one finds

\[
\gamma_{\text{hard}}(E) \simeq \frac{e^2 M^2}{32\pi} (E - \mu)^2 \left( \frac{1}{q^*^3} - \frac{1}{q_{\text{max}}^3} \right), \tag{22}
\]

where \(q_{\text{max}} \simeq \mu\) is the maximum momentum transfer that it is allowed by kinematics.

The total decay rate is found just by adding the soft and hard contributions. Then one finds that the dependence on the scale \(q^*\) cancels, as it should. The result is

\[
\gamma(E) \simeq \frac{e^2}{24\pi} (E - \mu) + \frac{e^2 M^2}{32\pi} (E - \mu)^2 \left( \frac{\pi}{2M^3} - \frac{1}{q_{\text{max}}^3} \right). \tag{23}
\]

In conclusion, we have been able to compute the damping rate of a quasiparticle in a degenerate ultrarelativistic plasma, when the fermion energy \(E\) is just above the Fermi energy \(\mu\). This damping rate is dominated by transverse photon (or gluon) exchange and proportional to \((E - \mu)\). This behavior arises from the combined effect of dynamical screening and phase space restrictions due to Pauli blocking. The lifetime \(\tau\) is related to \(\gamma\) by \(\tau \sim 1/\gamma\), and therefore the lifetime becomes infinite as the fermion energy approaches the Fermi energy, so that the Fermi sea is stable.
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FIG. 1. Resummed one-loop self-energy of the fermion.