CP Violation in B-Decays:  
The Standard Model and Beyond

Michael Gronau  
Department of Physics  
Technion - Israel Institute of Technology, 32000 Haifa, Israel

Abstract

We review the subject of CP violation in $B$ decays in the Standard Model (SM) and beyond the SM. We describe some of the present most promising ways of testing the Cabibbo-Kobayashi-Maskawa (CKM) origin of CP violation through a determination of the three angles of the CKM matrix unitarity triangle. Other sources of CP nonconservation violate SM constraints on the unitarity triangle. We show that different models of physics beyond the SM can be distinguished by combining their effects in CP asymmetries and in rare flavor-changing $B$ decays.

\footnote{Invited talk given at Beauty 96, Rome, June 17–21, 1996, to appear in the proceedings}
1 Introduction

One of the remaining goals in the present era of particle physics is to study the origin of CP violation. In the Standard Model (SM), CP nonconservation is due to a nonzero complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V$, describing the interaction of the three families of left-handed quarks with the charged gauge boson. This unitary matrix can be approximated by the following two useful forms [1]:

$$V \approx \begin{pmatrix}
1 - \frac{1}{2}s_{12}^2 & s_{12} & s_{13}e^{-i\gamma} \\
-\overline{s_{12}} & 1 - \frac{1}{2}s_{12}^2 & s_{23} \\
s_{12}\overline{s_{23}} - s_{13}e^{i\gamma} & -s_{23} & 1
\end{pmatrix} \approx \begin{pmatrix}
1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^2(\rho - i\eta) \\
-\overline{\lambda} & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\
A\lambda^2(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}. \quad (1)

The measured values of the three Euler-like mixing angles, $\theta_{ij}$, and the phase $\gamma$ are given by [2]:

$$s_{12} \equiv \sin \theta_{12} \approx |V_{us}| = 0.220 \pm 0.002,$$
$$s_{23} \equiv \sin \theta_{23} \approx |V_{cb}| = 0.039 \pm 0.003,$$
$$s_{13} \equiv \sin \theta_{13} \equiv |V_{ub}| = 0.0031 \pm 0.0009,$$
$$35^0 \leq \gamma \equiv \text{Arg}(V_{ub}^*) \leq 145^0. \quad (2)$$

The only information about a nonzero value of $\gamma$ comes from CP violation in the $K^0 - \overline{K}$ system.

Unitarity of $V$ implies triangle relations such as

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \quad (3)$$

which is shown in Fig. 1. The angle $\alpha$ of the unitarity triangle has rather crude bounds, qualitatively similar to those of $\gamma$,

$$20^0 \leq \alpha \leq 120^0, \quad (4)$$

whereas $\beta$ is somewhat better determined

$$10^0 \leq \beta \leq 35^0. \quad (5)$$
In addition to the constraints on $\alpha$, $\beta$ and $\gamma$, pairs of these angles are correlated. Due to the rather limited range of $\beta$, the angles $\alpha$ and $\gamma$ are almost linearly correlated through $\alpha + \gamma = \pi - \beta$ [3]. A special correlation exists also between small values of $\sin 2\beta$ and large values of $\sin 2\alpha$ [4].

A precise determination of the three angles $\alpha$, $\beta$ and $\gamma$, which would provide a test of the CKM origin of CP violation, depends crucially on measuring CP asymmetries in $B$ decays. This will be one of the central subjects of this paper. Since much of this material was reviewed in last year Beauty 95, I will be quite brief on most topics. The reader is referred to ref. [5] for details and further references. I will rather concentrate on new methods which were developed since last year. The second subject of this paper will be manifestations of physics beyond the standard model in CP asymmetries, on the one hand, and in rare flavor-changing $B$-decays, on the other hand.

Section 2 is a quick run through the by-now standard methods of measuring $\alpha$, $\beta$ and $\gamma$ in neutral and charged $B$ decays. The role of isospin symmetry in resolving “penguin pollution” is briefly discussed. Section 3 studies flavor SU(3) and first-order SU(3) breaking in $B$ decays to two pseudoscalar mesons. The power of this analysis is demonstrated by an example in which both $\alpha$ and $\gamma$ can be measured in $B$ decays to kaons and charged pions. The use of $\eta$ and $\eta'$ in final states is briefly discussed. Section 4 reviews CP asymmetries beyond the SM, while Section 5 concludes.

2 ”Standard” methods of measuring $\alpha$, $\beta$, $\gamma$

2.1 Neutral $B$ Decays to CP-eigenstates

The most frequently discussed method of measuring weak phases is based on neutral $B$ decays to final states $f$ which are common to $B^0$ and $\bar{B}^0$. CP violation is induced by $B^0 - \bar{B}^0$ mixing through the interference of the two amplitudes $B^0 \to f$ and $B^0 \to \bar{B}^0 \to f$. When $f$ is a CP-eigenstate, and when a single weak amplitude (or rather a single weak phase) dominates the decay process, the time-dependent asymmetry

$$A(t) \equiv \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)}$$

(6)
obtains the simple form [6]
\[ \mathcal{A}(t) = \xi \sin 2(\phi_M + \phi_f) \sin(\Delta m t) . \] (7)
\( \xi \) is the CP eigenvalue of \( f \), \( 2\phi_M \) is the phase of \( B^0 - \overline{B}^0 \) mixing, \( (\phi_M = \beta, 0 \) for \( B_d^0, B_s^0 \), respectively), \( \phi_f \) is the weak phase of the \( B^0 \to f \) amplitude, and \( \Delta m \) is the neutral \( B \) mass-difference.

The two very familiar examples are:
(i) \( B_d^0 \to \psi K_S \), where \( \xi = -1, \phi_f = \text{Arg}(V_{cb}^* V_{cs}) = 0, \)
\[ \mathcal{A}(t) = -\sin 2\beta \sin(\Delta m t) , \] (8)
and
(ii) \( B_d^0 \to \pi^+ \pi^- \), where \( \xi = 1, \phi_f = \text{Arg}(V_{ub}^* V_{ud}) = \gamma, \)
\[ \mathcal{A}(t) = -\sin 2\alpha \sin(\Delta m t) . \] (9)
Thus, the two asymmetries measure the angles \( \beta \) and \( \alpha \).

2.2 Decays to other states

A similar method can also be applied to measure weak phases when \( f \) is a common decay mode of \( B^0 \) and \( \overline{B}^0 \), but not necessarily a CP eigenstate. In this case one measures four different time-dependent decay rates, \( \Gamma_f(t), \Gamma_f(t), \Gamma_f(t), \Gamma_f(t) \), corresponding to initial \( B^0 \) and \( \overline{B}^0 \) decaying to \( f \) and its charge-conjugate \( \overline{f} \) [7]. The four rates depend on four unknown quantities, \( |A|, |\overline{A}|, \sin(\Delta \delta_f + \Delta \phi_f + 2\phi_M), \sin(\Delta \delta_f - \Delta \phi_f - 2\phi_M) \). \( A \) and \( \overline{A} \) are the decay amplitudes of \( B^0 \) and \( \overline{B}^0 \) to \( f \), \( \Delta \delta_f \) and \( \Delta \phi_f \) are the the strong and weak phase-differences between these amplitudes. Thus, the four rate measurements allow a determination of the weak CKM phase \( \Delta \phi_f + 2\phi_M \). This method can be applied to measure \( \alpha \) in \( B_d^0 \to \rho^+ \pi^- \), and to measure \( \gamma \) in \( B_s^0 \to D_s^+ K^- \) [8]. Other ways of measuring \( \gamma \) in \( B_s^0 \) decays were discussed recently in Ref. [9].

2.3 “Penguin pollution”

All this assumes that a single weak phase dominates the decay \( B^0(\overline{B}^0) \to f \). As a matter of fact, in a variety of decay processes, such as in \( B_d^0 \to \pi^+ \pi^- \),
there exists a second amplitude due to a “penguin” diagram in addition
to the usual “tree” diagram [10]. As a result, CP is also violated in the
direct decay of a $B^0$, and one faces a problem of separating the two types of
asymmetries. This can only be partially achieved through the more general
time-dependence

$$A(t) = \frac{(1 - |A/A|^2) \cos(\Delta mt) - 2\text{Im}(e^{-2i\phi M A/A}) \sin(\Delta mt)}{1 + |A/A|^2}. \quad (10)$$

Here the $\cos(\Delta mt)$ term implies direct CP violation, and the coefficient of
$\sin(\Delta mt)$ obtains a correction from the penguin amplitude. The two terms
have a different dependence on $\Delta \delta$, the final-state phase-difference between
the tree and penguin amplitudes. The coefficient of $\cos(\Delta mt)$ is proportional
to $\sin(\Delta \delta)$, whereas the correction to the coefficient of $\sin(\Delta mt)$ is propor-
tional to $\cos(\Delta \delta)$. Thus, if $\Delta \delta$ were small, this correction might be large in
spite of the fact that the $\cos(\Delta mt)$ term were too small to be observed.

### 2.4 Isospin resolution of penguin pollution

The above “penguin pollution” may lead to dangerously large effects in
$B^0_d(t) \rightarrow \pi^+\pi^-$ decay, which would avoid a clean determination of $\alpha$ [11].
One way to remove this effect is by measuring also the (time-integrated)
rates of $B^0_d \rightarrow \pi^0\pi^0$, $B^+ \rightarrow \pi^+\pi^0$ and their charge-conjugates [12]. One
uses the different isospin properties of the penguin ($\Delta I = 1/2$) and tree
($\Delta I = 1/2, 3/2$) operators and the well-defined weak phase of the tree op-
erator. This enables one to determine the correction to $\sin 2\alpha$ in the second
term of Eq. (10). Electroweak penguin contributions could, in principle, spoil
this method, since unlike the QCD penguins they are not pure $\Delta I = 1/2$
[13]. These effects are, however, very small and consequently lead to a tiny
uncertainty in determining $\alpha$ [14]. The difficult part of this method seems
to be the necessary observation of the decay to two neutral pions which is
expected to be color-suppressed. Other methods of resolving the “penguin
pollution” in $B^0_d \rightarrow \pi^+\pi^-$, which do not rely on decays to neutral pions, will
be described in Sec. 3.
2.5 Measuring $\gamma$ in $B^\pm \rightarrow DK^\pm$

In $B^\pm \rightarrow DK^\pm$, where $D$ may be either a flavor state ($D^0$, $\bar{D}^0$) or a CP-eigenstate ($D^0_1$, $D^0_2$), one can measure separately the magnitudes of two interfering amplitudes leading to direct CP violation. This enables a measurement of $\gamma$, the relative weak phase between these two amplitudes [15]. This method is based on a simple quantum mechanical relation among the amplitudes of three different processes,

$$\sqrt{2}A(B^+ \rightarrow D^0_1 K^+) = A(B^+ \rightarrow D^0 K^+) + A(B^+ \rightarrow \bar{D}^0 K^+).$$

(11)

The CKM factors of the two terms on the right-hand-side, $V_{ub}^* V_{cs}$ and $V_{cb}^* V_{us}$, involve the weak phases $\gamma$ and zero, respectively. A similar triangle relation can be written for the charge-conjugate processes. Measurement of the rates of these six processes, two pairs of which are equal, enables a determination of $\gamma$. The present upper limit on the branching ratio of $B^+ \rightarrow D^0 K^+$ [16] is already very close to the value expected in the SM. The major difficulty of this method may be in measuring $B^+ \rightarrow D^0 K^+$ which is expected to be color-suppressed. For further details and a feasibility study see Ref. [17].

3 Flavor SU(3) and SU(3) breaking

3.1 The general formalism

One may use approximate flavor SU(3) symmetry of strong interactions to relate all two body processes of the type $B \rightarrow \pi\pi$, $B \rightarrow \pi K$ and $B \rightarrow K\bar{K}$. Since SU(3) is expected to be broken by effects of order 20%, such as in $f_K/f_\pi$, one must introduce SU(3) breaking terms in such an analysis. This approach has recently received special attention [18, 19, 20, 21, 22, 23]. In the present section we will discuss two applications of this analysis to a determination of weak phases. Early applications of SU(3) to two-body $B$ decays can be found in Ref. [24].

The weak Hamiltonian operators associated with the transitions $\bar{b} \rightarrow \pi uq$ and $\bar{b} \rightarrow \bar{q}$ ($q = d$ or $s$) transform as a $3^*$, $6$ and $15^*$ of SU(3). The $B$ mesons are in a triplet, and the symmetric product of two final state pseudoscalar octets in an S-wave contains a singlet, an octet and a 27-plet. Thus, all these processes can be described in terms of five SU(3) amplitudes: $\langle 1 \parallel 3^* \parallel 3 \rangle$, $\langle 8 \parallel 3^* \parallel 3 \rangle$, $\langle 8 \parallel 6 \parallel 3 \rangle$, $\langle 8 \parallel 15^* \parallel 3 \rangle$, $\langle 27 \parallel 15^* \parallel 3 \rangle$. 
An equivalent and considerably more convenient representation of these amplitudes is given in terms of an overcomplete set of six quark diagrams occurring in five different combinations. These diagrams are denoted by $T$ (tree), $C$ (color-suppressed), $P$ (QCD-penguin), $E$ (exchange), $A$ (annihilation) and $PA$ (penguin annihilation). The last three amplitudes, in which the spectator quark enters into the decay Hamiltonian, are expected to be suppressed by $f_B/m_B$ ($f_B \approx 180$ MeV) and may be neglected to a good approximation.

The presence of higher-order electroweak penguin contributions introduces no new SU(3) amplitudes, and in terms of quark graphs merely leads to a substitution [14]

$$T \to t = T + P_{ECW}^C, \quad C \to c = C + P_{EW}, \quad P \to p = P - \frac{1}{3} P_{ECW}^C,$$

where $P_{EW}$ and $P_{ECW}^C$ are color-favored and color-suppressed electroweak penguin amplitudes. $\Delta S = 0$ amplitudes are denoted by unprimed quantities and $|\Delta S| = 1$ processes by primed quantities. Corresponding ratios are given by ratios of CKM factors

$$\frac{T'}{T} = \frac{C'}{C} = \frac{V_{us}}{V_{ud}}, \quad \frac{P'}{P} = \frac{P_{EW}'}{P_{EW}} = \frac{V_{ts}}{V_{td}}.$$

$t$-dominance was assumed in the ratio $P'/P$. The effect of $u$ and $c$ quarks in penguin amplitudes can sometimes be important [25].

The expressions of all two body decays to two light pseudoscalars in the SU(3) limit are given in Table 1. The vanishing of three of the amplitudes, associated with $B^0 \to K^+K^-$, $B^0_s \to \pi^+\pi^-$, $B^0_s \to \pi^0\pi^0$, follows from the assumption of negligible exchange ($E$) amplitudes. This can be used to test our assumption which neglects final state rescattering effects.

First-order SU(3) breaking corrections can be introduced in a most general manner through parameters describing mass insertions in the above quark diagrams [26]. The interpretation of these corrections in terms of ratios of decay constants and form factors is model-dependent. There is, however, one case in which such interpretation is quite reliable. Consider the tree amplitudes $T$ and $T'$. In $T$ the $W$ turns into a $u\bar{d}$ pair, whereas in $T'$ it turns into $u\bar{s}$. One may assume factorization for $T$ and $T'$, which is supported by data on $B \to D\pi$ [27], and is justified for $B \to \pi\pi$ and
\[ B \rightarrow \pi K \] by the high momentum with which the two color-singlet mesons separate from one another. Thus,

\[ \frac{T'}{T} = \frac{V_{us} f_K}{V_{ud} f_\pi}. \tag{14} \]

Similar assumptions for \( C'/C \) and \( P'/P \) cannot be justified.

### 3.2 \( \alpha \) and \( \gamma \) from \( B \) decays to kaons and charged pions

Table 1 and Eq. (14) can be used to separate the penguin term from the tree amplitude in \( B^0_d \rightarrow \pi^+\pi^- \), and thereby determine simultaneously both the angles \( \alpha \) and \( \gamma \). In the present subsection we outline in a schematic way a method which, for a practical purpose, uses only final states with kaons and charged pions. The reader is referred to Ref. [28] for more details. A few alternative ways to learn the penguin effects in \( B^0_d \rightarrow \pi^+\pi^- \) were suggested in Ref. [29].

Consider the amplitudes of the three processes \( B^0_d \rightarrow \pi^+\pi^- \), \( B^0_d \rightarrow \pi^-K^+ \), \( B^+ \rightarrow \pi^+K^0 \) given by

\[ A(B^0_d \rightarrow \pi^+\pi^-) = -t - p, \quad A(B^0_d \rightarrow \pi^-K^+) = -t' - p', \quad A(B^+ \rightarrow \pi^+K^0) = p'. \tag{15} \]

One measures the time-dependence of the first process and the decay rates of the other two self-tagging modes. Using these measurements, and the ones corresponding to the charge-conjugate processes, one can form the following six measurables:

\[
\begin{align*}
&\Gamma(B^0_d(t) \rightarrow \pi^+\pi^-) + \Gamma(\overline{B}^0_d(t) \rightarrow \pi^+\pi^-) = Ae^{-t/\tau_B} \\
&\Gamma(B^0_d(t) \rightarrow \pi^+\pi^-) - \Gamma(\overline{B}^0_d(t) \rightarrow \pi^+\pi^-) = |B\cos(\Delta mt) + C\sin(\Delta mt)|e^{-t/\tau_B} \\
&\Gamma(B^0_d \rightarrow \pi^-K^+) + \Gamma(\overline{B}^0_d \rightarrow \pi^+K^-) = D \\
&\Gamma(B^0_d \rightarrow \pi^-K^+) - \Gamma(\overline{B}^0_d \rightarrow \pi^+K^-) = E \\
&\Gamma(B^+ \rightarrow \pi^+K^0) = \Gamma(B^- \rightarrow \pi^-K^0) = F
\end{align*}
\tag{16} \]

The six quantities \( A, B, \ldots, F \) can all be expressed in terms six parameters, consisting of the magnitudes and the weak and strong phases of the tree and penguin amplitudes. Let’s count these parameters:
• The $\Delta S = 0$ tree amplitude $T$ has magnitude $T$, weak phase $\gamma$ and strong phase $\delta_T$.

• The $\Delta S = 0$ penguin amplitude $P + (2/3)P_{EW}^C$ has magnitude $P$, weak phase $-\beta$ and strong phase $\delta_P$.

• The $\Delta S = 1$ tree amplitude $T'$ has magnitude $(V_{us}/V_{ud})(f_K/f_\pi)T$, weak phase $\gamma$ and strong phase $\delta_T$.

• The $\Delta S = 1$ penguin amplitude $P' + (2/3)P_{EW}^C$ has magnitude $P'$, weak phase $\pi$ and strong phase $\delta_P$. (Since the strong phase difference $\delta_T - \delta_P$ is expected to be small, we neglect SU(3) breaking effects in this phase).

Therefore, the six measurables $A, B, \ldots F$ can be expressed in terms of the six parameters $T, P, P', \delta \equiv \delta_T - \delta_P, \gamma$ and $\alpha \equiv \pi - \beta - \gamma$. This enables a determination of both $\alpha$ and $\gamma$, with some remaining discrete ambiguity associated with the size of final-state phases. A sample of events corresponding to about $100B_0^0 \to \pi^+\pi^-$, $100B_0^0 \to \pi^0K^\pm$ events and a somewhat smaller number of detected $B^\pm \to \pi^\pm K_S$ events is sufficient to reduce the presently allowed region in the $(\alpha, \gamma)$ plane by a considerable amount.

### 3.3 $\gamma$ from charged $B$ decays: the use of $\eta$ and $\eta'$

The use of $\eta$ and $\eta'$ allows a determination of $\gamma$ from decays involving charged $B$ decays alone [30]. When considering final states involving $\eta$ and $\eta'$ one encounters one additional penguin diagram (a so-called “vacuum cleaner” diagram), contributing to decays involving one or two flavor SU(3) singlet pseudoscalar mesons [31]. This amplitude ($P_1$) appears in a fixed combination with a higher-order electroweak penguin contribution in the form $p_1 \equiv P_1 - (1/3)P_{EW}$.

Writing the physical states in terms of the SU(3) singlet and octet states

$$\eta = \eta_s \cos \theta - \eta_1 \sin \theta, \quad \eta' = \eta_s \sin \theta + \eta_1 \cos \theta, \quad \sin \theta \approx \frac{1}{3}, \quad (17)$$

one finds the following expressions for the four possible $\Delta S = 1$ amplitudes of charged $B$ decays to two charmless pseudoscalars:

$$A(B^+ \to \pi^+K^0) = p', \quad A(B^+ \to \pi^0K^+) = \frac{1}{\sqrt{2}}(-p' - t' - c'),$$
\[ A(B^+ \rightarrow \eta K^+) = \frac{1}{\sqrt{3}}(-t' - c' - p'_1), \quad A(B^+ \rightarrow \eta' K^+) = \frac{1}{\sqrt{6}}(3p' + t' + c' + 4p'_1). \]  

These amplitudes satisfy a quadrangle relation

\[ \sqrt{6}A(B^+ \rightarrow \pi^+ K^0) + \sqrt{3}A(B^+ \rightarrow \pi^0 K^+) \]

\[ -2\sqrt{2}A(B^+ \rightarrow \eta K^+) - A(B^+ \rightarrow \eta' K^+) = 0. \]  

(19)

A similar quadrangle relation is obeyed by the charge-conjugate amplitudes, and the relative orientation of the two quadrangles holds information about weak phases. However, it is clear that each of the two quadrangles cannot be determined from its four sides given by the measured amplitudes. A closer look at the expressions of the amplitudes shows that the two quadrangles share a common base, \( A(B^+ \rightarrow \pi^+ K^0) = A(B^- \rightarrow \pi^- \bar{K}^0) \), and the two sides opposite to the base (involving \( \eta \)) intersect at a point lying 3/4 of the distance from one vertex to the other. This fixes the shapes of the quadrangles up to discrete ambiguities. Finally, the phase \( \gamma \) can be determined by relating these amplitudes to that of \( B^+ \rightarrow \pi^+ \pi^0 \)

\[ |A(B^+ \rightarrow \pi^0 K^+) - A(B^- \rightarrow \pi^0 K^-)| = 2\frac{V_{us}}{V_{ud}} \frac{f_K}{f_\pi} |A(B^+ \rightarrow \pi^+ \pi^0)| \sin \gamma. \]  

(20)

Note that in this method we neglect SU(3) breaking terms associated with the difference between creation of nonstrange and strange quark pairs in the final state of penguin amplitudes.

4 Beyond the standard model

4.1 CP asymmetries and the unitarity triangle

The above discussion assumes that the only source of CP violation is the phase of the CKM matrix. Models beyond the SM involve other phases, and consequently the measurements of CP asymmetries may violate SM constraints on the three angles of the unitarity triangle [32]. Furthermore, even in the absence of new CP violating phases, these angles may be affected by new contributions to the sides of the triangles. The three sides (Fig. 1), \( V_{cd}V_{cb}^* \), \( V_{ud}V_{ub}^* \) and \( V_{td}V_{tb}^* \) are measured in \( b \rightarrow cl\nu \), \( b \rightarrow ul\nu \) and in \( B_d^0 - \bar{B}_d^0 \)
mixing, respectively. A variety of models beyond the SM provide new contributions to $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixing, but only very rarely do such models involve new amplitudes which can compete with the $W$-mediated tree-level $b$ decays. Therefore, whereas two of the sides of the unitarity triangle are usually stable under new physics effects, the side involving $V_{td}V_{tb}^*$ can be modified by such effects. In certain models, such as a four generation model and models involving $Z$-mediated flavor-changing neutral currents (to be discussed below), the unitarity triangle turns into a quadrangle.

In the phase convention of Eq.(1) the three angles $\alpha$, $\beta$, $\gamma$ are defined as follows:

$$\gamma \equiv \text{Arg}(V_{ud}V_{ub}^*) , \quad \beta \equiv \text{Arg}(V_{tb}V_{td}^*) , \quad \alpha \equiv \pi - \beta - \gamma . \quad (21)$$

Assuming that new physics affects only $B_d^0 - \bar{B}_d^0$ mixing, one can make the following simple observations about CP asymmetries beyond the SM:

1. The asymmetry in $B_d^0 \rightarrow \psi K_S$ measures the phase of $B_d^0 - \bar{B}_d^0$ mixing and is given by $2\beta'$, which in general can be different from $2\beta$.

2. The asymmetry in $B_d^0 \rightarrow \pi^+\pi^-$ measures the phase of $B_d^0 - \bar{B}_d^0$ mixing plus twice the phase of $V_{ub}^*$, and is given by $2\beta' + 2\gamma \equiv 2\pi - 2\alpha'$, where $\alpha' \neq \alpha$.

3. The time-dependent rates of $B_s^0/\bar{B}_s^0 \rightarrow D_s^\pm K^\mp$ determine a phase $\gamma'$ given by the phase of $B_s^0 - \bar{B}_s^0$ mixing plus the phase of $V_{ub}^*$; in this case $\gamma' \neq \gamma$.

4. The processes $B^\pm \rightarrow D^0 K^\pm$, $B^\pm \rightarrow \bar{D}^0 K^\pm$, $B^\pm \rightarrow D_{1(2)}^0 K^\pm$ measure the phase of $V_{ub}^*$ given by $\gamma$.

That is, a measurement of the phase $\gamma$ through the last method will obey the triangle relation $\alpha' + \beta' + \gamma = \pi$ with the phases of $B_d^0 \rightarrow \psi K_S$ and $B_d^0 \rightarrow \pi^+\pi^-$, irrespective of contributions from new physics. On the other hand, the phase $\gamma'$ measured by the third method violates this relation. This demonstrates the importance of measuring phases in a variety of independent ways.

Let us note in passing that in certain models, such as multi-Higgs doublet models with natural flavor conservation (to be discussed below), in spite of
new contributions to $B^0_d - \bar{B}^0_d$ and $B^0_s - \bar{B}^0_s$ mixing, the phases measured in $B^0_d \to \psi K_S$ and in $B^0_s/\bar{B}^0_s \to D^+_s K^\mp$ are unaffected, $\beta' = \beta$, $\gamma' = \gamma$. The values measured for these phases may, however, be inconsistent with the CP conserving measurements of the sides of the unitarity triangle.

4.2 CP asymmetries versus penguin decays

Models in which CP asymmetries in $B$ decays are affected by new contributions to $B^0 - \bar{B}^0$ mixing will usually also have new amplitudes contributing to rare flavor-changing $B$ decays, such as $b \to sX$ and $b \to dX$. We refer to such processes, involving a photon, a pair of leptons or hadrons in the final state, as “penguin” decays.

In the SM both $B^0 - \bar{B}^0$ mixing and penguin decays are governed by the CKM parameters $V_{ts}$ and $V_{td}$. Unitarity of the CKM matrix implies [2] $|V_{ts}/V_{cb}| \approx 1, 0.11 < |V_{td}/V_{cb}| < 0.33$, and $B^0_d - \bar{B}^0_d$ mixing only improves this constraint slightly due to large hadronic uncertainties, $0.15 < |V_{td}/V_{cb}| < 0.33$.

The addition of contributions from new physics to $B^0 - \bar{B}^0$ mixing relaxes the above constraints in a model-dependent manner. The new contributions depend on new couplings and new mass scales which appear in the models. These parameters also determine the rate of penguin decays. A recent comprehensive study [33], updating previous work, showed that the values of the new physics parameters, which yield significant effects in $B^0 - \bar{B}^0$ mixing, will also lead in a variety of models to large deviations from the SM predictions for certain penguin decays. Here we wish to briefly summarize the results of this model-by-model analysis:

1. **Four generations**: The magnitude and phase of $B^0 - \bar{B}^0$ mixing can be substantially changed due to new box-diagram contributions involving internal $t'$ quarks. For such a region in parameter space, one expects an order-of-magnitude enhancement (compared to the SM prediction) in the branching ratio of $B^0_d \to l^+l^-$ and $B^+ \to \phi\pi^+$. 

2. **$Z$-mediated flavor-changing neutral currents**: The magnitude and phase of $B^0 - \bar{B}^0$ mixing can be altered by a tree-level $Z$-exchange. If this effect is large, then the branching ratios of the penguin processes $b \to
\[ s l^+ l^-, \ B^0_s \rightarrow l^+ l^-, \ B_s^0 \rightarrow \phi \pi^0 \] (\[b \rightarrow dl^+ l^-\, B^0_d \rightarrow l^+ l^-\, B^+ \rightarrow \phi \pi^+\]) can be enhanced by as much as one (two) orders-of-magnitude.

3. **Multi-Higgs doublet models with natural flavor conservation:** New box-diagram contributions to \(B^0 - \bar{B}^0\) mixing with internal charged Higgs bosons affect the magnitude of the mixing amplitude but not its phase (measured, for instance, in \(B^0_d \rightarrow \psi K_S\)). When this effect is large, the branching ratios of \(B^0_d, B^0_s \rightarrow l^+ l^-\) are expected to be larger than in the SM by up to a factor 5.

4. **Multi-Higgs doublet models with flavor-changing neutral scalars:** Both the magnitude and phase of \(B^0 - \bar{B}^0\) mixing can be changed due to a tree-level exchange of a neutral scalar. In this case one expects no significant effects in penguin decays.

5. **Left-right symmetric models:** Unless one fine-tunes the right-handed quark mixing matrix, there are no significant new contributions in \(B^0 - \bar{B}^0\) mixing and in penguin \(B\) decays.

6. **Minimal supersymmetric models:** There are a few new contributions to \(B^0 - \bar{B}^0\) mixing, all involving the same phase as in the SM. Branching ratios of penguin decays are not changed significantly. However, certain energy asymmetries, such as the \(l^+ l^-\) energy asymmetry in \(b \rightarrow s l^+ l^-\) can be largely affected.

7. **Non-minimal supersymmetric models:** In non-minimal SUSY models with quark-squark alignment, the SUSY contributions to \(B^0 - \bar{B}^0\) mixing and to penguin decays are generally small. In other models, in which all SUSY parameters are kept free, large contributions with new phases can appear in \(B^0 - \bar{B}^0\) mixing and can affect considerably SM predictions for penguin decays. However, due to the many parameters involved, such schemes have little predictivity.

We see that measurements of CP asymmetries and rare penguin decays give complementary information and can distinguish among the different models. For instance, in models 3 and 6 one expects \(\beta' = \beta, \ \gamma' = \gamma\), whereas in models of type 1, 2 and 4 one has \(\beta' \neq \beta, \ \gamma' \neq \gamma\). In the latter case, one expects different measurements of \(\gamma\) in \(B^\pm \rightarrow DK^\pm\) and in
The three models 1, 2 and 4 can then be distinguished by their different predictions for branching ratios of penguin decays. To distinguish between models 3 and 6, one would have to rely on detailed dilepton energy distributions.

5 Conclusion

We addressed certain theoretical issues related to three of the main goals of future $B$ physics experiments, which are:

- Observation of CP asymmetries in $B_d^0, B^\pm, B_s^0$ decays.
- Determination of $\alpha, \beta, \gamma$ from these asymmetries and from $B$ decay rates.
- Detection of deviations from Standard Model asymmetry predictions, which would provide clues to a more complete theory when combined with information about rare flavor-changing $B$ decays.

Acknowledgements

It is a pleasure to thank A. Dighe, O. Hernández, D. London, J. Rosner and D. Wyler for very enjoyable collaborations on various topics presented here. This work was supported in part by the United States-Israel Binational Science Foundation under Research Grant Agreement 94-00253/2, and by the Fund for Promotion of Research at the Technion.
References


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Table 1: Decomposition of amplitudes for $B$ decays to two light pseudoscalars
Figure 1: The CKM unitarity triangle