Vector Condensate Model of Electroweak Interactions∗

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Abstract
Motivated by the fact that the Higgs is not seen, we have proposed a version of the standard model where the scalar doublet is replaced by a vector doublet and its neutral member forms a nonvanishing condensate. Gauge fields are coupled to the new vector fields $B$ in a gauge invariant way leading to mass terms for the gauge fields by condensation. $B$-particles become massive because of their self interactions. Fermion and gauge field couplings are standard. Low energy charged current phenomenology fixes the condensate. Fermion masses are coming from the condensation and $B$-particle–fermion couplings. The Kobayashi-Maskawa description is unchanged. The model has a low momentum scale of about 2 TeV. For instance, from tree-graph unitarity at a scale of 1 TeV the minimum mass of a charged $B$-particle is 369 GeV. Such $B$-particles are shown to copiously produced at high–energy linear $e^+e^-$ colliders. The model survives the test of oblique radiative corrections. To each momentum scale there exists a range of $B$ masses where the $S$, $T$ parameters are compatible with the experiment. For instance, at a scale of 1 TeV from the $S$ parameter the minimum mass of a charged (neutral) $B$ is 200–350 GeV (400–550 GeV).

∗Dedicated to Prof. G. Marx on his 70th birthday.
1 Introduction

A popular description of the symmetry breaking sector of the standard model is through a weakly interacting system (Higgs, $M_H \leq 1\,\text{TeV}$). Another possibility is a symmetry breaking system interacting strongly with the longitudinal weak vector bosons. This idea has been realised in the DHT model [1] based on a chiral Lagrangian approach. An alternative description of the strongly interacting symmetry breaking system has been proposed in the BESS Model [2] through nonlinear realisations.

Recently, top-quark condensation has also been suggested for describing the electroweak symmetry breaking [3] which has resulted in several interesting studies (e.g. [4]).

In the present note we start with the usual Lagrangian of the standard model of electroweak interactions, but instead of the Higgs-doublet a Y=1 vector-doublet $B_\mu$ is introduced whose neutral component forms a condensate. This creates a mechanism of dynamical symmetry breaking, and through the interaction of $B_\mu$ and the gauge fields one gets nonvanishing masses for W and Z, as well as a vanishing photon mass. Identifying $(-6d)^{1/2} = 246\,\text{GeV}$ from the low energy charged weak current interaction yields the standard description of weak vector boson masses.

A quartic, invariant self-coupling gives mass to $B^{0,+}$. In a cutoff field theory, however, the fixed value of the condensate confines considerably the region of validity of the model [5]: $\Lambda, m_{B^0} \leq 2.6\,\text{TeV}$ for $\Lambda \geq m_{B^0}$.

Fermion mass generation by a $B^0$–condensate is possible only if we assume a noninvariant interaction as a start. In this case the usual Kobayashi–Maskawa parametrisation immediately emerges.

The spin–one particle B has pair interactions with $\bar{f}f, WW, ZZ, Z, \bar{B}B$ etc. $\bar{f}f B^0$ is weaker than $\bar{f}f H$ (Higgs), but both of them are proportional to $m_f$. In coupling strengths $\bar{B}^0 B^0 WW(ZZ) = H H WW(ZZ), \bar{B}^0 B^0 Z \simeq \bar{f}f Z$.

From tree–graph unitarity the allowed region of $B^+(B^0)$ mass is estimated as $m_+ \geq 369\,\text{GeV} \; m_0 \geq 410\,\text{GeV}$ at $\Lambda = 1\,\text{TeV}$ [6]. Such B’s are copiously produced at high-energy linear $e^+e^-$ colliders [7].

As for the oblique radiative corrections, to each momentum scale there exist a domain of the masses of charged and neutral vector bosons where S is compatible with the experiments. At a scale of 1 TeV this requires vector
boson masses of at least $m_0 \approx 400\text{–}500 \text{ GeV}$, $m_+ \approx 200\text{–}350 \text{ GeV}$ [8]. The model survives also the test of the $\rho$ parameter [9]. For a fixed $\Lambda$ and $m_0$ the test of $\rho$ increases the minimum $m_+$ coming from $S$.

The model is outlined in Section 2 while Section 3 contains implications of the model.

2 The model

We replace the standard model Higgs-doublet by a $Y=1$ doublet of vector fields,

$$B_\mu = \begin{pmatrix} B_\mu^{(+)} \\ B_\mu^{(0)} \end{pmatrix},$$

and assume that $B_\mu^{(0)}$ forms a nonvanishing condensate $d$,

$$\langle B_\mu^{(0)+}(x)B_\nu^{(0)}(x) \rangle_0 = g_{\mu\nu}d, \quad \langle B_\mu^{(+)+}B_\nu^{(+)} \rangle_0 = 0$$

$B_\mu$ is coupled to itself and the SU(2) and U(1) gauge fields $A_\mu$ and $C_\mu$, respectively, in a gauge invariant way. In the Lagrangian of the standard model the H–A–C sector is replaced by $L_0(DB) - V(B)$ added to the Lagrangian of gauge fields, where

$$L_0(DB) = -\frac{1}{2} \left( D_\mu B_\nu - D_\nu B_\mu \right)^+ \left( D^\mu B_\nu - D^\nu B^\mu \right),$$

$$D_\mu = \partial_\mu - \frac{1}{2} ig_j A_{j,\mu} - \frac{1}{2} ig_j' C_\mu,$$

$$V(B) = \lambda \left( B_\mu^{(+)}B_\nu^{(+)} \right)^2 - \mu_0^2 B_\mu^{(+)}B_\nu^{(+)}, \lambda > 0.$$  

Now, one can get bilinear mass terms either in the Lagrangian or in the equations of motion of two-point functions once the condensate (2) is assumed. In the present case the $W^\pm$ mass is determined by the total $B$-condensate, while the two neutral combinations are proportional to $B_\mu^{(+)+}B_\nu^{(+)}$ and $B_\mu^{(0)+}B_\nu^{(0)}$, respectively. Therefore, a vanishing photon mass goes together with the assumption that $B_\nu^{(+)}$ does not form a condensate. This leads to the predictions

$$m_A = 0, \quad m_W = \frac{1}{2} g \sqrt{-6d}, \quad m_Z = \frac{1}{2} g \cos \theta_W \sqrt{-6d},$$

$$m_0 \approx 400\text{–}500 \text{ GeV}, \quad m_+ \approx 200\text{–}350 \text{ GeV} [8].$$
where $d$ fixes the $B^{0}$–condensate. $(-d)^{1/2}$ plays the role of the vacuum expectation value of the Higgs field. We have from charged current phenomenology

$$d = -(6\sqrt{2}G_{F})^{-1}$$  \hspace{1cm} (5)$$

Breaking the gauge symmetry by the $B^{0}$–condensate gives rise to a mass term also for $B^{0}_{\mu}$:

$$m_{B^{0}}^{2} = -10d\lambda + \mu_{0}^{2}. \hspace{1cm} (6)$$

In what follows let us assume $\mu_{0} = 0$, but we remark that for $\mu_{0}^{2} < 0$ (6) would give $-2d\lambda$ since from the minimum of $V$, $\mu_{0}^{2} = 8\lambda d$ (case of spontaneous symmetry breaking). Since the field $B^{(+)}_{\mu}$ cannot be transformed out, it represents a physical field which gets its bare mass from the self–interactions of $B_{\nu}$,

$$m_{B^{+}}^{2} = -8d\lambda, \left(\frac{m_{B^{+}}}{m_{B^{0}}}\right)^{2} = \frac{4}{5}. \hspace{1cm} (7)$$

Fermions are assigned to the gauge group in the standard manner. Since the four-vector $\Psi_{L}B_{\nu}\Psi_{R}$ is invariant under gauge transformations, the condensate $d$ will generate a fermion mass term only if a noninvariant interaction is introduced:

$$g_{ij}^{d}i\bar{\Psi}_{iL}B_{\nu}B^{0}_{\nu} + g_{ij}^{d}i\bar{\Psi}_{iL}d_{jR}B^{0}_{\nu} + h.c., \hspace{0.5cm} \Psi_{iL} = \begin{pmatrix} u_{i} \\ d_{i} \end{pmatrix}, B_{\nu}^{C} = \begin{pmatrix} B_{\nu}^{(+)} \\ B_{\mu}^{(0)} \end{pmatrix}. \hspace{1cm} (8)$$

This leads to the Kobayashi-Maskawa description, too. A typical lepton or quark mass is

$$m_{f} = -4g_{f}d. \hspace{1cm} (9)$$

For a fermion of mass $m_{f}$ the coupling strengths from (9) and the standard description are

$$g_{f} = \frac{3}{\sqrt{2}}m_{f}G_{F}, \hspace{0.5cm} g_{f}^{SM} = m_{f}(2\sqrt{2}G_{F})^{1/2}. \hspace{1cm} (10)$$

The trilinear interactions of $Z$, $W^{\pm}$ and $B$ are derived from (3) as

$$L \left( B^{0} \right) = \frac{ig}{2\cos\Theta_{W}}\partial^{\mu}B^{(0)\mu^{+}} \left( Z_{\mu}B_{\nu}^{(0)} - Z_{\nu}B_{\mu}^{(0)} \right) + h.c.,$$

$$L \left( B^{+}B^{-}Z \right) = -(\cos2\Theta_{W}) \cdot L \left( B^{(0)} \rightarrow B^{(+)} \right). \hspace{1cm} (11)$$
\[ L \left( B^0 B^\pm W \right) = \frac{ig}{\sqrt{2}} \left[ \partial^\mu B^{(+)\nu} \left( W_{\mu}^+ B^{(0)}_{\nu} - W_{\nu}^+ B^{(0)}_{\mu} \right) + \partial^\mu B^{(0)\nu} \left( W^-_{\mu} B^{(+)}_{\nu} - W^-_{\nu} B^{(+)}_{\mu} \right) \right] + h.c. \]

The quartic interactions coming from (3) are self-couplings of \( B^{+,0} \) and couplings of the type \( \gamma \gamma B^+ B^-, ZZB^+ B^-, \gamma W^+ B^- B^0, ZW^+ B^- B^0, W^- W^+ B^- B^+, W^- W^- B^0 B^0, ZZB^0 B^0 \). For instance, the \( V V B^0 \) couplings are

\[ L = -B^{(0)\nu} B^{(0)\nu} \left( \frac{1}{2} g^2 W^- W^+ + \frac{g^2}{4 \cos^2 \theta_w} Z \mu Z \mu \right) + B^{(0)\nu} B^{(0)\nu} \left( \frac{1}{2} g^2 W^- W^+ + \frac{g^2}{4 \cos^2 \theta_w} Z \nu Z \mu \right). \]

(12)

\( B^0 B^0 Z \) is the strongest interaction (\( \simeq f f Z \)). \( B^0 B^0 V V \) is weaker than \( VVH \) and as strong as \( HHVV, V=W,Z \). Similarly \( f f H \) is stronger than \( B^0 B^0 f f \), while \( (B^0 B^0)^2 \) may be weak or strong depending on \( m_B \).

### 3 Implications of the model

From precision measurements of the Z width and the form of \( \Gamma(Z \rightarrow B^0 \overline{B}^0) \) we get \( m_{B^0} \geq 43 \text{ GeV} \) [5]. High energy \( e^+ e^- \) colliders provide excellent opportunities for studying B bosons. At planned luminosities the yield of B’s is large in \( e^+ e^- \rightarrow B \overline{B}, B \overline{B} Z \) up to near the maximum kinematically possible \( m_B \)’s [7]. The cross section of the \( B^+ B^- \) final state is 0.29 times that of \( B^0 \overline{B}^0 \) at equal masses and energies. From (11) we get

\[ \sigma(e^+ e^- \rightarrow B^0 \overline{B}^0) = \frac{4 \left( 4 \sin^2 \theta_w - 1 \right)^2 + 1}{3072 \pi \cos^4 \theta_w} \frac{(s^2 - 4m_{B^0}^2)^2(s^2 + 3m_{B^0}^2)}{\sqrt{m_{B^0}^2 \left( s - m_{B^0}^2 + \frac{\Gamma_Z^2}{4} \right)^2 + m_{B^0}^2 \Gamma_Z^2}}. \]

(13)

With increasing \( m_{B^0} \) after threshold the rise of the cross section is slower and at \( s \gg m_{B^0}^2 \) \( \sigma \) is proportional to \( m_{B^0}^{-2} \).

At the linear collider of \( s^{1/2} = 500 \text{ GeV} \) (\( m_{B^0} \leq 250 \text{ GeV} \)) and taking the popular luminosity of 10fb\(^{-1}\) it follows that even a high \( B^0 \) mass results in a large number of events. For instance, for \( m_{B^0} \leq 200-240 \text{ GeV} \) we get more
than 800-200 events. At NLC (next linear collider) even higher masses can be searched for. At \( s^{1/2} = 1.5 \) TeV and with 10 (100)fb\(^{-1}\) one gets more than 200 (1000) events for \( m_{B^0} \leq 500(700) \) GeV, and the yield is growing with decreasing \( m_{B^0} \). Studying B production in hadron collisions is in progress. Oblique radiative corrections due to B-loops to the \( \rho \) parameter have been calculated in ref. [9]. The contribution \( \Delta \rho \) due to B-loops to \( \rho \) is

\[
\Delta \rho = \alpha T, \tag{14}
\]

where \( T \) is one of the three parameters constrained by precision experiments. The analysis in Ref. 10 finds for beyond the standard model \( \Delta \rho = -(0.09 \pm 0.25) \times 10^{-2} \) at \( m_t = 130 \) GeV, \( m_H = m_Z \).

The parameter \( T \) is defined by

\[
\alpha T = \frac{e^2}{s^2 c^2 m_Z^2} (\Pi_{ZZ}(0) - \Pi_{WW}(0)) \tag{15}
\]

with \( s = \sin \theta_W, c = \cos \theta_W \), and it is calculated in one B-loop order. \( \Pi_{ik} \) is expressed by the \( g_{\mu \nu} \) terms of the vacuum polarization contributions \( \Pi_{ik} \) due to B-loops as

\[
\Pi_{AA} = e^2 \Pi_{AA}, \quad \Pi_{ZZ} = \frac{e^2}{s^2 c^2} \Pi_{ZZ}, \quad \Pi_{WW} = \frac{e^2}{s^2} \Pi_{WW}. \tag{16}
\]

In a renormalizable theory \( T \) is finite. In the present model, however, it remains a function of \( \Lambda \), but the cutoff \( \Lambda \) is not restricted by experimental comparison.

Numerical analysis shows that for a given \( \Lambda \) there is always an \( (m_0 = m_{B^0}, m_+ = m_{B^+}) \) region where \( \Delta \rho \) is in agreement with the experimental limits. For fixed \( \Lambda \), at decreasing \( m_0 \), the \( m_+ \) range corresponding to the experimental \( \Delta \rho \) error bars shrinks. For instance for \( \Lambda = 1 \) TeV and \( m_0 = (100, 400, 800, 1000) \) GeV the 1\( \sigma \) \( m_+ \) region is \((263.6–263.9, 629.7–635.6, 950–990, 1219–1450) \) GeV, respectively. In general \( \Delta \rho \) can be written in the form of

\[
\Delta \rho = \frac{\Lambda^2}{m_Z^2} f \left( \frac{\Lambda^2}{m_0^2}, \frac{\Lambda^2}{m_+^2} \right) \]

thus we get similar \( \Delta \rho = 0 \) curves for different \( \Lambda \)’s by scaling the masses by a factor \( \frac{\Lambda}{\Lambda} \). For higher \( \Lambda \) the allowed mass region shrinks, for example at \( \Lambda = 1 \) TeV, \( m_0 = 600 \) GeV : \( m_+ = 846–868 \) GeV; = 1.5 TeV, \( m_0 = 900 \) GeV : \( m_+ = 1275–1290 \) GeV; = 5 TeV, \( m_0 = 3000 \) GeV : \( m_+ = 4264–4268 \) GeV. This has been checked up to \( \Lambda = 15 \) TeV.
Turning to the S parameter we define [8]
\[ \alpha S = 4e^2(\Pi'_{ZZ}(0) - (c^2 - s^2)\Pi'_{ZA}(0) - s^2 c^2 \Pi'_{AA}(0)), \]
\[ \Pi_{ik}(0) = \frac{d}{dq^2} \Pi_{ik}(q^2)|_{q^2=0}. \quad (17) \]

An analysis [11] of precision experiments shows that \( S_{new} < 0.09(0.23) \) at 90 (95)% C.L. for \( m_H^{ref} = 300\text{GeV} \) and assuming \( m_t = 174\text{GeV} \) (CDF value). Requiring \( S_{new} \geq 0 \), the corresponding constraints are \( S_{new} < 0.38(0.46) \) [5]. Since a Higgs of 300 GeV is absent in the present model, its contribution, 0.063, must be removed. In this way for the contribution of B we have \( S < 0.15(0.29) \) at 90 (95)% C.L. For \( m_+, m_0 \geq 1.90\Lambda \) this is fulfilled, in particular, \( S \rightarrow 0 \) for \( \frac{\Lambda}{m_+}, \frac{\Lambda}{m_0} \rightarrow 0 \) [8]. Since S is invariant multiplying \( \Lambda, m_0, m_+ \) by a common factor, allowed regions for scales different from 1 TeV easily follow. Higher \( \Lambda \) attracts higher minimum masses.

The allowed regions by S are tightened by T. For example, at \( \Lambda = 1 \) TeV, \( m_0 = 400 \) (600) GeV, the \( m_+ \) range allowed by S, T is \( m_+ = 630\text{–}636 \) (846–868) GeV. For higher \( \Lambda \) the allowed \( m_+ \) region shrinks at the same \( m_0 \). In general, \( \Lambda \) remains unrestricted and suitable, heavy \( B^+, B^0 \) provide small radiative corrections. \( \Lambda \) can be restricted by taking into account unitarity requirements [6].

In the vector condensate model there exist many \( BB \rightarrow BB, VV BV \rightarrow BV \) type processes with \( B = B^0, \bar{B}^0, B^\pm, V = W^\pm, Z \). We consider them for longitudinally polarized external particles and calculate the J=0 partial-wave amplitudes, \( a_0 \), from contact and one–particle exchange graphs. Unitarity requires \( |Rea_0| \leq 1/2 \). We have shown that the strongest lower bounds (200–400 GeV) are coming from B–B scatterings. Here the dominant contributions are derived from contact graphs. For example, in case of \( B^0B^0 \rightarrow B^0B^0 \), the contribution of the Z-exchange graph to the lower bound of 317 GeV (\( \Lambda = 1 \) TeV) is 4 GeV.

One finds the best bounds in \( B^+B^- \rightarrow B^0\bar{B}^0 \) leading to the s-wave amplitude
\[ a_0 = -\frac{3}{16\sqrt{10}} G_F \left( \frac{s^2}{2m_0m_+} - \left( \frac{m_0}{m_+} + \frac{m_+}{m_0} \right) s + \frac{1}{2} m_0m_+ \left( \frac{m_0}{m_+} + \frac{m_+}{m_0} \right)^2 \right). \quad (18) \]

Applying the requirement of unitarity at the maximum possible energy \( \Lambda \)
\[ \Lambda = 1.0\text{TeV} : \quad m_0 \geq 410\text{GeV}, \quad m_+ \geq 369\text{GeV} \]
\[ \Lambda = 1.5\text{TeV} : \quad m_0 \geq 741\text{GeV}, \quad m_+ \geq 667\text{GeV} \quad (19) \]
\[ \Lambda = 2.0\text{TeV} : \quad m_0 \geq 1091\text{GeV}, \quad m_+ \geq 980\text{GeV}. \]

It follows that in this approximation the momentum scale cannot reach 2 TeV and the B bosons are heavy particles. The bounds from \( B^+B^{+} \rightarrow B^+B^+ \) are very close to (19) and they are in turn \( m_+ \geq 332 \text{ GeV}, 615 \text{ GeV}, 960 \text{ GeV} \). The above bounds imposed by the unitarity are similar to those obtained from the S parameter.

In conclusion, the vector condensate model cannot be renormalized perturbatively, its scattering amplitudes contain polynomials in s, so that partial-wave unitarity provides a maximum energy. In tree-graph approximation this is \( \Lambda \simeq 2 \text{ TeV} \). A rough interpretation of the condensate parameter with a \( B^0 \)-propagator yields \( \Lambda \leq 2.6 \text{ TeV} \).

At the same time, the B–particles must be heavy and B–masses cannot be far from \( \Lambda \). Indeed, for \( \Lambda \gg m_{+,0} \) the S parameter becomes too large, while the unitarity argument provides low masses and \( \Lambda \) below \( \Lambda = 1 \text{ TeV} \).

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References


