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Local BRST Cohomology in Minimal D=4, N=1 Supergravity

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Abstract

The local BRST cohomology is computed in old and new minimal supergravity, including the coupling to Yang-Mills gauge multiplets. This covers the determination of all gauge invariant local actions for these models, the classification of all the possible counterterms that are invariant on-shell, of all candidate gauge anomalies, and of the possible nontrivial (continuous) deformations of the standard actions and gauge transformations. Among others it is proved that in old minimal supergravity the most general gauge invariant action can indeed be constructed from well-known superspace integrals, whereas in new minimal supergravity there are only a few additional (but important) contributions which cannot be constructed in this way without further ado. Furthermore the results indicate that supersymmetry itself is not anomalous in minimal supergravity and show that the gauge transformations are extremely stable under consistent deformations of the models. There is however an unusual deformation converting new into old minimal supergravity with local R-invariance which is reminiscent of a duality transformation.

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1 Introduction

1.1 Topics and motivation

This paper presents the first exhaustive analysis of the local BRST cohomology in supergravity. More precisely we will analyze the two most popular formulations of four dimensional $N = 1$ supergravity known as old [1] and new [2] minimal supergravity, including their coupling to Yang–Mills gauge multiplets. The inclusion of further (matter) multiplets will be briefly discussed too.

To begin with, let me recall some general features of the local BRST cohomology. By this I mean the cohomology of the BRST operator[1] in the space of local functionals of the fields and antifields. It allows to analyze various physically relevant aspects of gauge theories in a unified framework. Historically it was above all the anomaly problem that initiated the interest in this cohomology when it became clear that the possible gauge anomalies define cohomology classes at ghost number one. This was first shown in abelian Higgs–Kibble and Yang–Mills theories [4, 5] and later generalized to arbitrary gauge theories [6].

The classification of possible anomalies is however only one instance that can be efficiently investigated by means of the local BRST cohomology. Further well-known applications of this cohomology are the construction of gauge invariant actions and the classification of the possible "on-shell counterterms". Both the actions and these counterterms are local functionals of the classical fields, the difference being that actions have to be gauge invariant off-shell whereas the counterterms need to be gauge invariant only on-shell. Gauge invariant actions and on-shell counterterms define BRST cohomology classes at ghost number zero.

Possibly still less known are other applications of the local BRST cohomology which were realized more recently and will be therefore briefly described in a little more detail in the following.

One of these applications concerns the problem whether or not a given gauge theory, defined through a particular gauge invariant action, can be consistently and continuously deformed in a nontrivial manner. Here a deformation is called *consistent* when the deformed action is still gauge invariant but under possibly modified gauge transformations, *continuous* when it can be parametrized by a deformation parameter g such that one recovers the original theory for $g = 0$, and *nontrivial* when the deformation cannot be removed through mere field redefinitions. Such deformations concern thus simultaneously the action *and* the gauge transformations. To first order in g they are determined by the local BRST cohomology at ghost number zero [7]. At higher order in g they might get additionally obstructed by the local BRST cohomology at ghost number one [8].

Last but not least the local BRST cohomology provides, at *negative* ghost numbers, the dynamical conservation laws of a theory [9]. These conservation laws are

[1] Throughout the paper, the word "BRST operator" describes the strictly nilpotent antiderivation which acts nontrivially on both the fields and the antifields and is generated in the antibracket by the solution of the master equation [3], see section 2 for details.

described in terms of totally antisymmetric local functions $j^{\mu_1\cdots\mu_k}$ of the classical fields with on-shell vanishing divergence ($\partial_{\mu_1} j^{\mu_1\cdots\mu_k} \approx 0$), defined modulo trivial conservation laws which are on-shell of the form $\partial_{\mu_0} k^{[\mu_0\cdots\mu_k]}$. As shown recently in [10], *each* dynamical conservation law corresponds to a global (= rigid) symmetry of the solution of the master equation [11, 3] and gives rise to a corresponding Ward identity. This generalizes Noether's theorem on the correspondence of dynamically conserved currents (= conservation laws of order $k = 1$) and the global symmetries of the *classical* action.

To summarize, the local BRST cohomology contains physically relevant information at *all* ghost numbers ≤ 1 (a physical interpretation of the cohomology classes at ghost numbers > 1 has not yet been found). Therefore it is worthwhile to analyze this cohomology at least for these ghost numbers.

The analysis of the local BRST cohomology in minimal supergravity carried out here will allow us to answer questions which in part have been discussed already in the early days of supergravity without having been answered exhaustively so far. For instance, the problem whether the local supersymmetry transformations can be modified nontrivially was raised already in [12] soon after supergravity was invented in [13]. Shortly after that, the discussion of the possible on-shell counterterms was started [14], mainly to clarify whether or up to which loop order supergravity may be finite in the conventional quantum field theoretical approach. The construction of supergravity actions has been discussed extensively in the literature because it is much more involved than in standard gravity. In particular various superspace techniques have been invented for this purpose, see e.g. the textbooks [15, 16]. In this paper we will clarify whether or to what extend such methods provide the *most general* local action functional for old and new minimal supergravity. Anomalies in supergravity have been already discussed in the literature using the BRST approach, see e.g. [17]. However, the work on this topic concentrated up to now mainly on the determination of chiral anomalies in supergravity, leaving open among others the important question whether or not local supersymmetry itself can be anomalous.

1.2 Sketch of the approach

To compute the local BRST cohomology for old and new minimal supergravity, one has to solve

$$s\omega_4 + d\omega_3 = 0 \tag{1.1}$$

where ω_4 is the integrand of a local functional, written as a differential 4-form, ω_3 is some local 3-form, s is the nilpotent BRST operator for the supergravity theories studied here, and d denotes the spacetime exterior derivative. The searched for solutions ω_4 are defined modulo trivial solutions of the form $s\eta_4 + d\eta_3$. In particular ω_4 itself is of course required to be nontrivial,

$$\omega_4 \neq s\eta_4 + d\eta_3. \tag{1.2}$$

We will investigate (1.1) and (1.2) for all ghost numbers but spell out the results in detail only for the physically most important cases, i.e. for ghost numbers ≤ 1.

The investigation will be truly general, i.e. it will not use restrictive assumptions on the form of the solutions. For instance, a restriction on the number (and order) of derivatives of the fields and antifields occurring in ω_4, ω_3, η_4 and η_3 will not be imposed, apart from requiring this number to be finite[2]. Furthermore, the solutions will not be assumed to be covariant in whatever sense from the outset. In particular it will neither be assumed that they transform as true differential forms under spacetime diffeomorphisms, nor that any group indices of the fields occurring in them are "correctly" contracted. Also, we will of course not assume that the solutions can be constructed in whatsoever fashion from superfields (superfields and superspace techniques will nowhere be used in this paper!). The cohomological analysis itself will therefore reveal to what extend the solutions have such properties.

Our only inputs will be the field content and the BRST transformations. We will use the standard set of auxiliary fields to close the gauge algebra off-shell and the standard actions and gauge transformations to construct the BRST operator. These gauge transformations are of course general coordinate transformations (= spacetime diffeomorphisms), $N = 1$ supersymmetry transformations, Lorentz and Yang–Mills transformations, as well as the reducible gauge transformations of the 2-form gauge potential present in new minimal supergravity.

The use of the auxiliary fields is in principle not necessary as the antifield formalism allows to treat also open gauge algebras [3]. Nevertheless it has some advantages to close the gauge algebra off-shell by means of auxiliary fields. For instance this will facilitate to distinguish nontrivial deformations of the gauge transformations from trivial ones, and counterterms which are invariant only on-shell from those which can even be completed to off-shell invariants by means of the auxiliary fields. As a matter of fact, the solutions of (1.1) and (1.2) in a formulation without auxiliary fields can be easily obtained from those given here. To that end one must eliminate the auxiliary fields using their 'generalized equations of motion' resulting from the solution of the master equation rather than from the classical action, see [18, 19] for details.

To analyze (1.1) and (1.2) systematically we will use a general framework described in [20] (see also [21]). This reduces the computation of the BRST cohomology (locally) to a particular "covariant" cohomological problem for the operator $\tilde{s} = s + d$ in the space of *local total forms* depending on suitably defined *tensor fields* and *generalized connections*. Local total forms are simply formal sums of local differential forms with different form degrees and ghost numbers. To solve the reduced problem we will then combine results and techniques (e.g. Lie algebra cohomology) used successfully already in nonsupersymmetric theories [22, 23, 24, 25, 26] and methods developed in [27, 28] to deal with supersymmetry.

[2]More precisely we require finiteness of the number of derivatives in every term occurring in an expansion of the forms according to the number of antifields. This defines "local forms" here.

1.3 Outline of the paper

We will first define the problem precisely by fixing the field content and the BRST transformations in section 2. There we will also introduce already the relevant generalized connections and tensor fields. It will be crucial for the analysis to determine an appropriate complete set of these tensor fields such that there are no identities (including "differential" ones) relating the elements of this set off-shell. This will be done in section 3.

We will then be prepared to compute the local BRST cohomology. This computation will be carried out in two separate steps. In section 4 we will first compute the *restricted BRST cohomology* defined through (1.1) and (1.2) in the space of antifield independent local forms. Thanks to the use of the auxiliary fields, this problem is well-defined and reduces locally to the corresponding $\tilde{s}$-cohomology in the space of antifield independent total local forms depending only on tensor fields and generalized connections. Its solution will provide in particular the most general action functionals for old and new minimal supergravity spelled out in section 5.

In section 6 we will then analyze the *full BRST cohomology*, i.e. the solutions of (1.1) and (1.2) in the space of local forms which may depend on antifields as well. This problem can (and will) be also traced back to a cohomological problem defined in the space of total local forms depending only on the tensor fields and generalized connections (but not on antifields); however, this time the relevant cohomology is the *weak (= on-shell) $\tilde{s}$-cohomology* in this space [20]. A comparision of the results for the restricted and the weak $\tilde{s}$-cohomology will finally enable us to distinguish those solutions which necessarily involve antifields from those which can be written entirely in terms of the fields when the auxiliary fields are used (note however that the elimination of the auxiliary fields introduces in general additional antifield dependence [18]). The results for the local conservation laws, on-shell counterterms, consistent deformations and for the candidate gauge anomalies are presented in sections 7, 8 and 9.

In section 10 we discuss the modifications of the results when further (matter) multiplets are included or a more complicated action is used from the start. Section 11 briefly comments on topological aspects which are neglected in the remainder of the paper. A summary of the main results with some concluding comments is given in section 12, followed by several appendices containing conventions used in this paper, details concerning the realization of supersymmetry in minimal supergravity which underlie crucially the results, and the derivation of two theorems used within the computation.

2 Field content and BRST algebra

Old and new minimal supergravity differ both in their field content and gauge symmetries. The field content of old minimal supergravity in a formulation with closed gauge algebra, including Yang–Mills gauge multiplets and all the ghosts, is summarized in table 2.1 where gh(Φ), $\varepsilon(\Phi)$ and $dim(\Phi)$ denote the ghost number, Grassmann parity and dimension of Φ respectively (the dimension assignments are the 'natural' ones).

Φ	gh(Φ)	$\varepsilon(\Phi)$	$dim(\Phi)$	
${e_\mu}^a$	0	0	0	vierbein
ψ_μ	0	1	1/2	gravitino
M	0	0	1	complex Lorentz scalar (aux.)
B_a	0	0	1	real Lorentz vector (aux.)
C^μ	1	1	-1	diffeomorphism ghosts
ξ	1	0	$-1/2$	supersymmetry ghosts
C^{ab}	1	1	0	Lorentz ghosts
${A_\mu}^i$	0	0	1	Yang-Mills gauge fields
λ^i	0	1	3/2	gauginos
D^i	0	0	2	real Lorentz scalars (aux.)
C^i	1	1	0	Yang-Mills ghosts

Table 2.1: Field content of old minimal supergravity

The new minimal supergravity multiplet contains instead of the auxiliary fields M and B_a a 2-form gauge potential and a gauge field for R-transformations, with corresponding ghosts and a ghost for ghosts, cf. table 2.2. Accordingly the gauge symmetries of new minimal supergravity include the reducible gauge transformations of the 2-form gauge potential and local R-invariance. In contrast, old minimal supergravity may or may not be locally R-invariant (both cases are covered by our analysis). The Yang–Mills gauge multiplets have in new minimal supergravity the same field content $({A_\mu}^i, \lambda^i, D^i)$ as in old minimal supergravity, except for the missing gaugino and D-field for R-transformations (when old minimal supergravity with local R-symmetry is considered, the corresponding gauge field, gaugino and D-field count among the ${A_\mu}^i, \lambda^i, D^i$ in table 2.1).

Φ	gh(Φ)	$\varepsilon(\Phi)$	$dim(\Phi)$	
$t_{\mu\nu}$	0	0	0	2-form gauge potential
Q_μ	1	1	-1	ghosts associated with $t_{\mu\nu}$
Q	2	0	-2	ghost for the ghosts Q_μ
${A_\mu}^{(r)}$	0	0	1	gauge field for R-transformations
$C^{(r)}$	1	1	0	ghost for R-transformations

Table 2.2: Fields in new minimal supergravity replacing M and B_a

In order to define the cohomological problem we need to specify the BRST transformations of all the fields in tables 2.1 and 2.2 and of their antifields. Since a formulation with a closed gauge algebra is used, the BRST transformations of the 'classical fields' (i.e. those fields with ghost number 0) can be obtained directly from their gauge transformations by replacing the gauge parameters with the corresponding ghosts, using the standard field variations under diffeomorphisms, Lorentz and Yang–Mills transformations, the supersymmetry transformations given e.g. in [1, 2, 15], as well as the extra gauge transformations in new minimal supergravity. For instance the BRST transformation of the vierbein fields reads

$$s e_\mu{}^a = C^\nu \partial_\nu e_\mu{}^a + (\partial_\mu C^\nu) e_\nu{}^a + C_b{}^a e_\mu{}^b + 2i(\xi \sigma^a \bar{\psi}_\mu - \psi_\mu \sigma^a \bar{\xi}). \tag{2.1}$$

The BRST transformation of the ghosts (and the ghost for ghosts) are then chosen such that the BRST operator is nilpotent. In old minimal supergravity this yields

$$\begin{aligned} sC^\mu &= C^\nu \partial_\nu C^\mu + 2i\,\xi\sigma^\mu\bar{\xi}, && (2.2)\\ s\xi^\alpha &= C^\mu \partial_\mu \xi^\alpha + \tfrac{1}{2} C^{ab} \sigma_{ab\,\beta}{}^\alpha \xi^\beta + i\, C^{(r)} \xi^\alpha - 2i\,\xi\sigma^\mu\bar{\xi}\,\psi_\mu{}^\alpha, && (2.3)\\ sC^i &= C^\mu \partial_\mu C^i + \tfrac{1}{2} f_{jk}{}^i C^k C^j - 2i\,\xi\sigma^\mu\bar{\xi}\,A_\mu{}^i, && (2.4)\\ sC^{ab} &= C^\mu \partial_\mu C^{ab} + C^{ca} C_c{}^b - 2i\,\xi\sigma^\mu\bar{\xi}\,\omega_\mu{}^{ab} && (2.5)\\ &\quad + \tfrac{1}{2}\,\xi\sigma^{ab}\xi\,\bar{M} + \tfrac{1}{2}\,\bar{\xi}\bar{\sigma}^{ab}\bar{\xi}\,M + 2i\,\varepsilon^{abcd}\xi\sigma_c\bar{\xi}\,B_d && (2.6) \end{aligned}$$

where $f_{jk}{}^i$ are the structure constants of the Lie algebra of the Yang–Mills gauge group and $\omega_\mu{}^{ab}$ is the usual gravitino dependent spin connection, cf. appendix B.

The BRST transformations obtained in this way for old minimal supergravity with local R-symmetry turn into those for new minimal supergravity by setting M to zero (off-shell) and identifying B_a with

$$B^a \equiv \tfrac{1}{6}\,\varepsilon^{abcd} H_{bcd}, \quad H_{\mu\nu\rho} = 3\partial_{[\mu} t_{\nu\rho]} + 6i\psi_{[\mu}\sigma_\nu\bar{\psi}_{\rho]}\ . \tag{2.7}$$

Furthermore one gets the following BRST transformations of $t_{\mu\nu}$, Q_μ and Q:

$$\begin{aligned} st_{\mu\nu} &= \partial_\nu Q_\mu - \partial_\mu Q_\nu + C^\rho \partial_\rho t_{\mu\nu} + (\partial_\mu C^\rho)\, t_{\rho\nu} + (\partial_\nu C^\rho)\, t_{\mu\rho} \\ &\quad - i\,(\xi\sigma_\mu\bar{\psi}_\nu - \xi\sigma_\nu\bar{\psi}_\mu + \psi_\mu\sigma_\nu\bar{\xi} - \psi_\nu\sigma_\mu\bar{\xi}), && (2.8)\\ sQ_\mu &= \partial_\mu Q + C^\nu \partial_\nu Q_\mu + (\partial_\mu C^\nu) Q_\nu - 2i\,\xi\sigma^\nu\bar{\xi}\,t_{\mu\nu} - i\,\xi\sigma_\mu\bar{\xi}, && (2.9)\\ sQ &= C^\mu \partial_\mu Q - 2i\,\xi\sigma^\mu\bar{\xi}\,Q_\mu\ . && (2.10) \end{aligned}$$

Finally, the BRST transformation of the antifields are obtained from the respective solution $\mathcal{S}$ of the (classical) master equation [3] through

$$s\Phi^*_A = \frac{\delta^R \mathcal{S}}{\delta \Phi^A}\ . \tag{2.11}$$

Thanks to the closure of the gauge algebra, $\mathcal{S}$ itself is simply given by

$$\mathcal{S} = \mathcal{S}_{cl} - \int d^4x\, (s\Phi^A)\Phi^*_A \tag{2.12}$$

where $\mathcal{S}_{cl}$ denotes the classical action and $\{\Phi^A\}$ is the set of fields given in tables 2.1 and 2.2.[3]

The somewhat tedious construction of the BRST algebra outlined above can in fact be streamlined considerably. Namely it can be obtained more easily and elegantly from the gauge covariant supergravity algebra which is well-known in the literature and summarized in appendix B. In particular this provides the BRST algebra directly in a compact form which is best suited for the cohomological analysis [20]. The gauge covariant algebra reads

$$[\mathcal{D}_A, \mathcal{D}_B\} = -T_{AB}{}^C\mathcal{D}_C - F_{AB}{}^I\delta_I, \quad [\delta_I, \mathcal{D}_A] = -g_{IA}{}^B\mathcal{D}_B, \quad [\delta_I, \delta_J] = f_{IJ}{}^K\delta_K \quad (2.13)$$

where $[\cdot,\cdot\}$ is the graded commutator, $\{\mathcal{D}_A\}$ denotes collectively the super-covariant derivatives $\mathcal{D}_a$ and the supersymmetry transformations $\mathcal{D}_\alpha$ and $\bar{\mathcal{D}}_{\dot\alpha}$, and $\{\delta_I\}$ contains the independent elements of the direct sum $\mathcal{G} = \mathcal{G}_L + \mathcal{G}_{YM}$ of the Lorentz algebra $\mathcal{G}_L$ and of the (reductive) Lie algebra $\mathcal{G}_{YM}$ of the Yang–Mills gauge group whose elements are denoted by $l_{ab} = -l_{ba}$ and δ_i respectively,

$$\{\mathcal{D}_A\} = \{\mathcal{D}_a,\, \mathcal{D}_\alpha,\, \bar{\mathcal{D}}_{\dot\alpha}\}, \quad \{\delta_I\} = \{\delta_i,\, l_{ab}:\ a > b\}.$$

In (2.13) the $f_{IJ}{}^K$ are the structure constants of $\mathcal{G}$, the $g_{IA}{}^B$ are the entries of the matrices representing $\mathcal{G}$ on the $\mathcal{D}_A$, and the $T_{AB}{}^C$ and $F_{AB}{}^I$ are torsions and curvatures spelled out explicitly in appendix B. (2.13) is nonlinearly realized on tensor fields, i.e. in old minimal supergravity on

$$M,\, \bar{M},\, B_a,\, \lambda^i_\alpha,\, \bar\lambda^i_{\dot\alpha},\, T_{ab}{}^\alpha,\, T_{ab}{}^{\dot\alpha},\, D^i,\, F_{ab}{}^I \quad (2.14)$$

and all their super-covariant derivatives ($\mathcal{D}_{a_1}\dots\mathcal{D}_{a_k}M$ etc.). The corresponding set of tensor fields in new minimal supergravity is obtained by setting M to zero and using the identification (2.7).

The BRST transformations of *all* the fields can then be compactly written in the form

$$\begin{aligned}
\tilde{s}\,\mathcal{T} &= (\tilde\xi^A\mathcal{D}_A + \tilde{C}^I\delta_I)\,\mathcal{T}, && (2.15)\\
\tilde{s}\,\tilde\xi^A &= \tilde{C}^I g_{IB}{}^A\tilde\xi^B - \tfrac{1}{2}(-)^{\varepsilon_B}\tilde\xi^B\tilde\xi^C T_{CB}{}^A, && (2.16)\\
\tilde{s}\,\tilde{C}^I &= \tfrac{1}{2}f_{KJ}{}^I\tilde{C}^J\tilde{C}^K - \tfrac{1}{2}(-)^{\varepsilon_A}\tilde\xi^A\tilde\xi^B F_{BA}{}^I, && (2.17)\\
\tilde{s}\,\tilde{Q} &= \tfrac{1}{6}\,\tilde\xi^a\tilde\xi^b\tilde\xi^c H_{abc} + i\,\tilde\xi^\alpha\tilde\xi_{\alpha\dot\alpha}\tilde\xi^{\dot\alpha} && (2.18)
\end{aligned}$$

where $\tilde{s}$ is the sum of the BRST operator and the spacetime exterior derivative $d = dx^\mu\partial_\mu$,

$$\tilde{s} = s + d\ , \quad (2.19)$$

[3]Since antighosts and the corresponding Nakanishi-Lautrup (Lagrange multiplier) fields and their antifields contribute only via trivial solutions to the cohomological problem (see e.g. [22, 9]), they are completely neglected in this paper without loss of generality.

ε_A is the Grassmann parity of $\mathcal{D}_A$ ($\varepsilon_a = 0$, $\varepsilon_\alpha = \varepsilon_{\dot\alpha} = 1$), $\mathcal{T}$ denotes an arbitrary tensor field, and $\tilde\xi^A$, $\tilde C^I$ and $\tilde Q$ are "generalized connections" defined by

$$\begin{aligned}
\tilde\xi^a &= (C^\mu + dx^\mu)\, e_\mu{}^a, \\
\tilde\xi^\alpha &= \xi^\alpha + (C^\mu + dx^\mu)\, \psi_\mu{}^\alpha, \\
\tilde\xi^{\dot\alpha} &= \bar\xi^{\dot\alpha} - (C^\mu + dx^\mu)\, \bar\psi_\mu{}^{\dot\alpha}, \\
\tilde C^{ab} &= C^{ab} + (C^\mu + dx^\mu)\, \omega_\mu{}^{ab}, \\
\tilde C^i &= C^i + (C^\mu + dx^\mu)\, A_\mu{}^i, \\
\tilde Q &= Q + (C^\mu + dx^\mu)\, Q_\mu + \tfrac{1}{2}(C^\mu + dx^\mu)(C^\nu + dx^\nu)\, t_{\mu\nu}\ .
\end{aligned} \tag{2.20}$$

It should be noted that the equations (2.15)–(2.18) decompose into parts with different ghost numbers and form degrees. The reader may check that this decomposition reproduces at nonvanishing ghost numbers indeed the standard BRST transformations for supergravity, such as (2.1) and (2.2)–(2.10). Furthermore the ghost number 0 parts of (2.15)–(2.18) provide the explicit form of the super-covariant derivatives $\mathcal{D}_a$, of the spin connection, and of the super-covariantized field strengths $T_{ab}{}^\alpha$, $T_{ab}{}^{\dot\alpha}$, $F_{ab}{}^I$ and H_{abc} respectively (see appendix B for details). The consistency of all these formulae and the nilpotency of the BRST transformations is guaranteed by the algebra (2.13) and the Jacobi and Bianchi identities implied by it.

Remark: I stress again that the $\mathcal{D}_A$ do *not* act in a superspace here. Rather, they are algebraically (and nonlinearly) realized on the tensor fields which are *not* superfields (see appendix B for more details concerning the approach used here). One can "promote" this realization to superspace but this would not be useful for the cohomological analysis, see e.g. [29].

3 Off-shell basis for the tensor fields

It will be crucial for the analysis to determine an appropriate "basis" for the tensor fields, i.e. for the fields (2.14) and all their super-covariant derivatives. Here "basis" is not used in the vector space sense but for a subset $\{\mathcal{T}^r\}$ of these tensor fields which allows to express any tensor field uniquely in terms of the elements of this subset. Later, when the full BRST cohomology is computed, we will construct an analogous on-shell basis. Here we will construct an off-shell basis[4]. To that end we first decompose the torsions $T_{ab}{}^\alpha$ and $T_{ab}{}^{\dot\alpha}$ ($\equiv$ gravitino field strengths) and the curvatures $F_{ab}{}^I$ ($\equiv$ Lorentz- and Yang–Mills field strengths) into Lorentz irreducible multiplets, using $T_{\alpha\dot\alpha\,\beta\dot\beta\,\gamma} = \sigma^a_{\alpha\dot\alpha}\sigma^b_{\beta\dot\beta}T_{ab\gamma}$ etc.,

$$T_{\alpha\dot\alpha\,\beta\dot\beta\,\gamma} = \varepsilon_{\alpha\beta}U_{\dot\alpha\dot\beta\gamma} + \varepsilon_{\dot\beta\dot\alpha}(W_{\alpha\beta\gamma} + \tfrac{2}{3}\varepsilon_{\gamma(\alpha}S_{\beta)}), \tag{3.1}$$

$$F_{\alpha\dot\alpha\,\beta\dot\beta}{}^i = \varepsilon_{\alpha\beta}\bar G_{\dot\alpha\dot\beta}{}^i + \varepsilon_{\dot\alpha\dot\beta}G_{\alpha\beta}{}^i, \tag{3.2}$$

$$\begin{aligned}
F_{\alpha\dot\alpha\,\beta\dot\beta\,\gamma\dot\gamma\,\delta\dot\delta} &= \varepsilon_{\dot\alpha\dot\beta}\varepsilon_{\dot\gamma\dot\delta}[X_{\alpha\beta\gamma\delta} - \tfrac{1}{6}(\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta} + \varepsilon_{\beta\gamma}\varepsilon_{\alpha\delta})\mathcal{R}] - \varepsilon_{\alpha\beta}\varepsilon_{\dot\gamma\dot\delta}Y_{\gamma\delta\dot\alpha\dot\beta} \\
&\quad + \varepsilon_{\alpha\beta}\varepsilon_{\gamma\delta}[\bar X_{\dot\alpha\dot\beta\dot\gamma\dot\delta} - \tfrac{1}{6}(\varepsilon_{\dot\alpha\dot\gamma}\varepsilon_{\dot\beta\dot\delta} + \varepsilon_{\dot\beta\dot\gamma}\varepsilon_{\dot\alpha\dot\delta})\mathcal{R}] - \varepsilon_{\dot\alpha\dot\beta}\varepsilon_{\gamma\delta}Y_{\alpha\beta\dot\gamma\dot\delta}\ .
\end{aligned} \tag{3.3}$$

[4]Analogous constructions can be found in chapters 4 and 5 of [30].

Here the components of U, W, G, X and Y are completely symmetric in all their undotted and dotted spinor indices respectively. X, Y and $\mathcal{R}$ are the super-covariantized Weyl tensor, trace-free Ricci tensor and curvature scalar in spinor notation respectively (one has $\mathcal{R} = F_{ab}{}^{ba}$, $Y_{ab} = F_{acb}{}^{c} + \frac{1}{4}\eta_{ab}\mathcal{R}$).

We now construct a basis for all the super-covariant derivatives of the tensor fields (2.14). To that end we introduce the short hand notation $\mathcal{T}^m_n$ for a Lorentz irreducible multiplet of tensor fields whose components have m dotted and undotted n spinor indices and are completely symmetric in them respectively,

$$\mathcal{T}^m_n \equiv \{\mathcal{T}^{\dot\alpha_1\cdots\dot\alpha_m}_{\alpha_1\cdots\alpha_n}\}, \quad \mathcal{T}^{\dot\alpha_1\cdots\dot\alpha_m}_{\alpha_1\cdots\alpha_n} = \mathcal{T}^{(\dot\alpha_1\cdots\dot\alpha_m)}_{(\alpha_1\cdots\alpha_n)} \ .$$

We now define the following operations $\mathcal{D}^+_+, \ldots, \mathcal{D}^-_-$, using $\mathcal{D}_{\alpha\dot\alpha} = \sigma^a_{\alpha\dot\alpha}\mathcal{D}_a$:

$$\begin{aligned}
\mathcal{D}^+_+\mathcal{T}^m_n &\equiv \{\mathcal{D}^{(\dot\alpha_0}_{(\alpha_0}\mathcal{T}^{\dot\alpha_1\cdots\dot\alpha_m)}_{\alpha_1\cdots\alpha_n)}\},\\
\mathcal{D}^-_+\mathcal{T}^m_n &\equiv \{m\,\mathcal{D}_{\dot\alpha_m(\alpha_0}\mathcal{T}^{\dot\alpha_1\cdots\dot\alpha_m}_{\alpha_1\cdots\alpha_n)}\},\\
\mathcal{D}^+_-\mathcal{T}^m_n &\equiv \{n\,\mathcal{D}^{\alpha_n(\dot\alpha_0}\mathcal{T}^{\dot\alpha_1\cdots\dot\alpha_m)}_{\alpha_1\cdots\alpha_n}\},\\
\mathcal{D}^-_-\mathcal{T}^m_n &\equiv \{mn\,\mathcal{D}^{\alpha_n}_{\dot\alpha_m}\mathcal{T}^{\dot\alpha_1\cdots\dot\alpha_m}_{\alpha_1\cdots\alpha_n}\}.
\end{aligned} \tag{3.4}$$

Using the algebra (2.13) it is straightforward to verify that these operations satisfy

$$\begin{aligned}
[\mathcal{D}^\pm_\pm, \mathcal{D}^\mp_\pm]\mathcal{T}^m_n &= O(2),\\
[\mathcal{D}^\pm_\pm, \mathcal{D}^\pm_\mp]\mathcal{T}^m_n &= O(2),\\
[\mathcal{D}^+_+, \mathcal{D}^-_-]\mathcal{T}^m_n &= (m+n+2)\Box\mathcal{T}^m_n + O(2),\\
[\mathcal{D}^+_-, \mathcal{D}^-_+]\mathcal{T}^m_n &= (m-n)\Box\mathcal{T}^m_n + O(2),\\
\mathcal{D}^-_+\mathcal{D}^+_-\mathcal{T}^m_n &= \tfrac{1}{2}\,n(m+2)\,\Box\mathcal{T}^m_n + \mathcal{D}^+_+\mathcal{D}^-_-\mathcal{T}^m_n + O(2).
\end{aligned} \tag{3.5}$$

Here $O(2)$ denotes terms which are at least quadratic in the tensor fields and we used the super-covariant d'Alembertian

$$\Box = \mathcal{D}_a\mathcal{D}^a = \tfrac{1}{2}\mathcal{D}_{\alpha\dot\alpha}\mathcal{D}^{\dot\alpha\alpha} \ .$$

The relations (3.5) imply that, up to terms $O(2)$, all super-covariant derivatives of an arbitrary $\mathcal{T}^m_n$ can be expressed as linear combinations of the following quantities:

$$\Box^p(\mathcal{D}^+_+)^q(\mathcal{D}^-_+)^r(\mathcal{D}^-_-)^s\mathcal{T}^m_n\ , \quad \Box^p(\mathcal{D}^+_+)^q(\mathcal{D}^+_-)^{r'}(\mathcal{D}^-_-)^{s'}\mathcal{T}^m_n \tag{3.6}$$

where $p, q \geq 0$, $(r+s) \leq m$, $s \leq n$, $(r'+s') \leq n$, $s' \leq m$ and $r' > 0$ (the last condition avoids a double counting of the terms with $r = 0$ and $r' = 0$). Note that none of the expressions (3.6) contains both $\mathcal{D}^-_+$ and $\mathcal{D}^+_-$ as a consequence of the last two relations in (3.5).

Up to terms $O(2)$, (3.6) provides already a basis for the super-covariant derivatives of the 'elementary' (non-composite) tensor fields M, B, λ^i and D^i. However, when applied to the field strengths, it yields an overcomplete set of super-covariant

derivatives because there are still algebraic identities relating the elements of this set. These remaining identities reflect the Bianchi identities

$$\mathcal{D}_{[a}T_{bc]}{}^{\alpha} = O(2), \quad \mathcal{D}_{[a}F_{bc]}{}^{I} = O(2) \tag{3.7}$$

and the super-covariant derivatives thereof. In order to remove this remaining redundancy, we first decompose the identities (3.7) into Lorentz irreducible parts. In the above notation the result is

$$\mathcal{D}_-^- U + 2\mathcal{D}_-^+ S = O(2), \tag{3.8}$$
$$2\mathcal{D}_-^+ W + 2\mathcal{D}_+^+ S - 3\mathcal{D}_+^- U = O(2), \tag{3.9}$$
$$\mathcal{D}_-^+ X - 2\mathcal{D}_+^- Y = O(2), \tag{3.10}$$
$$\mathcal{D}_-^- Y - 2\mathcal{D}_+^+ \mathcal{R} = O(2), \tag{3.11}$$
$$\mathcal{D}_-^+ G^i + \mathcal{D}_+^- \bar{G}^i = O(2) \tag{3.12}$$

and the complex conjugation of (3.8)–(3.10) (the remaining identities (3.11) and (3.12) are real). Now, (3.8), all its super-covariant derivatives and the corresponding complex conjugated identities show that all super-covariant derivatives (3.6) of U and $\bar{U}$ containing the operation $\mathcal{D}_-^-$ can be expressed in terms of super-covariant derivatives of S and $\bar{S}$ up to terms $O(2)$. Analogously (3.9)–(3.12) (and their super-covariant derivatives and complex conjugations) allow to eliminate all super-covariant derivatives (3.6) of W, $\bar{W}$, X, $\bar{X}$ and $\bar{G}^i$ except for those with $(p, q, r, s) = (0, q, 0, 0)$, and all super-covariant derivatives of Y containing the operation $\mathcal{D}_-^-$, up to terms of $O(2)$. As the $O(2)$ terms are composed of tensor fields that have lower dimension than the respective considered quantity, one can easily prove by induction that in old minimal supergravity the following list provides a basis $\{\mathcal{T}^r\}$ for the tensor fields in the above sense:

$$\begin{array}{rl}
M, \bar{M}: & \Box^p(\mathcal{D}_+^+)^q\{M,\, \bar{M}\}; \\
B: & \Box^p(\mathcal{D}_+^+)^q\{B,\, \mathcal{D}_+^- B,\, \mathcal{D}_-^+ B,\, \mathcal{D}_-^- B\}; \\
S, \bar{S}: & \Box^p(\mathcal{D}_+^+)^q\{S,\, \bar{S},\, \mathcal{D}_-^+ S,\, \mathcal{D}_+^- \bar{S}\}; \\
U, \bar{U}: & \Box^p(\mathcal{D}_+^+)^q\{U,\, \bar{U},\, \mathcal{D}_-^+ U,\, \mathcal{D}_+^- U,\, \mathcal{D}_-^+ \bar{U},\, \mathcal{D}_+^- \bar{U},\, (\mathcal{D}_+^-)^2 U,\, (\mathcal{D}_-^+)^2 \bar{U}\}; \\
W, \bar{W}: & (\mathcal{D}_+^+)^q\{W,\, \bar{W}\}; \\
\mathcal{R}: & \Box^p(\mathcal{D}_+^+)^q \mathcal{R}; \\
Y: & \Box^p(\mathcal{D}_+^+)^q\{Y,\, \mathcal{D}_-^+ Y,\, \mathcal{D}_+^- Y,\, (\mathcal{D}_-^+)^2 Y,\, (\mathcal{D}_+^-)^2 Y\}; \\
X, \bar{X}: & (\mathcal{D}_+^+)^q\{X,\, \bar{X}\}; \\
\lambda^i, \bar{\lambda}^i: & \Box^p(\mathcal{D}_+^+)^q\{\lambda^i,\, \bar{\lambda}^i,\, \mathcal{D}_-^+ \lambda^i,\, \mathcal{D}_+^- \bar{\lambda}^i\}; \\
G^i, \bar{G}^i: & (\mathcal{D}_+^+)^q \bar{G}^i, \quad \Box^p(\mathcal{D}_+^+)^q\{G^i,\, \mathcal{D}_-^+ G^i,\, (\mathcal{D}_-^+)^2 G^i\}; \\
D^i: & \Box^p(\mathcal{D}_+^+)^q D^i
\end{array} \tag{3.13}$$

where $p, q = 0, 1, \ldots$ and we used the notation

$$\Box^p(\mathcal{D}_+^+)^q\{M,\, \bar{M}\} \equiv \{\Box^p(\mathcal{D}_+^+)^q M,\, \Box^p(\mathcal{D}_+^+)^q \bar{M}\} \quad \textit{etc.}\ .$$

I stress that all the tensor fields listed in (3.13) are algebraically independent, i.e. a function $f(\mathcal{T})$ vanishes off-shell if and only if it vanishes identically in terms of the elementary fields and their derivatives.

An analogous off-shell basis for the tensor fields in new minimal supergravity is obtained from the above list by discarding M, $\bar{M}$ and B (and, if present, the tensor fields associated with R-transformations) and adding to it

$$\begin{aligned}
&\Box^p(\mathcal{D}_+^+)^q\{\tilde{H},\, \mathcal{D}_+^-\tilde{H},\, \mathcal{D}_-^+\tilde{H}\},\\
&(\mathcal{D}_+^+)^q\bar{G}^{(r)}, \quad \Box^p(\mathcal{D}_+^+)^q\{G^{(r)},\, \mathcal{D}_-^+G^{(r)},\, (\mathcal{D}_-^+)^2G^{(r)}\}
\end{aligned} \tag{3.14}$$

where $\tilde{H} \equiv \{\tilde{H}_\alpha^{\dot{\alpha}}\}$ denotes the dual of H_{abc}, $\tilde{H}^a = \frac{1}{6}\varepsilon^{abcd}H_{bcd}$ (one has $\mathcal{D}_-^-\tilde{H} = 0$ and therefore (3.14) contains no terms with $\mathcal{D}_-^-$).

4 Restricted BRST cohomology

4.1 Reduction to the covariant cohomology of $\tilde{s}$

To compute the restricted BRST cohomology we have to solve (1.1) and (1.2) for antifield independent local forms ω_4, ω_3, η_4 and η_3. The first step towards the solution of this problem reduces it (locally) to the cohomology of $\tilde{s} = s + d$ on local functions depending only on the tensor fields $\mathcal{T}^r$ given in the previous section and on the generalized connections (2.20). To show this we use the fact that (1.1) implies descent equations for s and d,

$$s\omega_p + d\omega_{p-1} = 0, \quad p = 0, \dots, 4 \quad (\omega_{-1} \equiv 0), \tag{4.1}$$

which are compactly written in the form

$$\tilde{s}\,\omega = 0, \quad \omega = \sum_{p=0}^{4} \omega_p \ . \tag{4.2}$$

The nontriviality condition (1.2) translates into

$$\omega \neq \tilde{s}\,\eta + \mathit{constant} \tag{4.3}$$

where η is a sum of local forms like ω. Such sums are the *local total forms* mentioned in the introduction. Thanks to the closure of the gauge algebra, $\tilde{s}$ is strictly nilpotent on antifield independent forms and therefore the restricted local BRST cohomology can be obtained from the cohomology of $\tilde{s}$ in the space of antifield independent local total forms.

The analysis can be considerably simplified by a suitable choice of local "jet coordinates" in terms of which all antifield independent total forms can be expressed, cf. [20] for details. An appropriate set of local jet coordinates is $\{\mathcal{U}^l, \mathcal{V}^l, \mathcal{W}^i\}$ where

$$\{\mathcal{U}^l\} \;=\; \{x^\mu,\, \partial_{(\mu_1\dots\mu_k}e_{\mu_{k+1})}{}^a,\, \partial_{(\mu_1\dots\mu_k}\omega_{\mu_{k+1})}{}^{ab},$$

$$
\begin{aligned}
&\partial_{(\mu_1\ldots\mu_k}\psi_{\mu_{k+1})}{}^{\alpha},\ \partial_{(\mu_1\ldots\mu_k}\bar{\psi}_{\mu_{k+1})}{}^{\dot{\alpha}},\\
&\partial_{(\mu_1\ldots\mu_k}t_{\mu_{k+1})\nu},\ \partial_{(\mu_1\ldots\mu_k}Q_{\mu_{k+1})},\\
&\partial_{(\mu_1\ldots\mu_k}A_{\mu_{k+1})}{}^{i}:\ k=0,1,\ldots\},
\end{aligned}
\tag{4.4}
$$

$$
\{\mathcal{V}^l\} = \{\tilde{s}\,\mathcal{U}^l\} \tag{4.5}
$$

$$
\{\mathcal{W}^i\} = \{\tilde{C}^I,\, \tilde{\xi}^A,\, \tilde{Q},\, \mathcal{T}^r\}. \tag{4.6}
$$

These jet coordinates satisfy

$$
\tilde{s}\,\mathcal{U}^l = \mathcal{V}^l, \quad \tilde{s}\,\mathcal{W}^i = R^i(\mathcal{W}) \tag{4.7}
$$

which implies by Künneth's formula that the $\tilde{s}$-cohomology on local total forms $\omega(\mathcal{U},\mathcal{V},\mathcal{W})$ can be obtained from the $\tilde{s}$-cohomologies on local total forms $\alpha(\mathcal{U},\mathcal{V})$ and $\alpha(\mathcal{W})$ according to

$$
\tilde{s}\,\omega(\mathcal{U},\mathcal{V},\mathcal{W}) = 0 \ \Rightarrow\ \omega(\mathcal{U},\mathcal{V},\mathcal{W}) = a_{ai}\,\alpha^i(\mathcal{U},\mathcal{V})\,\alpha^a(\mathcal{W}) + \tilde{s}\,\eta(\mathcal{U},\mathcal{V},\mathcal{W}). \tag{4.8}
$$

Here the a_{ai} are constants and $\alpha^i(\mathcal{U},\mathcal{V})$ and $\alpha^a(\mathcal{W})$ represent nontrivial $\tilde{s}$-cohomology classes in the space of local total forms $\alpha(\mathcal{U},\mathcal{V})$ and $\alpha(\mathcal{W})$ respectively.

Now, in the manifold of the $\mathcal{U}$'s and $\mathcal{V}$'s the $\tilde{s}$-cohomology is *locally* trivial thanks to (4.7) and thus locally represented by a constant. Hence, locally the restricted BRST cohomology reduces to the $\tilde{s}$-cohomology on local total forms $\alpha(\mathcal{W})$ and can be obtained from the solutions of

$$
\tilde{s}\alpha(\mathcal{W}) = 0, \quad \alpha(\mathcal{W}) \neq \tilde{s}\beta(\mathcal{W}) + \mathit{constant}. \tag{4.9}
$$

The modifications due to global (topological) aspects of the manifold of the $\mathcal{U}$'s and $\mathcal{V}$'s are roughly sketched in section 11 along the lines of [26].

For later purpose I add a few comments.

1. If ω_4 has ghost number g, then ω has *total degree* $G = g + 4$ where the total degree (totdeg) is given by the sum of the form degree (formdeg) and the ghost number (gh),
$$
\text{totdeg} = \text{gh} + \text{formdeg}. \tag{4.10}
$$
Hence, the (restricted) BRST cohomology at ghost number g is obtained from the (restricted) $\tilde{s}$-cohomology at total degree $G = g + 4$. The total degrees of the $\mathcal{W}$'s are given by
$$
\text{totdeg}(\mathcal{T}^r) = 0, \quad \text{totdeg}(\tilde{\xi}^A) = \text{totdeg}(\tilde{C}^I) = 1, \quad \text{totdeg}(\tilde{Q}) = 2. \tag{4.11}
$$

2. We can assume without loss of generality that the solutions to (4.9) are real. Indeed, as $\tilde{s}$ is a real operator (see appendix A.2 for the conventions concerning complex conjugation used here), $\tilde{s}\alpha = 0$ implies that both the real and the imaginary part of α are separately $\tilde{s}$-invariant, and $\alpha = \tilde{s}\beta$ implies that both parts are $\tilde{s}$-exact.

3. (4.7) implies that $\alpha(\mathcal{W})$ is $\tilde{s}$-exact in the space of local total forms $\alpha(\mathcal{W})$ whenever it is $\tilde{s}$-exact in the space of all local total forms,

$$\alpha(\mathcal{W}) = \tilde{s}\,\eta(\mathcal{U},\mathcal{V},\mathcal{W}) \quad \Rightarrow \quad \exists\,\beta:\ \alpha(\mathcal{W}) = \tilde{s}\,\beta(\mathcal{W}). \tag{4.12}$$

4. In the following the analysis will be carried out for old and new minimal supergravity simultaneously. It should be kept in mind however that $\tilde{Q}$ and the super-covariant field strength H_{abc} of $t_{\mu\nu}$ occur only in new minimal supergravity, whereas the auxiliary fields M and B_a are present only in old minimal supergravity.

5. The space of local total forms $\alpha(\mathcal{W})$ to be considered for the computation of the restricted $\tilde{s}$-cohomology is actually the space of polynomials in the $\mathcal{W}$'s. This follows from the fact that $\tilde{s}$ has total degree 1 and vanishing dimension on the $\mathcal{W}$'s. Hence, the restricted $\tilde{s}$-cohomology can be computed separately for each subspace of total forms with given dimension and total degree. As all tensor fields have positive dimension, *local* total forms $\alpha(\mathcal{W})$ with fixed dimension and total degree are necessarily polynomials in the $\mathcal{W}$'s since we do not allow them to involve negative powers of the auxiliary fields (because otherwise they may become nonlocal or even ill-defined after eliminating the auxiliary fields through their equations of motion)[5].

4.2 Decomposition of the problem

To solve (4.9), we expand it in $\tilde{Q}$ and in the $\tilde{C}^I$, using

$$\alpha(\mathcal{W}) = \sum_{k=0}^{m} \alpha_k(\mathcal{W}), \quad N_{CQ}\,\alpha_k = k\,\alpha_k \tag{4.13}$$

where N_{CQ} is the counting operator for the $\tilde{C}^I$ and $\tilde{Q}$,

$$N_{CQ} = \tilde{C}^I\,\frac{\partial}{\partial \tilde{C}^I} + \tilde{Q}\,\frac{\partial}{\partial \tilde{Q}}\ . \tag{4.14}$$

Note that m in (4.13) cannot exceed the total degree of α because α does not involve antifields (hence, m is always finite). $\tilde{s}$ decomposes on the $\mathcal{W}$'s into three parts,

$$\tilde{s}\,\alpha(\mathcal{W}) = (\tilde{s}_{lie} + \tilde{s}_{susy} + \tilde{s}_{curv})\,\alpha(\mathcal{W}), \tag{4.15}$$

which have N_{CQ}-degrees $1, 0, -1$ respectively,

$$[N_{CQ}, \tilde{s}_{lie}] = \tilde{s}_{lie}\,, \quad [N_{CQ}, \tilde{s}_{susy}] = 0\,, \quad [N_{CQ}, \tilde{s}_{curv}] = -\tilde{s}_{curv}\ . \tag{4.16}$$

[5] All the arguments go through also for formal local series' in the $\mathcal{W}$'s. This can become important when matter multiplets are included because then it might not be justified anymore to restrict the investigation to polynomials in the $\mathcal{W}$'s.

Note that this implies the following anticommutation relations due to $\tilde{s}^2 = 0$:

$$(\tilde{s}_{lie})^2 = (\tilde{s}_{curv})^2 = \{\tilde{s}_{lie}, \tilde{s}_{susy}\} = \{\tilde{s}_{curv}, \tilde{s}_{susy}\} = 0,$$
$$\{\tilde{s}_{lie}, \tilde{s}_{curv}\} + (\tilde{s}_{susy})^2 = 0. \tag{4.17}$$

The three parts of $\tilde{s}$ are spelled out explicitly in table 4.1 where $\mathcal{F}^I$ and $\mathcal{H}$ are total super-curvature forms given by

$$\mathcal{F}^I = -\tfrac{1}{2}(-)^{\varepsilon_A}\tilde{\xi}^A\tilde{\xi}^B F_{BA}{}^I, \tag{4.18}$$
$$\mathcal{H} = \tfrac{1}{6}\,\tilde{\xi}^a\tilde{\xi}^b\tilde{\xi}^c H_{abc} + i\,\tilde{\xi}^\alpha\tilde{\xi}_{\alpha\dot{\alpha}}\tilde{\xi}^{\dot{\alpha}}\ . \tag{4.19}$$

It is evident from table 4.1 that the three parts of $\tilde{s}$ have well-distinguished interpretations indicated by their subscripts: $\tilde{s}_{lie}$ encodes the Lie algebra of $\mathcal{G}$, $\tilde{s}_{susy}$ contains the supersymmetry transformations and the super-covariant derivatives, and $\tilde{s}_{curv}$ transforms the $\tilde{C}^I$ and $\tilde{Q}$ into the corresponding total super-curvature forms.

$\mathcal{W}$	$\tilde{s}_{lie}\mathcal{W}$	$\tilde{s}_{susy}\mathcal{W}$	$\tilde{s}_{curv}\mathcal{W}$
$\tilde{Q}$	0	0	$\mathcal{H}$
$\tilde{C}^I$	$\frac{1}{2}\tilde{C}^J\tilde{C}^K f_{KJ}{}^I$	0	$\mathcal{F}^I$
$\tilde{\xi}^A$	$\tilde{C}^I g_{IB}{}^A\tilde{\xi}^B$	$-\frac{1}{2}(-)^{\varepsilon_B}\tilde{\xi}^B\tilde{\xi}^C T_{CB}{}^A$	0
$\mathcal{T}$	$\tilde{C}^I\delta_I\mathcal{T}$	$\tilde{\xi}^A\mathcal{D}_A\mathcal{T}$	0

Table 4.1: Decomposition of $\tilde{s}$

$\tilde{s}\alpha = 0$ thus decomposes into

$$0 = \tilde{s}_{lie}\alpha_m\ , \tag{4.20}$$
$$0 = \tilde{s}_{susy}\alpha_m + \tilde{s}_{lie}\alpha_{m-1}\ , \tag{4.21}$$
$$0 = \tilde{s}_{curv}\alpha_m + \tilde{s}_{susy}\alpha_{m-1} + \tilde{s}_{lie}\alpha_{m-2}\ , \tag{4.22}$$
$$\vdots$$

Now, the cohomology of $\tilde{s}_{lie}$ is well-known. It is just the Lie algebra cohomology of $\mathcal{G}$. Since all the $\tilde{\xi}^A$ and the tensor fields transform according to finite dimensional linear representations of $\mathcal{G}$ and since $\mathcal{G}$ is reductive by assumption, the cohomology of $\tilde{s}_{lie}$ is represented by functions of the form $f_i(\tilde{\xi}, \mathcal{T})P^i(\tilde{\theta}, \tilde{Q})$ where the f_i are $\mathcal{G}$-invariant and the $P^i(\tilde{\theta}, \tilde{Q})$ are linearly independent monomials in $\tilde{Q}$ and in the primitive elements $\tilde{\theta}_K$ of the Lie algebra cohomology of $\mathcal{G}$. The $\tilde{\theta}_K$ themselves are polynomials in the $\tilde{C}^I$ and correspond to the independent Casimir operators of $\mathcal{G}$. Their number therefore equals the rank of $\mathcal{G}$ ($K = 1, \ldots, rank(\mathcal{G})$). They can be constructed using appropriate matrix representations $\{T_I^{(K)}\}$ of $\mathcal{G}$ (see e.g. [31]),

$$\tilde{\theta}_K = (-)^{m_K+1}\frac{m_K!(m_K-1)!}{(2m_K-1)!}\,Tr(\tilde{C}^{2m_K-1}), \quad \tilde{C} = \tilde{C}^I T_I^{(K)} \tag{4.23}$$

where m_K denotes the order of the Casimir operator corresponding to $\tilde{\theta}_K$.

(4.20) implies thus

$$\alpha_m = f_i(\tilde{\xi}, \mathcal{T})\, P^i(\tilde{\theta}, \tilde{Q}), \quad \delta_I f_i(\tilde{\xi}, \mathcal{T}) = 0 \tag{4.24}$$

where

$$\delta_I \tilde{\xi}^A = g_{IB}{}^A \tilde{\xi}^B. \tag{4.25}$$

In (4.24) a contribution to α_m of the form $\tilde{s}_{lie}\beta_{m-1}$ has been neglected without loss of generality because such a contribution can be always removed from α_m by subtracting the trivial piece $\tilde{s}\beta_{m-1}$ from α.

Inserting (4.24) in (4.21) results in $(\tilde{s}_{susy} f_i)P^i + \tilde{s}_{lie}\alpha_{m-1} = 0$. Using the Lie algebra cohomology again, we conclude

$$\tilde{s}_{susy} f_i(\tilde{\xi}, \mathcal{T}) = 0. \tag{4.26}$$

Furthermore, we can assume without loss of generality that none of the f_i is of the form $\tilde{s}h(\tilde{\xi}, \mathcal{T})$. Namely otherwise we could remove that particular f_i by subtracting the trivial piece $\tilde{s}(hP^i)$ from α. Notice that such a subtraction does not clash with the other redefinitions made so far. In particular it does not reintroduce a term $\tilde{s}_{lie}\beta_{m-1}$ in (4.24) (note that in general a similar reasoning would *not* apply if h would depend on one of the $\tilde{C}^I$).

Hence, the f_i are determined by the cohomology of $\tilde{s}_{susy}$ in the space of $\mathcal{G}$-invariant local total forms depending only on the $\tilde{\xi}$ and $\mathcal{T}$ (note that $\tilde{s}_{susy}$ is nilpotent in this space). Now, the latter cohomology is completely equivalent to the $\tilde{s}$-cohomology in the space of local total forms $f(\tilde{\xi}, \mathcal{T})$. Indeed (4.24) and (4.26) require f_i to be $\tilde{s}$-invariant because $\delta_I f_i = 0$ implies $\tilde{s}_{lie} f_i = 0$ and because $\tilde{s}_{curv} f_i = 0$ holds trivially as f_i depends neither on the $\tilde{C}^I$ nor on $\tilde{Q}$. Conversely, $\tilde{s} f_i(\tilde{\xi}, \mathcal{T}) = 0$ requires f_i to be $\mathcal{G}$-invariant and $f_i(\tilde{\xi}, \mathcal{T}) = \tilde{s}h(\tilde{\xi}, \mathcal{T})$ requires h to be $\mathcal{G}$-invariant (this is evident from expanding these equations in $\tilde{C}^I$). The f_i can thus be assumed to solve

$$\tilde{s} f_i(\tilde{\xi}, \mathcal{T}) = 0, \quad f_i(\tilde{\xi}, \mathcal{T}) \neq \tilde{s}h_i(\tilde{\xi}, \mathcal{T}). \tag{4.27}$$

Note that this does not necessarily imply that f_i solves (4.9) too. Namely one might have $f_i = \tilde{s}\beta(\mathcal{W})$ but nevertheless $f_i \neq \tilde{s}h(\tilde{\xi}, \mathcal{T})$.

4.3 Cohomology of $\tilde{s}$ on local total forms $f(\tilde{\xi}, \mathcal{T})$

The solution of (4.27) is the most involved part and the cornerstone of the computation of the restricted BRST cohomology. The result is the following:

Nonconstant solutions to (4.27) exist only at total degrees 2, 3 and 4,

$$\tilde{s} f(\tilde{\xi}, \mathcal{T}) = 0, \quad \text{totdeg}(f) = G$$

$$\Leftrightarrow \quad f(\tilde{\xi}, \mathcal{T}) = \begin{cases} \textit{constant} & \text{for } G = 0 \\ \tilde{s}h(\mathcal{T}) & \text{for } G = 1 \\ a_{i_a}\mathcal{F}^{i_a} + \tilde{s}h(\tilde{\xi}, \mathcal{T}) & \text{for } G = 2 \\ a\,\mathcal{H} + \tilde{s}h(\tilde{\xi}, \mathcal{T}) & \text{for } G = 3 \\ a^\Delta \mathcal{P}_\Delta + \tilde{s}h(\tilde{\xi}, \mathcal{T}) & \text{for } G = 4 \\ \tilde{s}h(\tilde{\xi}, \mathcal{T}) & \text{for } G > 4. \end{cases} \tag{4.28}$$

Here the a's are constants, the h's are $\mathcal{G}$-invariant, the $\mathcal{F}^{i_a}$ are the *abelian* total curvature forms (4.18), $\mathcal{H}$ is the total curvature form corresponding to $t_{\mu\nu}$ given in (4.19), and the $\mathcal{P}_\Delta$ are reminiscent of superspace integrals,

$$\mathcal{P}_\Delta = (\hat{\mathcal{D}}_{\dot{\alpha}}\hat{\mathcal{D}}^{\dot{\alpha}} - M)\,\Xi\,\{\mathcal{A}_\Delta(\bar{M}, \bar{W}, \bar{\lambda}) + (\mathcal{D}^\alpha \mathcal{D}_\alpha - \bar{M})\mathcal{B}_\Delta(\mathcal{T})\} + c.c. \tag{4.29}$$

where

$$\Xi = -\tfrac{1}{24}\,\varepsilon_{abcd}\,\tilde{\xi}^a\tilde{\xi}^b\tilde{\xi}^c\tilde{\xi}^d\ , \tag{4.30}$$

$$\hat{\mathcal{D}}_{\dot{\alpha}} = \tilde{\xi}^B T_{B\dot{\alpha}}{}^A \frac{\partial}{\partial \tilde{\xi}^A} + \bar{\mathcal{D}}_{\dot{\alpha}}\ , \tag{4.31}$$

$$\delta_I \mathcal{P}_\Delta = 0. \tag{4.32}$$

In (4.29) the operators $\hat{\mathcal{D}}_{\dot{\alpha}}$ act on everything on their right (i.e. $\hat{\mathcal{D}}_{\dot{\alpha}}\hat{\mathcal{D}}^{\dot{\alpha}}$ acts on $\Xi\{\cdots\}$, not just on the "total volume form" Ξ). The $T_{AB}{}^C$ in (4.31) are the torsions of the supergravity algebra (2.13) spelled out explicitly in appendix B. As indicated by its arguments, $\mathcal{A}_\Delta$ is a polynomial in the *undifferentiated* fields $\bar{M}$, $\bar{W}_{\dot{\alpha}\dot{\beta}\dot{\gamma}}$, $\bar{\lambda}^i_{\dot{\alpha}}$ ($W_{\alpha\beta\gamma}$ is the chiral part of the gravitino field strength, see section 3). (4.32) requires $\mathcal{B}_\Delta$ to be $\mathcal{G}$-invariant and $\mathcal{A}_\Delta$ to be $\mathcal{G}$-invariant except under R-transformations (if R-transformations are gauged, $\mathcal{A}_\Delta$ must have R-charge -2 as $\hat{\mathcal{D}}_{\dot{\alpha}}$ carries R-weight 1 according to the conventions used here, see appendix B). I stress that the $\mathcal{A}_\Delta$ and $\mathcal{B}_\Delta$ need not satisfy any additional requirement in order to ensure $\tilde{s}\mathcal{P}_\Delta = 0$ (but of course we require them to be local functions). The term $a\mathcal{H}$ occurring in (4.28) for $G = 3$ contributes only in new minimal supergravity and in (4.29) the field M has to be set to zero in new minimal supergravity.

In the remainder of this section I sketch the basic ideas used to derive the above result, relegating details of the computation to the appendices. Let us first consider the cases $G < 4$. In these cases we can use the fact that *any* $\tilde{s}$-closed local total form that does not depend on antifields (!) is necessarily (locally) $\tilde{s}$-exact in the space of antifield independent local total forms except for the constants,

$$\begin{gathered} \tilde{s}\,\omega(\mathcal{U}, \mathcal{V}, \mathcal{W}) = 0,\ \ \mathrm{totdeg}(\omega) = G < 4 \\ \Rightarrow \quad \omega = \begin{cases} \textit{constant} & \text{for } G = 0 \\ \tilde{s}\eta(\mathcal{U}, \mathcal{V}, \mathcal{W}) & \text{for } 0 < G < 4. \end{cases} \end{gathered} \tag{4.33}$$

This follows from very general arguments which are not restricted to the theories under study but hold analogously whenever the gauge algebra closes off-shell. Indeed,

recall that $\tilde{s}\omega = 0$ decomposes into descent equations (4.1) and that the nontrivial (and nonconstant) solutions of these equations in the space of antifield independent local forms correspond (locally) one-to-one to the nontrivial solutions of (1.1) in the space of antifield independent local volume forms. Now, if ω has total degree < 4 and does not depend on antifields, it does not contain a nontrivial solution to (1.1), simply because it cannot contain a volume form. This implies (4.33).

In particular we conclude from (4.33) that $\tilde{s}f(\tilde{\xi}, \mathcal{T}) = 0$ implies $f = \tilde{s}\eta$ for $0 < G < 4$ and $f = constant$ for $G = 0$. However, this does not yet solve our problem (4.27) for $G < 4$ because to that end we have still to investigate whether or not η can be assumed to depend only on the $\tilde{\xi}$ and $\mathcal{T}$. This investigation is performed in appendix D within a derivation of the "super-covariant Poincaré lemma" which is the counterpart of the "covariant Poincaré lemma" proved in [23] for standard gravity coupled to Yang–Mills fields[6]. The result is that one can indeed choose η to be of the form $h(\tilde{\xi}, \mathcal{T})$ except that in the case $G = 2$ it may involve a linear combination of the abelian $\tilde{C}$'s and in the case $G = 3$ it may contain a term proportional to $\tilde{Q}$. This provides the results for $G < 4$ in (4.28) due to $\tilde{s}\tilde{C} = \mathcal{F}$ for the abelian $\tilde{C}$'s and $\tilde{s}\tilde{Q} = \mathcal{H}$.

The investigation of (4.27) is much more involved in the cases $G \geq 4$. In particular the supersymmetric structure of the theory plays an essential role in these cases. The strategy to attack this problem is based on spectral sequence techniques using the degree in the tensor fields as filtration. That is to say, we decompose the equation $\tilde{s}f(\tilde{\xi}, \mathcal{T}) = 0$ into parts with definite degree in the tensor fields and analyze it starting from the part with lowest degree. The decomposition is unique and thus well-defined thanks to the completeness and algebraic independence of the $\mathcal{T}$'s, cf. section 3. The decomposition of $\tilde{s}$ takes the form

$$\tilde{s} = \sum_{k \geq 0} \tilde{s}_{(k)} \ , \quad [N_{\mathcal{T}}, \tilde{s}_{(k)}] = k\tilde{s}_{(k)} \tag{4.34}$$

where $N_{\mathcal{T}}$ is the counting operator for the $\mathcal{T}$'s,

$$N_{\mathcal{T}} = \mathcal{T}^r \frac{\partial}{\partial \mathcal{T}^r} \ . \tag{4.35}$$

$f(\tilde{\xi}, \mathcal{T})$ decomposes into

$$f(\tilde{\xi}, \mathcal{T}) = \sum_{k \geq \ell} f_{(k)}(\tilde{\xi}, \mathcal{T}), \quad N_{\mathcal{T}} f_{(k)}(\tilde{\xi}, \mathcal{T}) = k f_{(k)}(\tilde{\xi}, \mathcal{T}). \tag{4.36}$$

$\tilde{s}f = 0$ requires, at lowest order in the tensor fields,

$$\tilde{s}_{(0)} f_{(\ell)}(\tilde{\xi}, \mathcal{T}) = 0. \tag{4.37}$$

Furthermore we can remove any piece of the form $\tilde{s}_{(0)} h_{(\ell)}(\tilde{\xi}, \mathcal{T})$ from $f_{(\ell)}$ without changing the cohomology class of f by substracting $\tilde{s}h_{(\ell)}$ from f. Hence, $f_{(\ell)}$ is

[6]See also [32, 22, 33] for analogous results in standard gravity and Yang–Mills theory.

actually determined by the $\tilde{s}_{(0)}$-cohomology in the space of local total forms $f(\tilde{\xi}, \mathcal{T})$. In particular we can assume

$$f_{(\ell)} \neq \tilde{s}_{(0)} h_{(\ell)}(\tilde{\xi}, \mathcal{T}). \tag{4.38}$$

The analysis of (4.37) and (4.38) proceeds along the lines of an analogous investigation performed in [28] for rigid supersymmetry. This is possible thanks to the structure of $\tilde{s}_{(0)}$ which splits into

$$\tilde{s}_{(0)} = \tilde{s}_{lie} + \delta_{susy}\ . \tag{4.39}$$

Here δ_{susy} is nothing but the linearized part of $\tilde{s}_{susy}$ (see table 4.1) and acts according to

$$\delta_{susy}\tilde{\xi}^a \ = \ 2i\,\tilde{\xi}^\alpha \tilde{\xi}^{\dot\alpha} \sigma^a_{\alpha\dot\alpha}\ , \tag{4.40}$$

$$\delta_{susy}\tilde{\xi}^\alpha \ = \ \delta_{susy}\tilde{\xi}^{\dot\alpha} \ = \ 0, \tag{4.41}$$

$$\delta_{susy}\mathcal{T}^r \ = \ \tilde{\xi}^A D_A \mathcal{T}^r \tag{4.42}$$

where $D_A\mathcal{T}$ is that part of $\mathcal{D}_A\mathcal{T}$ which is linear in the $\mathcal{T}$'s. The D_A are thus *linearly* realized on the tensor fields, in contrast to the $\mathcal{D}_A$, and satisfy an algebra involving only structure constants (rather than field dependent structure functions). This algebra is the linearized version of the covariant supergravity algebra of the $\mathcal{D}_A$. It reads simply

$$\{D_\alpha, \bar{D}_{\dot\alpha}\} = -2i\sigma^a_{\alpha\dot\alpha} D_a\ , \quad [D_A, D_B\} = 0 \quad \text{otherwise.} \tag{4.43}$$

(4.37) requires $f_{(\ell)}$ in particular to be $\mathcal{G}$-invariant in consequence of the presence of $\tilde{s}_{lie}$ in $\tilde{s}_{(0)}$ (recall that the $\tilde{s}$-cohomology on total forms $f(\tilde{\xi}, \mathcal{T})$ is nothing but the $\tilde{s}_{susy}$-cohomology on $\mathcal{G}$-invariant total forms $f(\tilde{\xi}, \mathcal{T})$, see section 4.2). Hence, (4.37) and (4.38) are equivalent to

$$\delta_{susy} f_{(\ell)}(\tilde{\xi}, \mathcal{T}) = 0, \quad f_{(\ell)} \neq \delta_{susy} h_{(\ell)}(\tilde{\xi}, \mathcal{T}), \quad \delta_I f_{(\ell)} = \delta_I h_{(\ell)} = 0. \tag{4.44}$$

$f_{(\ell)}$ is thus determined by the δ_{susy}-cohomology on $\mathcal{G}$-invariant local total forms $f(\tilde{\xi}, \mathcal{T})$. This problem is indeed analogous to the one investigated in [28] because (4.43) is of course nothing but the familiar algebra of rigid supersymmetry analyzed there. The only difference to the investigation performed in [28] is that in our case (4.43) is represented on tensor fields whereas in [28] it was represented on ordinary fields where D_a reduces to ∂_a. This difference does not prevent us from using the methods of [28]. Namely the only property needed to adopt the analysis and results of [28] is that the representation of the subalgebra $\{D_\alpha, D_\beta\} = 0$ of (4.43) has *QDS-structure* in the terminology of [28], both in old and in new minimal supergravity. This is explained in detail in appendix C. It is quite remarkable that this property alone allows us to solve first (4.44) and then (4.27) completely in the cases $G \geq 4$.

Indeed, as in section 6 of [28][7] one proves by means of the QDS-structure of minimal supergravity that all the solutions to (4.37) with $G > 4$ are $\tilde{s}_{(0)}$-exact, while

[7] See also appendix E of the present paper where the derivation of the analogous result for the on-shell cohomology is sketched.

the nontrivial solutions with $G = 4$ are linearized counterparts of (4.29) given by

$$P_\Delta = \hat{D}_{\dot{\alpha}} \hat{D}^{\dot{\alpha}}\, \Xi\, \{\mathcal{A}_\Delta(\bar{M}, \bar{W}, \bar{\lambda}) + D^\alpha D_\alpha \mathcal{B}_\Delta(\mathcal{T})\} + c.c. \tag{4.45}$$

with Ξ as in (4.30) and

$$\hat{D}_{\dot{\alpha}} = 2i\tilde{\xi}^\alpha \sigma^a_{\alpha\dot{\alpha}} \frac{\partial}{\partial \tilde{\xi}^a} + \bar{D}_{\dot{\alpha}} \ . \tag{4.46}$$

The results (4.28) for $G \geq 4$ follow then from standard arguments of spectral sequence techniques and from the fact that any $\mathcal{G}$-invariant local total form (4.29) is indeed $\tilde{s}$-invariant as shown in [34].

4.4 Completion of the analysis

The results of sections 4.2 and 4.3 imply that any solution to $\tilde{s}\alpha(\mathcal{W}) = 0$ is, up to a trivial solution $\tilde{s}\beta(\mathcal{W})$, an $\tilde{s}$-invariant completion of a local total form

$$f_i(\tilde{\xi}, \mathcal{T})\, P^i(\tilde{\theta}, \tilde{Q}), \quad f_i(\tilde{\xi}, \mathcal{T}) \in \{1, \mathcal{F}^{i_a}, \mathcal{H}, \mathcal{P}_\Delta\}. \tag{4.47}$$

We are therefore left with the following problem: which total forms (4.47) have a local $\tilde{s}$-invariant completion and which of these completions are inequivalent in the restricted BRST cohomology?

To answer these questions I show first that each of the following local total forms can be completed to a local solution of $\tilde{s}\alpha = 0$:

$$P(\tilde{\theta}) + \mathcal{H}\hat{P}(\tilde{\theta}, \tilde{Q}) + \mathcal{P}_\Delta P^\Delta(\tilde{\theta}, \tilde{Q}) \tag{4.48}$$

where P^Δ and $\hat{P}$ are arbitrary polynomials in $\tilde{Q}$ and the $\tilde{\theta}_K$, whereas P depends only on those $\tilde{\theta}_K$ with $m_K > 2$ (cf. (4.23)),

$$\frac{\partial P(\tilde{\theta})}{\partial \tilde{\theta}_K} = 0 \quad \text{for} \quad m_K = 1, 2. \tag{4.49}$$

Note that the second term in (4.48) contributes only in new minimal supergravity.

In fact each term in (4.48) can separately be completed to a solution to $\tilde{s}\alpha = 0$. To show this it is useful to complete first $\tilde{\theta}_K$ to a "total super-Chern–Simons form" q_K, analogously to the standard construction in Yang–Mills theory (see e.g. [35]),

$$\begin{gathered} q_K = m_K \int_0^1 dt\, Tr\left[\tilde{C}\mathcal{F}_t^{m_K - 1}\right], \\ \mathcal{F}_t = t\mathcal{F} + (t^2 - t)\tilde{C}^2, \quad \tilde{C} = \tilde{C}^I T_I^{(K)}, \quad \mathcal{F} = \mathcal{F}^I T_I^{(K)} \end{gathered} \tag{4.50}$$

with $T_I^{(K)}$ as in (4.23). Due to (2.17), resp.

$$\tilde{s}\tilde{C} + \tilde{C}^2 = \mathcal{F},$$

the q_K satisfy

$$\tilde{s}\, q_K = Tr(\mathcal{F}^{m_K}) \equiv f_K. \tag{4.51}$$

The f_K are of course the supersymmetric counterparts of the familiar characteristic classes. Note however that they do *not* necessarily vanish for $m_K > 2$ because they are total forms decomposing into local differential forms with various form degrees. Hence, the q_K are not automatically $\tilde{s}$-invariant for $m_K > 2$, in contrast to their counterparts in Yang–Mills theory and standard gravity which provide for $m_K = 3$ directly the well-known nonabelian chiral anomalies. Nevertheless, every q_K with $m_K > 2$ has a local $\tilde{s}$-invariant completion. This follows immediately from the results of section 4.3. Indeed, as f_K is (a) $\tilde{s}$-closed due to $f_K = \tilde{s} q_K$ and $\tilde{s}^2 = 0$, (b) depends only on the $\tilde{\xi}$ and $\mathcal{T}$, and (c) has total degree $2m_K$, (4.28) implies

$$m_K > 2: \quad f_K = \tilde{s}\, p_K(\tilde{\xi}, \mathcal{T}) \tag{4.52}$$

for some local total form $p_K(\tilde{\xi}, \mathcal{T})$. Hence, the total forms

$$\tilde{q}_K = q_K - p_K \tag{4.53}$$

are $\tilde{s}$-invariant,

$$\tilde{s}\, \tilde{q}_K = 0 \quad (m_K > 2). \tag{4.54}$$

Note that $\tilde{q}_K$ does not vanish as p_K depends only on the $\tilde{\xi}$ and $\mathcal{T}$ whereas q_K involves the $\tilde{C}$ too. Any polynomial $P(\tilde{\theta})$ satisfying (4.49) can thus indeed be completed to an $\tilde{s}$-invariant total form $P(\tilde{q})$ by replacing $\tilde{\theta}_K$ with the corresponding $\tilde{q}_K$. In particular the $\tilde{q}_K$ with $m_K = 3$, given explicitly in [27, 34], provide the supersymmetrized versions of the nonabelian chiral anomalies spelled out in section 9.

Similar arguments prove that the remaining terms in (4.48) can be completed to local $\tilde{s}$-invariants. This will be now shown for the second term in (4.48) (the third term can be treated in a completely analogous way). In a first step we complete $\mathcal{H}\hat{P}(\tilde{\theta}, \tilde{Q})$ to

$$\alpha' = \mathcal{H}\hat{P}(q, \tilde{Q})$$

by replacing in $\hat{P}$ all $\tilde{\theta}_K$ with the corresponding q_K. Thanks to (4.51) and $\mathcal{H}^2 = 0$ (the latter holds as $\mathcal{H}$ is Grassmann odd), the $\tilde{s}$-transformation of α' reads

$$\tilde{s}\alpha' = -\mathcal{H} f_K \, \frac{\partial \hat{P}(q, \tilde{Q})}{\partial q_K} \ .$$

The total forms $\mathcal{H} f_K$ occurring on the r.h.s. of this equation are $\tilde{s}$-closed, depend only on the $\tilde{\xi}$ and $\mathcal{T}$ and have total degrees $2m_K + 3 > 4$. (4.28) therefore implies the existence of local total forms $h_K(\tilde{\xi}, \mathcal{T})$ such that

$$\mathcal{H} f_K = \tilde{s}\, h_K(\tilde{\xi}, \mathcal{T}).$$

In a second step we now consider

$$\alpha'' = \alpha' + h_K \, \frac{\partial \hat{P}(q, \tilde{Q})}{\partial q_K} \ .$$

Its $\tilde{s}$-transformation is given by

$$\tilde{s}\alpha'' = h_{[K}f_{L]}\,\frac{\partial^2\hat{P}(q,\tilde{Q})}{\partial q_L\partial q_K} + h_K\mathcal{H}\,\frac{\partial^2\hat{P}(q,\tilde{Q})}{\partial\tilde{Q}\partial q_K}$$

where the antisymmetrization in K and L in the first term on the r.h.s. is automatic thanks to the odd Grassmann parity of the q_K. $h_{[K}f_{L]}$ is $\tilde{s}$-closed due to $\tilde{s}(h_{[K}f_{L]}) = \mathcal{H}f_{[K}f_{L]} = 0$ (one has $f_{[K}f_{L]} = 0$ because the f_K are Grassmann even), depends only on the $\tilde{\xi}$ and $\mathcal{T}$, and has total degree $2+2m_K+2m_L > 5$. (4.28) therefore implies

$$h_{[K}f_{L]} = -\tilde{s}h_{KL}(\tilde{\xi},\mathcal{T})$$

for some $h_{KL} = -h_{LK}$. Analogous arguments (using $\mathcal{H}^2 = 0$ again) imply

$$h_K\mathcal{H} = -\tilde{s}\,g_K(\tilde{\xi},\mathcal{T})$$

for some g_K. We conclude that the $\tilde{s}$-transformation of

$$\alpha''' = \alpha'' + h_{KL}\,\frac{\partial^2\hat{P}(q,\tilde{Q})}{\partial q_L\partial q_K} + g_K\,\frac{\partial^2\hat{P}(q,\tilde{Q})}{\partial\tilde{Q}\partial q_K}$$

contains only third order derivatives of $\hat{P}(q,\tilde{Q})$ w.r.t. $\tilde{Q}$ and the q_K. The arguments can be iterated until $\tilde{Q}$ and all the q_K are differentiated away and one is left with an $\tilde{s}$-invariant completion $\alpha'^{\cdots\prime}$ of $\mathcal{H}\hat{P}(\tilde{\theta},\tilde{Q})$.

Those $P(\tilde{\theta},\tilde{Q})$ which involve $\tilde{Q}$ or one of the $\tilde{\theta}_K$ with $m_K = 1,2$ and the terms in (4.47) containing the $\mathcal{F}^{i_a}$ do not provide further solutions to $\tilde{s}\alpha(\mathcal{W}) = 0$. Namely either they cannot be completed to $\tilde{s}$-invariants or the respective $\tilde{s}$-invariants are cohomologically equivalent to solutions arising already from (4.48). This can be proved as the analogous statement in standard (non-supersymmetric) gravity, cf. [23] for details.

To summarize, an $\tilde{s}$-invariant local completion exists for any total form (4.48) satisfying (4.49) and these completions provide all the solutions to $\tilde{s}\alpha(\mathcal{W}) = 0$ up to trivial ones of the form $\tilde{s}\beta(\mathcal{W})$. The resulting list of solutions is still overcomplete because it still contains trivial solutions which may be removed at each total degree separately. The solutions with total degree 4 and 5 have been given explicitly in [34] and will be discussed in sections 5 and 9. They provide the supergravity Lagrangians and the antifield independent candidate gauge anomalies respectively. The other solutions (with higher total degrees) will not be further discussed here because a physical interpretation is not yet known for them (they can be found in [27]).

5 Invariant Actions

5.1 Old minimal supergravity

Supergravity actions which are invariant under the standard gauge resp. BRST transformations given in section 2 arise from those total forms (4.48) which have total

degree 4. In old minimal supergravity these are just linear combinations of the $\mathcal{P}_\Delta$ (with constant coefficients) because the first term in (4.48) provides only solutions with total degrees exceeding 4 due to (4.49) and the second term contributes only in new minimal supergravity. Hence, in old minimal supergravity the integrand of the most general real action $\int \omega_4$ that is invariant under the standard gauge transformations is the 4-form ω_4 contained in $\mathcal{P} = a^\Delta \mathcal{P}_\Delta$ where a^Δ are arbitrary real constant coefficients. It is an easy exercise to verify that the result is

$$\omega_4 = d^4x\, e\, L_{old}\ , \quad e = \det(e_\mu{}^a),$$

with

$$\begin{aligned} L_{old} &= (\bar{\mathcal{D}}^2 - 4i\psi_\mu\sigma^\mu\bar{\mathcal{D}} - 3M + 16\psi_\mu\sigma^{\mu\nu}\psi_\nu)\,\Omega + c.c.\ , \\ \Omega &= \mathcal{A}(\bar{M}, \bar{W}, \bar{\lambda}) + (\mathcal{D}^2 - \bar{M})\,\mathcal{B}(\mathcal{T}) \end{aligned} \tag{5.1}$$

where $\bar{\mathcal{D}}^2$ and $\mathcal{D}^2$ are shorthand notations for $\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}$ and $\mathcal{D}^\alpha\mathcal{D}_\alpha$ respectively, $\mathcal{B} \equiv a^\Delta\mathcal{B}_\Delta$ is $\mathcal{G}$-invariant whereas $\mathcal{A} \equiv a^\Delta\mathcal{A}_\Delta$ is $\mathcal{G}$-invariant except under R-transformations if the latter are to be gauged (then $\mathcal{A}$ must have R-charge -2). It is of course well-known that supergravity actions can be constructed from (5.1). For instance, (5.1) emerges from Eq. (15.28) of [36] when one identifies the function L occurring there with $\bar{\Omega}$ given above, and can also be obtained from superspace integrals à la [15]. The new result we have derived here is that (5.1) gives remarkably the *most general* local action for old minimal supergravity.

The supersymmetrized Einstein–Hilbert action arises from a contribution to $\mathcal{A}$ proportional to $\bar{M}$ yielding

$$\begin{aligned} M_{Pl}^{-2}L_{grav} &= \tfrac{1}{2}\mathcal{R} + 2D^{(r)} - 2i\psi_\mu\sigma^\mu(\bar{S} + i\bar{\lambda}^{(r)}) + 2i(S - i\lambda^{(r)})\sigma^\mu\bar{\psi}_\mu \\ &\quad -3B_aB^a - \tfrac{3}{16}M\bar{M} + \tfrac{3}{2}(\bar{M}\psi_\mu\sigma^{\mu\nu}\psi_\nu + M\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu). \end{aligned} \tag{5.2}$$

where $\lambda^{(r)}$ and $D^{(r)}$ contribute of course only if R-transformations are to be gauged (otherwise these fields simply have to be set to zero) and M_{Pl} is the Planck mass. I note that (5.2) is an unusual way to write L_{grav} but agrees in fact completely with more familiar expressions that can be found in the literature. For instance, in (5.2) the super-covariantized curvature scalar $\mathcal{R}$ contains gravitino dependent contributions that combine with the term $2iS\sigma^\mu\bar{\psi}_\mu + c.c.$ to the familiar kinetic term for the gravitino given already in [13]. Furthermore, all the terms linear in B, M and $\bar{M}$, i.e. those contained in $\mathcal{R}$, S and $\bar{S}$ and the last two terms in (5.2), cancel exactly.

The supersymmetrized Yang–Mills Lagrangian arises from the contribution $\frac{1}{16}\bar{\lambda}^i\bar{\lambda}_i$ to $\mathcal{A}$ (nonabelian indices i are lowered with the Cartan–Killing metric of the Yang–Mills gauge group and abelian ones with the unit matrix). It reads

$$\begin{aligned} L_{YM} = -\tfrac{1}{4}\,F_{\mu\nu}{}^iF^{\mu\nu}{}_i - \tfrac{1}{2}\,i\,(\lambda^i\sigma^\mu\nabla_\mu\bar{\lambda}_i + \bar{\lambda}^i\bar{\sigma}^\mu\nabla_\mu\lambda_i) + \tfrac{1}{2}\,D^iD_i + \tfrac{3}{2}\,\lambda^i\sigma^\mu\bar{\lambda}_iB_\mu \\ -\tfrac{1}{2}\,F_{\mu\nu}{}^i\varepsilon^{\mu\nu\rho\sigma}(\psi_\rho\sigma_\sigma\bar{\lambda}_i + \lambda_i\sigma_\sigma\bar{\psi}_\rho) + \psi_\mu\sigma^{\mu\nu}\psi_\nu\bar{\lambda}^i\bar{\lambda}_i + \bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu\lambda^i\lambda_i \end{aligned} \tag{5.3}$$

where $\varepsilon^{\mu\nu\rho\sigma} = E_a{}^\mu \cdots E_d{}^\sigma\varepsilon^{abcd}$ is vierbein dependent ($e\varepsilon^{\mu\nu\rho\sigma}$ is constant), ∇_μ is the usual covariant derivative (not the super-covariant one),

$$\nabla_\mu = \partial_\mu - A_\mu{}^i\delta_i - \tfrac{1}{2}\,\omega_\mu{}^{ab}l_{ab}\ , \tag{5.4}$$

and ${F_{\mu\nu}}^i$ is the super-covariantized Yang–Mills field strength,

$$ {F_{\mu\nu}}^i = \partial_\mu {A_\nu}^i - \partial_\nu {A_\mu}^i + {f_{jk}}^i {A_\mu}^j {A_\nu}^k + 2i\,(\lambda^i \sigma_{[\mu}\bar{\psi}_{\nu]} + \psi_{[\mu}\sigma_{\nu]}\bar{\lambda}^i). \tag{5.5} $$

Of course (5.1) can be used to construct supergravity actions that generalize the simple one arising from (5.2) and (5.3). In particular, a constant contribution m to $\mathcal{A}$ gives rise to

$$ L_{cosmo} = -3mM + 16m\psi_\mu \sigma^{\mu\nu}\psi_\nu + c.c. \tag{5.6} $$

which, when included in L_{old}, contributes to the cosmological constant. Note however that L_{cosmo} is neither locally nor globally R-invariant and is thus forbidden when global or local R-invariance is imposed, in contrast to L_{grav} and L_{YM}. Note also that the most general action contains at most one Fayet–Iliopoulos contribution, namely the one for R-transformations occurring in L_{grav}.

5.2 New minimal supergravity

In new minimal supergravity, both the second and third term in (4.48) contain total forms with total degree 4 and thus give contributions to the most general invariant action. The corresponding contributions to the Lagrangian are denoted by L_1 and L_2 respectively,

$$ L_{new} = L_1 + L_2 \ . \tag{5.7} $$

L_1 arises from the second term in (4.48) by choosing $\hat{P} = \mu_{i_a}\tilde{C}^{i_a}$ as a linear combination of the abelian $\tilde{C}$'s with constant coefficients μ_{i_a} (the $\tilde{C}^{i_a}$ are those $\tilde{\theta}_K$ with $m_K = 1$). Now, $\mathcal{H}\hat{P}$ is not yet $\tilde{s}$-invariant. We know however that it can be completed to an $\tilde{s}$-invariant local total form, see section 4.4. This completion has been computed already in [34] and is given by

$$ \mu_{i_a}\,(2\tilde{C}^{i_a}\mathcal{H} + \bar{\lambda}^{i_a}\bar{\eta} + \eta\lambda^{i_a} + \Xi\, D^{i_a}) \tag{5.8} $$

where Ξ is the "total volume form" (4.30), η^α is given by

$$ \eta^\alpha = -\tfrac{i}{6}\,\tilde{\xi}_{\dot{\beta}}\tilde{\xi}^{\dot{\beta}\beta}\tilde{\xi}_{\beta\dot{\alpha}}\tilde{\xi}^{\dot{\alpha}\alpha} \tag{5.9} $$

and it is understood that, in accordance with appendix B, the gaugino and D-field of R-transformations are identified with

$$ \lambda^{(r)}_\alpha \equiv -iS_\alpha\ , \quad D^{(r)} \equiv -\tfrac{1}{4}\,(\mathcal{R} + H_{abc}H^{abc}). \tag{5.10} $$

The appearance of the D-fields in (5.8) indicates already that L_1 contains Fayet–Iliopoulos terms for the abelian symmetries except for the R-symmetry. The latter exception is due to (5.10) which also shows that the terms in (5.8) corresponding to R-transformations provide the supersymmetrized Einstein–Hilbert action in this

case. With $M_{Pl}^2 = -\mu_{(r)}/2$, one obtains from the volume form contained in (5.8)[8]

$$
\begin{aligned}
L_1 &= L_{grav} + L_{FI}, &(5.11)\\
M_{Pl}^{-2} L_{grav} &= \tfrac{1}{2}\mathcal{R} + \tfrac{1}{2} H_{abc} H^{abc} + 2i(S\sigma^\mu \bar{\psi}_\mu - \psi_\mu \sigma^\mu \bar{S}) - 2\varepsilon^{\mu\nu\rho\sigma} A_\mu{}^{(r)} \partial_\nu t_{\rho\sigma}, &(5.12)\\
L_{FI} &= \sum_{i_a \neq (r)} \mu_{i_a} (D^{i_a} + \lambda^{i_a} \sigma^\mu \bar{\psi}_\mu + \psi_\mu \sigma^\mu \bar{\lambda}^{i_a} + \varepsilon^{\mu\nu\rho\sigma} A_\mu{}^{i_a} \partial_\nu t_{\rho\sigma}). &(5.13)
\end{aligned}
$$

Of course, by an appropriate choice of basis for the abelian gauge multiplets one can assume that at most one μ_{i_a} ($i_a \neq (r)$) is different from zero. Again, one may check that (5.12) agrees indeed completely with the action for new minimal supergravity that can be found in the literature, see e.g. [2, 37]. (5.12) and (5.13) cannot naturally be written as standard superspace integrals, unless one modifies the whole approach using an enlarged field content, c.f. [16, 37].

The remaining terms L_2 in the general Lagrangian (5.7) are analogous to the Lagrangian (5.1) of old minimal supergravity and can thus be written as superspace integrals. The difference to (5.7) is of course that the auxiliary field M is absent now. Therefore the supersymmetrized Einstein–Hilbert action does not arise from L_2 which reads

$$
\begin{aligned}
L_2 &= (\bar{\mathcal{D}}^2 - 4i\psi_\mu \sigma^\mu \bar{\mathcal{D}} + 16\psi_\mu \sigma^{\mu\nu} \psi_\nu)\, \Omega + c.c.\ , \\
\Omega &= \mathcal{A}(\bar{W}, \bar{\lambda}) + \mathcal{D}^2 \mathcal{B}(\mathcal{T}) &(5.14)
\end{aligned}
$$

where S counts among the λ's due to (5.10). The supersymmetrized Yang–Mills action arises from (5.14) through a contribution proportional to $\sum_{i\neq(r)} \bar{\lambda}^i \bar{\lambda}_i$ to $\mathcal{A}$, like in old minimal supergravity (the sum over i excludes (r) here in order to end up with the standard action for new minimal supergravity). Since R-transformations are gauged in new minimal supergravity, $\mathcal{A}$ must have R-charge -2 and therefore does not contain a constant piece in this case. Hence, L_{new} contains no contribution analogous to (5.6).

6 Full BRST cohomology

We will now analyze the full BRST cohomology based on the standard super-Einstein–Yang–Mills action $\int d^4x\, e\, L$ with Lagrangian

$$L = L_{grav} + L_{YM} \tag{6.1}$$

where L_{grav} is given for old and new minimal supergravity by (5.2) and (5.12) respectively, and L_{YM} as in (5.3). The specialization to a particular action is necessary because the BRST transformations of the antifields involve the variational derivatives of the action with respect to the fields. Choosing the simple action with Lagrangian (6.1) has the major advantage that the results can be easily generalized to more

[8] I remark that (5.12) and (5.13) correct a mistake in formula (3.29) of [34].

complicated actions by means of spectral sequence techniques, cf. remarks in section 10.

The analysis of the full BRST cohomology proceeds closely to that of the restricted BRST cohomology in section 4. It uses the fact that the full BRST cohomology (taking the antifields into account) can be obtained from the weak ("on-shell") $\tilde{s}$-cohomology involving the fields only (but not the antifields) [20].

6.1 On-shell basis for the tensor fields

It will be crucial to determine an appropriate *on-shell basis* for the tensor fields. Such a basis is a subset of the off-shell basis determined in section 3 taking the equations of motion into account. This makes sense because the equations of motion are equivalent to equations involving only the tensor fields [20]. The equations of motion can therefore be used to express some of the tensor fields $\mathcal{T}$ in terms of others (of course they may even set some $\mathcal{T}$'s to zero). The remaining $\mathcal{T}$'s form the searched for on-shell basis for the tensor fields which will be denoted by $\{\hat{\mathcal{T}}^r\}$.

Let me first illustrate the procedure for pure old minimal supergravity with Lagrangian (5.2) without gauged R-symmetry (i.e. for $\lambda^{(r)} = D^{(r)} = 0$). In this case the equations of motion simply set $\mathcal{R}$, Y, S, U, M and B to zero where the notation of section 3 is used and spinor indices are suppressed[9]. Using (3.13), one now easily verifies that an on-shell basis for the tensor fields in pure old minimal supergravity is given by W, $\bar{W}$, X, $\bar{X}$ and all their $\mathcal{D}_+^+$ derivatives defined in (3.4) (recall that X is the super-covariantized Weyl tensor and that W is the chiral part of the super-covariantized gravitino field strength).

When Yang–Mills multiplets are present, $\mathcal{R}$, Y, etc. do not vanish anymore on-shell but can still be expressed (nonlinearly) in terms of other tensor fields. In addition one has the equations of motion for the Yang–Mills multiplets which are analyzed analogously. One obtains that an on-shell basis for the tensor fields is given by

$$\{\hat{\mathcal{T}}^r\} = \{W_q,\, \bar{W}_q,\, X_q,\, \bar{X}_q,\, \lambda_q^i,\, \bar{\lambda}_q^i,\, G_q^i,\, \bar{G}_q^i:\ q = 0, 1, \ldots\} \tag{6.2}$$

where q indicates the number of $\mathcal{D}_+^+$ operations,

$$W_q \equiv (\mathcal{D}_+^+)^q\, W \quad etc. \tag{6.3}$$

In new minimal supergravity one finds analogously that (6.2) gives an on-shell basis for the tensor fields with the understanding that it does not contain any tensor field associated with local R-symmetry ($G^{(r)}$ and $\bar{G}^{(r)}$ are eliminated using the equations of motion for $t_{\mu\nu}$ while H_{abc} is eliminated through the equations of motion for $A_\mu{}^{(r)}$).

[9] This is a somewhat unusual but correct form of the equations of motion. E.g., the gravitino dependent terms that appear in more familiar versions of the equations of motion "on the r.h.s. of the Einstein equations" are indeed taken into account as $\mathcal{R}$ and Y are super-covariant.

6.2 Sketch of the computation

As shown in [20], the computation of the full BRST cohomology reduces (locally) to the determination of the weak cohomology of $\tilde{s}$ on local total forms depending only on the $\mathcal{W}$'s given in (4.6). The computation of this cohomology can therefore be further reduced to a problem involving only the tensor fields $\hat{\mathcal{T}}$ given in (6.2) and the generalized connections $\tilde{\xi}^A$, $\tilde{C}^I$ and $\tilde{Q}$ where it is understood that $G^{(r)}$ and $\bar{G}^{(r)}$ do not count among the $\hat{\mathcal{T}}$ in new minimal supergravity and that $\tilde{Q}$ is absent in old minimal supergravity. To formulate the problem on the remaining variables correctly, one has to express their weak $\tilde{s}$-transformations completely in terms of these variables again. To that end one must use the equations of motion to express those tensor fields $\mathcal{T} \notin \{\hat{\mathcal{T}}^r\}$ which occur in $\tilde{s}\hat{\mathcal{T}}$, $\tilde{s}\tilde{\xi}$, $\tilde{s}\tilde{C}$ and $\tilde{s}\tilde{Q}$ in terms of the $\hat{\mathcal{T}}$ as described in section 6.1. Denoting weak (= on-shell) equalities by $\approx$, we are thus left with the problem

$$\tilde{s}\,\alpha(\tilde{C},\tilde{\xi},\tilde{Q},\hat{\mathcal{T}}) \approx 0 \tag{6.4}$$

defined modulo trivial solutions which are weakly of the form $\tilde{s}\beta(\tilde{C},\tilde{\xi},\tilde{Q},\hat{\mathcal{T}}) +$ *constant*.

(6.4) is now analyzed analogously to (4.9). As the equations of motion do not interfere with the Lie algebra cohomology, one first shows as in section 4.2 that the nontrivial solutions of (6.4) are at highest degree in the $\tilde{C}$ and $\tilde{Q}$ given by

$$f_i(\tilde{\xi},\hat{\mathcal{T}})\,P^i(\tilde{\theta},\tilde{Q}) \tag{6.5}$$

where the f_i solve

$$\tilde{s}\,f_i(\tilde{\xi},\hat{\mathcal{T}}) \approx 0, \quad f_i(\tilde{\xi},\hat{\mathcal{T}}) \not\approx \tilde{s}\,h_i(\tilde{\xi},\hat{\mathcal{T}}). \tag{6.6}$$

The problem (6.6) is now analyzed using techniques similar to those described in section 4.3. There is however one important complication compared to the restricted cohomology. It consists in the fact that (6.4) has in general nontrivial solutions with total degree $G < 4$, in contrast to the analogous "strong" problem (4.27). Such solutions correspond to local conservation laws as they provide representatives of the local BRST cohomology at negative ghost numbers [9]. For $G < 3$ they can be computed using methods developed in [9] (see also [38]) which are not repeated here. One finds that there are no nontrivial solutions with total degrees $G < 1$, whereas the only nontrivial solutions at $G = 1$ and $G = 2$ correspond to solutions of (1.1) given by

$$G = 1 \quad \leftrightarrow \quad d^4x\,Q^*, \tag{6.7}$$

$$G = 2 \quad \leftrightarrow \quad d^4x\,C^*_{i_a} \quad (i_a \neq (r)), \tag{6.8}$$

where Q^* and $C^*_{i_a}$ are the antifields of Q and C^{i_a} respectively. It is very easy to verify that (6.7) and (6.8) solve (1.1). Indeed (2.11) gives directly

$$sQ^* = \partial_\mu(C^\mu Q^* + Q^{\mu *}); \quad i_a \neq (r): \; sC^*_{i_a} = \partial_\mu(C^\mu C^*_{i_a} - A^{\mu *}_{i_a}). \tag{6.9}$$

Of course, (6.7) occurs only in new minimal supergravity. There is no solution (6.8) corresponding to R-transformations because the gravitino and the gauginos have non-vanishing R-charges. Analogously $d^4xC^*_{i_a}$ disappears from the list of solutions when one includes further (matter) fields transforming nontrivially under the i_ath abelian gauge symmetry, see [9].

(6.9) ensures that (6.7) and (6.8) give rise to solutions of $\tilde{s}\alpha = 0$. The latter are obtained by evaluating the descent equations implied by (6.9). We denote these solutions by $\tilde{Q}^*$ and $\tilde{C}^*_{i_a}$ respectively. It is easy to verify that they are of the form

$$\tilde{Q}^* = \tfrac{1}{e}\,\Xi\, Q^* - \tfrac{1}{6e}\,\tilde{\xi}^a\tilde{\xi}^b\tilde{\xi}^c\varepsilon_{abcd}Q^{d*} + \ldots + 4\tilde{C}^{(r)}, \tag{6.10}$$

$$\tilde{C}^*_{i_a} = \tfrac{1}{e}\,\Xi\, C^*_{i_a} + \tfrac{1}{6e}\,\tilde{\xi}^a\tilde{\xi}^b\tilde{\xi}^c\varepsilon_{abcd}A^{d*}_{i_a} + \ldots \quad (i_a \neq (r)) \tag{6.11}$$

where Ξ is the total volume form (4.30). Furthermore one finds that the antifield independent part of (6.11) is given by

$$\tilde{C}^*_{i_a|} = \tfrac{1}{4}\,\tilde{\xi}^a\tilde{\xi}^b\varepsilon_{abcd}F^{cd}_{i_a} + \vartheta\lambda_{i_a} - \bar{\vartheta}\bar{\lambda}_{i_a} \tag{6.12}$$

with

$$\vartheta^\alpha = \tilde{\xi}_{\dot{\alpha}}\tilde{\xi}^{\dot{\alpha}\alpha}, \quad \bar{\vartheta}^{\dot{\alpha}} = \tilde{\xi}^{\dot{\alpha}\alpha}\tilde{\xi}_\alpha\ . \tag{6.13}$$

(6.12) thus solves (6.6) as it depends only on the $\tilde{\xi}$ and $\hat{\mathcal{T}}$. In contrast, $\tilde{Q}^*$ involves $\tilde{C}^{(r)}$ and therefore its antifield independent part solves (6.4) but *not* (6.6). The latter reflects the presence of the Chern–Simons like term $\varepsilon^{\mu\nu\rho\sigma}A_\mu{}^{(r)}\partial_\nu t_{\rho\sigma}$ in (5.12). The antifield independent parts of the $\tilde{C}^*_{i_a}$ are therefore the only nontrivial solutions to (6.6) with total degree $G < 3$ on top of those occurring already in (4.28). The latter remain indeed nontrivial even in the weak $\tilde{s}$-cohomology on local total forms $f(\tilde{\xi},\hat{\mathcal{T}})$, except for $\mathcal{F}^{(r)}$ in the case of new minimal supergravity where it vanishes weakly. This quite plausible statement can be proved rigorously by a technique used in appendix D. The proof parallels that of a corresponding result in standard gravity given in appendix E of [26] and is therefore not spelled out here.

Let us now turn to the discussion of (6.6) for total degrees $G \geq 3$. Similarly to the analogous "strong" problem for $G \geq 4$ in section 4.3 we decompose (6.6) according to the degree in the tensor fields $\hat{\mathcal{T}}$. At lowest degree, this yields the linearized problem

$$\delta_{susy}f_{(\ell)}(\tilde{\xi},\hat{\mathcal{T}}) \sim 0, \quad f_{(\ell)} \not\sim \delta_{susy}h_{(\ell)}(\tilde{\xi},\hat{\mathcal{T}}), \quad \delta_I f_{(\ell)} = \delta_I h_{(\ell)} = 0 \tag{6.14}$$

with δ_{susy} as in (4.40)–(4.42), and $\sim$ denoting "linearized weak equality" based on the linearization of the equations of motion in the tensor fields $\hat{\mathcal{T}}$.

Now, in the cases $G \geq 4$ the methods and results of [28] can be straightforwardly adapted to solve (6.14). Again, this is possible thanks to the QDS structure of the on-shell representation of the subalgebra $\{D_\alpha, D_\beta\} = 0$ of (4.43) proved in appendix C.3. This implies that, in the cases $G \geq 4$, the solution of (6.6) is analogous to the solution of the corresponding "strong" problem (4.44), see appendix E for details.

One is left with the case $G = 3$ which I have not been able to solve completely. Partial results are derived in appendix E where it is shown that all the solutions to

(6.14) with $G=3$ can be assumed to be of the form

$$G=3: \quad f_{(\ell)} = \{4\Theta - i\bar{\vartheta}_{\dot{\alpha}}\tilde{\xi}^{\dot{\alpha}\alpha}D_\alpha - i\vartheta^\alpha\tilde{\xi}_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}} + \tfrac{1}{12}\tilde{\xi}^{\dot{\alpha}\beta}\tilde{\xi}_{\beta\dot{\beta}}\tilde{\xi}^{\dot{\beta}\alpha}[D_\alpha,\bar{D}_{\dot{\alpha}}]\}\,R(\hat{\mathcal{T}}) \tag{6.15}$$

where Θ is the quantity

$$\Theta = \tilde{\xi}^\alpha\tilde{\xi}_{\alpha\dot{\alpha}}\tilde{\xi}^{\dot{\alpha}} \tag{6.16}$$

and $R(\hat{\mathcal{T}})$ are real functions solving

$$D^\alpha D_\alpha R(\hat{\mathcal{T}}) \sim 0, \quad R = \bar{R}. \tag{6.17}$$

One solution which is always present is of course $R = constant$ for which (6.15) becomes simply proportional to Θ. The latter can be completed to a solution of (6.6) corresponding to the Noether current for R-transformations in the case of old minimal supergravity (see section 7) and reproducing in new minimal supergravity the solution $\mathcal{H}$ already present in the restricted cohomology, cf. (4.28), where now of course $\mathcal{H}$ has to be replaced by its on-shell version (e.g. in pure new minimal supergravity $\mathcal{H}$ reduces on-shell to $i\Theta$). Recall also that $\mathcal{H}$ is $\tilde{s}$-exact, cf. (2.18), but it is not $\tilde{s}$-exact in the space of total forms $f(\tilde{\xi},\hat{\mathcal{T}})$ (not even weakly).

There might be further solutions to (6.6) with $G=3$, in particular when matter multiplets are included. Fortunately it will not matter in the following whether we know all these solutions explicitly. We denote them by N_τ, except for the special solution $\mathcal{H}$ present only in new minimal supergravity which is treated separately because it is the only one which occurs already in the restricted cohomology. We can then summarize the solution of (6.6) as follows:

$$\begin{aligned} &\tilde{s}\, f(\tilde{\xi},\hat{\mathcal{T}}) \approx 0, \quad \mathrm{totdeg}(f) = G \\ \Leftrightarrow \qquad & f(\tilde{\xi},\hat{\mathcal{T}}) \approx \begin{cases} constant & \text{for } G=0 \\ \tilde{s}h(\hat{\mathcal{T}}) & \text{for } G=1 \\ a_{i_a}\mathcal{F}^{i_a} + \sum_{i_a\neq(r)} b^{i_a}\tilde{C}^*_{i_a|} + \tilde{s}h(\tilde{\xi},\hat{\mathcal{T}}) & \text{for } G=2 \\ a\,\mathcal{H} + a^\tau N_\tau(\tilde{\xi},\hat{\mathcal{T}}) + \tilde{s}h(\tilde{\xi},\hat{\mathcal{T}}) & \text{for } G=3 \\ a^\Delta\hat{\mathcal{P}}_\Delta + \tilde{s}h(\tilde{\xi},\hat{\mathcal{T}}) & \text{for } G=4 \\ \tilde{s}h(\tilde{\xi},\hat{\mathcal{T}}) & \text{for } G>4 \end{cases} \end{aligned} \tag{6.18}$$

where the a's and the b^{i_a} are constants ($a_{(r)}$ can be assumed to vanish in new minimal supergravity as $\mathcal{F}^{(r)}$ vanishes on-shell in that case, see above), $\tilde{C}^*_{i_a|}$ was given in (6.12), and

$$\mathcal{P}_\Delta = \hat{\mathcal{D}}_{\dot{\alpha}}\hat{\mathcal{D}}^{\dot{\alpha}}\,\Xi\,\{\mathcal{A}_\Delta(\bar{W},\bar{\lambda}) + \mathcal{D}^\alpha\mathcal{D}_\alpha\mathcal{B}_\Delta(\hat{\mathcal{T}})\} + c.c. \tag{6.19}$$

with $\hat{\mathcal{D}}_{\dot{\alpha}}$ as in (4.31). In (6.18) it is understood that the on-shell version of all occurring total forms (esp. of $\mathcal{H}$) is used.

6.3 Result

We can now complete the analysis of the full BRST cohomology. To that end we have to find out which total forms

$$\hat{P} + \mathcal{F}^{i_a}\hat{P}_{i_a} + \sum_{i_a \neq (r)} \tilde{C}^*_{i_a} P^{i_a} + \mathcal{H}P_{\mathcal{H}} + N_\tau P^\tau + \hat{\mathcal{P}}_\Delta P^\Delta \tag{6.20}$$

can be completed to (inequivalent) $\tilde{s}$-invariants, where the P's and $\hat{P}$'s are polynomials in the $\tilde{\theta}_K$ and in $\tilde{Q}$. Without going into details I note that arguments analogous to those used in section 4.4 show that

1. $\hat{P}$ depends neither on $\tilde{Q}$ nor on those $\tilde{\theta}_K$ with $m_K = 1,2$ except on $\tilde{C}^{(r)}$ in new minimal supergravity. The latter exception reflects that $\tilde{C}^{(r)}$ can be completed to an $\tilde{s}$-invariant total form in new minimal supergravity, cf. (6.10). Hence, we get $\hat{P} = P(\tilde{\theta}) + \tilde{C}^{(r)} P_{new}(\tilde{\theta})$ where P_{new} is present only in new minimal supergravity and

$$\frac{\partial P(\tilde{\theta})}{\partial \tilde{\theta}_K} = \frac{\partial P_{new}(\tilde{\theta})}{\partial \tilde{\theta}_K} = 0 \quad \text{for} \quad m_K = 1,2. \tag{6.21}$$

2. The terms $\mathcal{F}^{i_a}\hat{P}_{i_a}$ either cannot be completed to $\tilde{s}$-invariants or they can be removed by subtracting $\tilde{s}$-exact total forms and redefining $P_{\mathcal{H}}$ and P^Δ appropriately.

3. The terms $\tilde{C}^*_{i_a} P^{i_a}$ $(i_a \neq (r))$ must be of the form

$$\sum_{i_a \neq (r)} \tilde{C}^*_{i_a} \frac{\partial P_*(\tilde{\theta}, \tilde{Q})}{\partial \tilde{C}^{i_a}} \tag{6.22}$$

where P_* is an arbitrary polynomial in the case of new minimal supergravity, whereas in old minimal supergravity it must not involve $\tilde{C}^{(r)}$,

$$\frac{\partial P_*(\tilde{\theta}, \tilde{Q})}{\partial \tilde{C}^{(r)}} = 0 \quad \text{in old min. supergravity.} \tag{6.23}$$

The remaining total forms (6.20), summarized in table 6.1, have $\tilde{s}$-invariant completions which provide (locally) a complete set of cohomology classes of the full $\tilde{s}$-cohomology. This set is actually still overcomplete because it still contains trivial (i.e. $\tilde{s}$-exact) total forms. The latter may be removed at each total degree separately. The results for the physically important cases (total degrees ≤ 5) are spelled out and discussed in the following sections. Note that the solutions of type III and VI are present only in new minimal supergravity and that the solutions I–III can be chosen so as not to involve antifields when the auxiliary fields are used (as they have

counterparts in the restricted cohomology, see section 4.4). The remaining solutions IV–VI necessarily involve antifields, whether or not the auxiliary fields are used.

Type	α	Remarks
I	$P(\tilde{\theta}) + \ldots$	P as in (6.21)
II	$\hat{\mathcal{P}}_\Delta P^\Delta(\tilde{\theta}, \tilde{Q}) + \ldots$	
III	$\mathcal{H} P_{\mathcal{H}}(\tilde{\theta}, \tilde{Q}) + \ldots$	
IV	$N_\tau P^\tau(\tilde{\theta}, \tilde{Q}) + \ldots$	
V	$\sum_{i_a \neq (r)} \tilde{C}^*_{i_a}\, \partial P_*(\tilde{\theta}, \tilde{Q})/\partial \tilde{C}^{i_a} + \ldots$	P_* as in (6.23)
VI	$\tilde{Q}^* P_{new}(\tilde{\theta}) + \ldots$	P_{new} as in (6.21)

Table 6.1: $\tilde{s}$-cohomology

7 Dynamical conservation laws

The local conservation laws are determined by the *weak cohomology of* d on local differential forms at form degrees $0 < p < 4$ (as the spacetime dimension is 4 and constant zero forms are not counted among the local conservation laws). In other words, they are the solutions of

$$d\, j_{4-k} \approx 0, \quad j_{4-k} \not\approx dk_{4-k-1} \quad (k = 1, 2, 3) \tag{7.1}$$

or, equivalently, of

$$\partial_{\mu_1} j^{\,[\mu_1 \cdots \mu_k]} \approx 0, \quad j^{\,\mu_1 \cdots \mu_k} \not\approx \partial_{\mu_0} k^{[\mu_0 \cdots \mu_k]} \tag{7.2}$$

where the $j^{\,\mu_1 \cdots \mu_k}$ are local functions of the classical fields and the j_{4-k} are local $(4-k)$-forms

$$j_{4-k} = \frac{1}{(4-k)!}\, dx^{\mu_4} \cdots dx^{\mu_{k+1}}\, \frac{1}{e}\, \varepsilon_{\mu_4 \cdots \mu_1} j^{\,\mu_k \cdots \mu_1}.$$

We call j_{4-k} a "dynamical conservation law of order k" when it solves (7.1) *locally*. Weakly d-closed forms which are locally, but not globally d-exact on-shell are called "topological conservation laws" instead and are briefly discussed in section 11. The dynamical conservation laws of order 1 are the Noether currents j^μ and thus correspond one-to-one to the nontrivial global symmetries of the classical action[10].

The weak d-cohomology at form degree $(4-k)$ can be shown to be (locally) isomorphic to the local BRST-cohomology at *negative* ghost number $(-k)$ [9]. The dynamical conservation laws of order 1,2,3 are thus obtained from the (full) $\tilde{s}$-cohomology

[10]A global symmetry $\delta_\epsilon \phi^i$ is called trivial if it equals a special gauge transformation (with special, possibly field dependent 'parameter') up to an on-shell vanishing part of the form $\mu^{ij} \delta \mathcal{S}_{cl} / \delta \phi^j$ with $\mu^{ij} = -(-)^{\varepsilon_i \varepsilon_j} \mu^{ji}$ (in de Witt's notation). Trivial global symmetries correspond to trivial Noether currents (satisfying $j^\mu \approx \partial_\nu k^{[\nu\mu]}$) and vice versa [9].

at total degree 3,2,1 respectively. The latter is obtained from table 6.1 in section 6.3. j_p is just the antifield independent part of the p-form contained in the corresponding $\tilde{s}$-invariant total form α with total degree p.

The results are summarized in table 7.1 where $\tilde{Q}^*$ is the $\tilde{s}$-invariant completion of $4\tilde{C}^{(r)}$ in new minimal supergravity, cf. eq. (6.10), and j^μ_τ denotes the Noether current corresponding to N_τ. I note that table 6.1 contains one more solution with total degree 3, namely the solution of type III given just by $\mathcal{H}$. The latter is however $\tilde{s}$-exact, cf. (2.18), and does therefore not provide a dynamical conservation law.

k	Type	α		$j^{\mu_1\cdots\mu_k}$
1	IV	N_τ		j^μ_τ
	Va	$\tilde{C}^{*[i_a}\tilde{C}^{i_b]}+\ldots$	$(i_a,i_b\neq(r))$	$e\,F^{\nu\mu[i_a}A_\nu{}^{i_b]}+\ldots$
	Vb	$\tilde{C}^*_{i_a}\tilde{Q}^*$	$(i_a\neq(r))$	$e\,F^{\nu\mu}_{i_a}A_\nu{}^{(r)}+\ldots$
2	V	$\tilde{C}^*_{i_a}$	$(i_a\neq(r))$	$\frac{1}{2}\,e\,F^{\mu\nu}_{i_a}+\ldots$
3	VI	$\tilde{Q}^*$		$\frac{2}{3}\,e\,\varepsilon^{\mu\nu\rho\sigma}A_\sigma{}^{(r)}+\ldots$

Table 7.1: Dynamical conservation laws

Let me briefly comment the result.

The third order conservation law occurs only in new minimal supergravity. In complete form it reads

$$j^{\mu\nu\rho}=e\,\varepsilon^{\mu\nu\rho\sigma}(\tfrac{2}{3}\,A_\sigma{}^{(r)}-\tfrac{1}{4}\,\lambda^i\sigma_\sigma\bar{\lambda}_i)+e\,H^{\mu\nu\rho} \tag{7.3}$$

with $H_{\mu\nu\rho}$ as in (2.7). This conservation law remains when new minimal supergravity is coupled to matter fields in the standard way (then $j^{\mu\nu\rho}$ just receives further terms involving the matter fields). Old minimal supergravity cannot possess a conservation law of third order as its gauge symmetries are irreducible [9].

The second order conservation laws read in complete form

$$j^{\mu\nu}_{i_a}=\tfrac{1}{2}\,e\,F^{\mu\nu}_{i_a}+\tfrac{1}{2}\,e\,\varepsilon^{\mu\nu\rho\sigma}(\psi_\rho\sigma_\sigma\bar{\lambda}_{i_a}+\lambda_{i_a}\sigma_\sigma\bar{\psi}_\rho),\quad i_a\neq(r) \tag{7.4}$$

where $F_{\mu\nu}{}^{i_a}$ are the abelian super-covariant field strengths (5.5). These conservation laws disappear when matter fields are included transforming nontrivially under the abelian gauge transformations, cf. remarks in the text after (6.9).

The Noether currents associated with the type-Va-solutions correspond to the global symmetry of the Lagrangian (5.3) under $SO(N)$ rotations of the N abelian gauge multiplets different from the one gauging R-transformations.

The type-Vb-solutions $\tilde{C}^*_{i_a}\tilde{Q}^*$ are present only in new minimal supergravity. They correspond to global symmetries of new minimal supergravity transforming for instance $t_{\mu\nu}$ among others into the dual of the field strength of $A_\mu{}^{i_a}$ and $A_\mu{}^{i_a}$ among others into a linear combination of $A_\mu{}^{(r)}$ and the dual of $H_{\mu\nu\rho}$ (this can be read off from the antifield dependent terms contained in $\tilde{C}^*_{i_a}\tilde{Q}^*$).

The remaining Noether currents arise from the N_τ. Explicitly we know only one of them, arising from the quantity Θ given in (6.16) and present only in old minimal supergravity when R-transformations are not gauged. In *pure* old minimal supergravity Θ is weakly $\tilde{s}$-invariant by itself. The 3-form contained in it is given by

$$dx^\mu dx^\nu dx^\rho\, \psi_\rho \sigma_\nu \bar{\psi}_\mu$$

and provides the Noether current corresponding to the global R-invariance of the Lagrangian (5.2) (with $\lambda^{(r)} = 0$ and $D^{(r)} = 0$). When old minimal supergravity is coupled to Yang–Mills multiplets via (5.3), Θ is not weakly $\tilde{s}$-invariant anymore by itself but has still an $\tilde{s}$-invariant completion which again provides the R-Noether current (containing now the gauginos as well) and remains nontrivial unless R-transformations are gauged.

8 On-shell counterterms and deformations

In this section we discuss the implications of the results for the possible on-shell counterterms and for the consistent and continuous deformations of old and new supergravity and their gauge symmetries, based on (6.1). In fact both issues are closely related.

The possible counterterms that are non-vanishing and gauge invariant on-shell are directly determined by the local BRST cohomology at ghost number 0 as the latter is equivalent to the weak BRST cohomology on local functionals constructed only out of the classical fields.

That the consistent and continuous deformations are also restricted by the local BRST cohomlogy at ghost number 0 was shown in [7] where a systematic method was outlined to obtain and classify these deformations. The basic idea of this method is to deform the (classical) master equation [11, 3]. More precisely one looks for a local solution to the master equation of the form

$$\mathcal{S}_g = \mathcal{S} + g\,\mathcal{S}^{(1)} + g^2\mathcal{S}^{(2)} + \dots \tag{8.1}$$

where $\mathcal{S}$ is the solution in the original (undeformed) theory and g is a deformation parameter. The master equation $(\mathcal{S}_g, \mathcal{S}_g) = 0$ is then expanded in powers of g which yields

$$\left(\mathcal{S}, \mathcal{S}^{(1)}\right) = 0, \quad \left(\mathcal{S}^{(1)}, \mathcal{S}^{(1)}\right) + 2\left(\mathcal{S}, \mathcal{S}^{(2)}\right) = 0, \quad \dots \tag{8.2}$$

As $(\mathcal{S},\ \cdot\ )$ generates the original BRST transformations, the first condition in (8.2) requires $\mathcal{S}^{(1)}$ to be invariant under the undeformed BRST transformations. To first order in g the deformations are thus indeed determined by the local BRST cohomology at ghost number 0. At higher orders in g the BRST cohomology at ghost number 1 can impose further obstructions [8]. The antifield independent part of $\mathcal{S}_g$ gives of course the deformation of the original action while the antifield dependent part provides the correspondingly deformed gauge transformations.

The BRST cohomology at ghost number 0 arises from the $\tilde{s}$-cohomology at total degree 4. The latter is obtained from section 6.3, leading to the results summarized in table 8.1. The latter sketches both the $\tilde{s}$-invariant total forms and the corresponding BRST invariant local functionals, writing the latter as $\int d^4x\, e\, L_{onshell}$ and giving only a characteristic term. Moreover we use the notation

$$\Omega_\Delta = \mathcal{A}_\Delta(\bar{\lambda}, \bar{W}) + D^2\mathcal{B}_\Delta(\hat{\mathcal{T}}), \quad D^2 = D^\alpha D_\alpha\,, \quad \bar{D}^2 = \bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}} \tag{8.3}$$

with $\hat{\mathcal{T}}$ as in section 6.1 and $D_\alpha\hat{\mathcal{T}}$ as in appendix C.3, and

$$J^\mu_\tau = \frac{1}{e}\, j^\mu_\tau \tag{8.4}$$

where j^μ_τ is the Noether current corresponding to N_τ. J^μ_τ is of course only covariantly conserved, in contrast to j^μ_τ which satisfies $\partial_\mu j^\mu_\tau \approx 0$.

Type	α		$L_{onshell}$
II	$\hat{\mathcal{P}}_\Delta$		$\bar{D}^2\Omega_\Delta + D^2\bar{\Omega}_\Delta + \ldots$
III	$\mathcal{H}\tilde{C}^{i_a} + \ldots$		$A_\mu{}^{i_a}\partial_\nu t_{\rho\sigma}\varepsilon^{\mu\nu\rho\sigma} + \ldots$
IV	$\tilde{C}^{i_a}N_\tau + \ldots$		$A_\mu{}^{i_a}J^\mu_\tau + \ldots$
Va	$\tilde{C}^{*[i_a}\tilde{C}^{i_b}\tilde{C}^{i_c]} + \ldots$	$(i_a, i_b, i_c \neq (r))$	$F^{\mu\nu[i_a}A_\nu{}^{i_b}A_\mu{}^{i_c]} + \ldots$
Vb	$\tilde{C}^{*[i_a}\tilde{C}^{i_b]}\tilde{Q}^* + \ldots$	$(i_a, i_b \neq (r))$	$4F^{\mu\nu[i_a}A_\nu{}^{i_b]}A_\mu{}^{(r)} + \ldots$
Vc	$\tilde{C}^*_{i_a}\tilde{Q} + \ldots$	$(i_a \neq (r))$	$-\frac{1}{2}F^{\mu\nu}_{i_a}t_{\mu\nu} + \ldots$

Table 8.1: Counterterms and deformations

Let me comment the result.

Type II and III: Using the auxiliary fields, all these functionals can be completed to off-shell invariants which do not involve antifields as they have counterparts in the restricted BRST cohomology, see section 5. The solutions of type II and III yield thus only deformations which do not change the gauge transformations nontrivially. I note that these solutions yield among others possible counterterms which were discussed already in [14]. For instance, the integrand of a counterterm arising from $\mathcal{B}_\Delta = W^2\bar{W}^2X^{2n}\bar{X}^{2n}$ contains a contribution proportional to $eX^{2(n+1)}\bar{X}^{2(n+1)}$, i.e. a term of order $4(n+1)$ in the Weyl tensor. The terms of type III are present only in new minimal supergravity and reproduce the supergravity Lagrangian (5.12) and the Fayet–Iliopoulos term (5.13).

Type IV and V: The remaining terms have no counterparts in the restricted $\tilde{s}$-cohomology. They yield therefore the possible counterterms that are invariant on-shell but cannot be completed to off-shell invariants, and provide the possible nontrivial deformations of the gauge transformations to first order in the deformation parameter. We have to ask ourselves what these deformations of the gauge symmetries might be.

The terms of type IV contain couplings of Noether currents to abelian gauge fields. This suggests that the resulting deformations just gauge global symmetries in the standard way.

The terms of type Va are reminiscent of the trilinear vertex of gauge fields in nonabelian Yang–Mills theory which suggests that the corresponding deformations convert abelian gauge multiplets into nonabelian ones. The complete antisymmetrization in the group indices then corresponds to the antisymmetry of the structure constants of the Lie algebra. The Jacobi identity for these structure constants arises at second order in g, cf. [39, 8] for a discussion in the nonsupersymmetric case.

The terms of type Vb occur only in new minimal supergravity (due to (6.23)) and are somewhat similar to those of type Va. The differences to the type-Va-terms are however that a) the antisymmetrization in the abelian indices excludes the R-transformation and b) the gauginos and the gravitino transform nontrivially under R-transformations, in contrast to the properties of the type-Va-terms. These differences reflect again that $\tilde{C}^{(r)}$ can be completed to an $\tilde{s}$-invariant total form in new minimal supergravity, cf. (6.10). I have not yet investigated whether this might lead to interesting unknown deformations.

The term of type Vc has no counterpart in standard Yang–Mills theory or gravity because it involves $t_{\mu\nu}$. Therefore it deserves special attention. Surprisingly it gives rise to a deformation which converts on-shell new minimal supergravity into old minimal supergravity with local R-invariance. This unusual feature is possible because fields which are gauge fields in new minimal supergravity mutate through the deformation to auxiliary fields or disappear for $g \neq 0$ completely from the theory after suitable local field redefinitions. The details of the computation are in principle straightforward but nevertheless somewhat involved. They will be given elsewhere [40]. Here I only illustrate the underlying mechanism in a nonsupersymmetric toy model in flat space.

The toy model involves two ordinary abelian gauge (vector) fields, denoted by A_μ and a_μ, and a 2-form gauge potential whose components are denoted again by $t_{\mu\nu}$. a_μ and $t_{\mu\nu}$ play roles analogous to the R-gauge field and the 2-form gauge potential in new minimal supergravity. The Lagrangian of the toy model is analogous to the "bosonic part" of the sum of (5.12) and (5.3). It is given by

$$L = \tfrac{1}{2}\, H_{\mu\nu\rho}H^{\mu\nu\rho} - \tfrac{2}{3}\, \varepsilon^{\mu\nu\rho\sigma} a_\mu H_{\nu\rho\sigma} - \tfrac{1}{4}\, F^{\mu\nu}F_{\mu\nu} \tag{8.5}$$

where $H_{\mu\nu\rho} = 3\partial_{[\mu}t_{\nu\rho]}$ and $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$ are the field strengths of $t_{\mu\nu}$ and A_μ respectively and all indices refer to flat space (hence, *here* we work with $\varepsilon^{\mu\nu\rho\sigma} \in \{0, 1, -1\}$). Observe first that the field redefinition $a'_\mu = 2a_\mu + \frac{1}{4}\varepsilon_{\mu\nu\rho\sigma}H^{\nu\rho\sigma}$ casts (8.5) in the simpler form

$$L = -\varepsilon^{\mu\nu\rho\sigma} a'_\mu \partial_\nu t_{\rho\sigma} - \tfrac{1}{4}\, F^{\mu\nu}F_{\mu\nu}\ . \tag{8.6}$$

The toy model is evidently invariant under the gauge resp. BRST transformations

Φ	$t_{\mu\nu}$	A_μ	a'_μ	Q_μ	C	c	Q
$s\Phi$	$\partial_\nu Q_\mu - \partial_\mu Q_\nu$	$\partial_\mu C$	$\partial_\mu c$	$\partial_\mu Q$	0	0	0

(8.7)

The first order deformation $\mathcal{S}^{(1)}$ of the toy model analogous to the type-Vc-term in

table 8.1 is easily verified to be

$$\mathcal{S}^{(1)} = \int d^4x\,(-\tfrac{1}{2}\,t_{\mu\nu}F^{\mu\nu} + A^{*\mu}Q_\mu + C^*\,Q)$$

and evidently nontrivial because $t_{\mu\nu}F^{\mu\nu}$ does not vanish on-shell modulo a total derivative. Elementary computations show that an $\mathcal{S}^{(2)}$ satisfying (8.2) is simply

$$\mathcal{S}^{(2)} = -\tfrac{1}{4}\int d^4x\,t_{\mu\nu}t^{\mu\nu}$$

and that

$$\mathcal{S}_g = \mathcal{S} + g\,\mathcal{S}^{(1)} + g^2\mathcal{S}^{(2)}$$

solves the master equation where $\mathcal{S}$ is the original solution whose integrand is $L - (s\Phi^A)\Phi^*_A$. The deformed Lagrangian L_g and BRST transformations s_g are now easily read off from $\mathcal{S}_g$,

$$L_g = -\varepsilon^{\mu\nu\rho\sigma}a'_\mu\partial_\nu t_{\rho\sigma} - \tfrac{1}{4}\,(F_{\mu\nu}+gt_{\mu\nu})(F^{\mu\nu}+gt^{\mu\nu}), \tag{8.8}$$

Φ	$t_{\mu\nu}$	A_μ	a'_μ	Q_μ	C	c	Q
$s_g\Phi$	$\partial_\nu Q_\mu - \partial_\mu Q_\nu$	$\partial_\mu C + gQ_\mu$	$\partial_\mu c$	$\partial_\mu Q$	$-gQ$	0	0

(8.9)

Evidently the deformed Lagrangian depends only on a'_μ and on $t'_{\mu\nu} = F_{\mu\nu} + gt_{\mu\nu}$. The latter is s_g-invariant and can be algebraically eliminated for $g \neq 0$ through its equation of motion. This turns L_g into the usual action for just one abelian gauge field,

$$L'_g = -\frac{1}{g^2}\,F'_{\mu\nu}F'^{\mu\nu}, \quad F'_{\mu\nu} = \partial_\mu a'_\nu - \partial_\nu a'_\mu\ . \tag{8.10}$$

Note that we started from an action for three gauge fields $A_\mu, a_\mu, t_{\mu\nu}$ and, after the deformation and elimination of $t'_{\mu\nu}$, ended up with an action for only one gauge field. What happened to the other gauge fields and gauge symmetries? One can take the following point of view: $t_{\mu\nu}$ mutated to an auxiliary field $t'_{\mu\nu}$ carrying no gauge symmetry anymore, whereas A_μ dropped out completely due to its deformed gauge transformation $s_gA_\mu = Q'_\mu \equiv \partial_\mu C + gQ_\mu$. We may also take the point of view that by deforming the model we have gauged the global shift symmetry $A_\mu \to A_\mu + \epsilon_\mu$ of L. In this perspective $t_{\mu\nu}$ and Q_μ are the gauge and ghost fields associated with the shift symmetry and $t'_{\mu\nu}$ is the new field strength of A_μ which is covariant (in fact invariant) with respect to the gauged shift symmetry.

This mechanism is somewhat reminiscent of a familiar implementation of duality transformations in abelian gauge theories, see e.g. [41]. In particular the first term in the Lagrangian (8.8) is analogous to the Lagrange multiplier term that one introduces in that approach. In fact duality transformations relating old and new minimal supergravity are well-known in the literature [42, 16]. It might therefore be worthwhile to study their relations to the deformation found here.

9 Candidate gauge anomalies

The candidate gauge anomalies are obtained from the (full) $\tilde{s}$-cohomology at total degree 5. The latter is summarized in table 9.1, with Ω_Δ and J^μ_τ as in (8.3) and (8.4) and writing the candidate anomalies as $\int d^4x\, e\, L_{ano}$. Furthermore, we use (solutions of type I and Va)

$$C = C^I T_I^{(K)} \ , \quad F_{\mu\nu} = F_{\mu\nu}{}^I T_I^{(K)} \tag{9.1}$$

for nonabelian ghost and curvature matrices constructed by means of the respective Lie algebra representations $\{T_I^{(K)}\}$ as in (4.23).

Type	α		L_{ano}
I	$\tilde{\theta}_K + \dots$	$(m_K = 3)$	$Tr(CF_{\mu\nu}F_{\rho\sigma})\varepsilon^{\mu\nu\rho\sigma} + \dots$
II	$\tilde{C}^{i_a}\hat{\mathcal{P}}_\Delta + \dots$		$C^{i_a}(\bar{D}^2\Omega_\Delta + D^2\bar{\Omega}_\Delta) + \dots$
III	$\mathcal{H}\tilde{C}^{i_a}\tilde{C}^{i_b} + \dots$		$C^{[i_a}A_\mu{}^{i_b]}\partial_\nu t_{\rho\sigma}\varepsilon^{\mu\nu\rho\sigma} + \dots$
IVa	$N_\tau\tilde{C}^{i_a}\tilde{C}^{i_b} + \dots$		$C^{[i_a}A_\mu{}^{i_b]}J^\mu_\tau + \dots$
IVb	$N_\tau\tilde{Q} + \dots$		$Q_\mu J^\mu_\tau + \dots$
Va	$\tilde{C}^*_{i_a}\tilde{\theta}_K + \dots$	$(m_K = 2, i_a \neq (r))$	$-\frac{1}{2}\,Tr(CF_{\mu\nu})F^{\mu\nu}_{i_a} + \dots$
Vb	$\tilde{C}^{*[i_a}\tilde{C}^{i_b}\tilde{C}^{i_c}\tilde{C}^{i_d]} + \dots$	$(i_a, \dots, i_d \neq (r))$	$F^{\mu\nu[i_a}A_\nu{}^{i_b}A_\mu{}^{i_c}C^{i_d]} + \dots$
Vc	$\tilde{C}^{*[i_a}\tilde{C}^{i_b}\tilde{C}^{i_c]}\tilde{Q}^* + \dots$	$(i_a, i_b, i_c \neq (r))$	$4C^{(r)}F^{\mu\nu[i_a}A_\nu{}^{i_b}A_\mu{}^{i_c]} + \dots$
Vd	$\tilde{C}^{*[i_a}\tilde{C}^{i_b]}\tilde{Q} + \dots$	$(i_a, i_b \neq (r))$	$C^{[i_a}F_{\mu\nu}{}^{i_b]}t^{\mu\nu} + \dots$
Ve	$\tilde{C}^*_{i_a}\tilde{C}^{(r)}\tilde{Q} + \dots$	$(i_a \neq (r))$	$-\frac{1}{2}\,C^{(r)}F^{\mu\nu}_{i_a}t_{\mu\nu} + \dots$

Table 9.1: Candidate anomalies

Note that the candidate anomalies of type III, IVb, Vc, Vd, Ve occur only in new minimal supergravity. Let me now briefly discuss and comment these results.

I. The solutions of type I are the supersymmetric version of the nonabelian chiral anomalies. In section 4.4 it has been shown already that they exist on general grounds, see text after (4.51). Remarkably they can be chosen so as to involve neither the antifields, nor the gravitino, nor any of the auxiliary fields. In this form they are given by [34]

$$\int Tr\Big\{Cd(AdA + \tfrac{1}{2}A^3) + 3i\,d^4x\,e\,(\bar{\xi}\bar{\lambda}\,\lambda\lambda + \xi\lambda\,\bar{\lambda}\bar{\lambda}) \\ +i(\xi\sigma\,\bar{\lambda} + \lambda\,\sigma\bar{\xi})(AdA + (dA)A + \tfrac{3}{2}A^3)\Big\} \tag{9.2}$$

where ξ and $\bar{\xi}$ are the supersymmetry ghosts, C are ghost matrices as in (9.1) and A, λ, $\bar{\lambda}$ and σ are the matrices

$$A = dx^\mu A_\mu{}^i T_i^{(K)},\ \lambda_\alpha = \lambda^i_\alpha T_i^{(K)},\ \bar{\lambda}_{\dot{\alpha}} = \bar{\lambda}^i_{\dot{\alpha}} T_i^{(K)},\ \sigma_{\alpha\dot{\alpha}} = dx^\mu\sigma_{\mu\,\alpha\dot{\alpha}}. \tag{9.3}$$

II. The solutions of type II are analogous to candidate anomalies in non-supersymmetric Yang–Mills theory whose integrands are of the form "abelian ghost × gauge invariant function". Their complete form has been given in [34] (see eq. (3.12) there). In particular they include the supersymmetric version of abelian chiral anomalies arising from $\Omega_\Delta = a\bar{\lambda}^i\bar{\lambda}_i + b\bar{W}\bar{W}$ where a is purely imaginary. It should be remarked however in this context that *in new minimal supergravity there is no abelian chiral anomaly involving the ghost of R-transformations* as the corresponding solutions are trivial. Indeed, in new minimal supergravity $4\tilde{C}^{(r)}$ can be completed to an $\tilde{s}$-invariant total form $\tilde{Q}^*$, see (6.10). A total form solving $\tilde{s}\alpha = 0$ and corresponding to abelian chiral anomalies involving the R-ghost would therefore read $\tilde{Q}^* Tr(\mathcal{F}\mathcal{F})$ with $\mathcal{F}$ as in (4.50). However, this total form is $\tilde{s}$-exact, $\tilde{Q}^* Tr(\mathcal{F}\mathcal{F}) = -\tilde{s}(\tilde{Q}^* q)$ where q is the total super-Chern–Simons form satisfying $\tilde{s}\, q = Tr(\mathcal{F}\mathcal{F})$, see section 4.4.

III. The candidate anomalies of type III are present only in new minimal supergravity. They can be chosen to be antifield independent when the auxiliary D-fields are used and read then in complete form [34]

$$a_{i_a i_b} \int d^4x\, e\, \Big\{ C^{i_a}(\varepsilon^{\mu\nu\rho\sigma} A_\mu{}^{i_b} \partial_\nu t_{\rho\sigma} + \lambda^{i_b}\sigma^\mu\bar{\psi}_\mu + \psi_\mu\sigma^\mu\bar{\lambda}^{i_b} + D^{i_b}) \\ - A_\mu{}^{i_a}(\lambda^{i_b}\sigma^\mu\bar{\xi} + \xi\sigma^\mu\bar{\lambda}^{i_b}) - i\varepsilon^{\mu\nu\rho\sigma} A_\mu{}^{i_a} A_\nu{}^{i_b}(\xi\sigma_\rho\bar{\psi}_\sigma + \psi_\rho\sigma_\sigma\bar{\xi}) \Big\} \qquad (9.4)$$

where the coefficients $a_{i_a i_b}$ must be antisymmetric,

$$a_{i_a i_b} = -a_{i_b i_a}\ , \qquad (9.5)$$

and the use of the identifications (5.10) is understood. Due to (9.5) these candidate anomalies occur only in presence of at least one abelian gauge symmetry in addition to R-symmetry. Note that they are somewhat similar to chiral anomalies, as the first term in the integrand of (9.4) can be written completely in terms of ghosts and connection- and curvature-forms. I remark that the total form $\mathcal{H}\tilde{Q}$ occurring in table 6.1 among the type III terms does not give rise to a candidate anomaly because it is trivial thanks to $\mathcal{H}\tilde{Q} = \tilde{s}(\frac{1}{2}\tilde{Q}^2)$.

IV. All type-IV-candidate anomalies involve a Noether current and are thus present only if the classical action has at least one nontrivial global symmetry. The solutions of type IVa have counterparts in Yang–Mills theory and standard gravity [19, 24, 25, 26] and occur only if there are at least two abelian gauge symmetries. The solutions of type IVb are present only in new minimal supergravity and involve among others the ghosts Q_μ corresponding to $t_{\mu\nu}$.

V. The candidate gauge anomalies of type V disappear as soon as one includes matter fields transforming nontrivially under the abelian gauge transformations, see remark in the text after equation (6.9). Therefore these solutions appear to be only of academic interest insofar as (true) anomalies are concerned. Recall however that abelian gauge fields couple in supergravity always to the gravitino and to the abelian gauginos via triple vertices present in (5.3).

VI. Finally I stress that in new minimal supergravity *any* nontrivial Noether current j^μ gives rise to a previously unknown candidate gauge anomaly of the form

$$\int d^4x\, Q_\mu j^\mu + \dots \tag{9.6}$$

This follows from tables 7.1 and 9.1 because all those solutions in table 7.1 providing the Noether currents (type IV, Va and Vb) occur in table 9.1 multiplied with $\tilde{Q}$.

10 Generalizations of the results

The investigation can of course be extended to the case that further (matter) multiplets are present. It is particularly easy to include matter multiplets which (a) have QDS-structure and (b) transform linearly under the Yang–Mills gauge group (and under the Lorentz group), provided one can restrict the investigation to local functionals which depend on the undifferentiated scalar fields of these multiplets only via formal (possibly infinite) series'. Indeed, under these assumptions the methods and results of the computation extend straightforwardly to the case that matter fields are included.

Requirement (a) is for instance satisfied off-shell for the matter multiplets most commonly used in supergravity theories, namely chiral multiplets $(\varphi, \chi_\alpha, F)$ with supersymmetry transformations given by

$$\begin{array}{lll} \mathcal{D}_\alpha\varphi = \chi_\alpha, & \mathcal{D}_\alpha\chi_\beta = \varepsilon_{\beta\alpha}F, & \mathcal{D}_\alpha F = -\frac{1}{2}\bar{M}\chi_\alpha, \\ \mathcal{D}_\alpha\bar{\varphi} = 0, & \mathcal{D}_\alpha\bar{\chi}_{\dot{\alpha}} = -2i\mathcal{D}_{\alpha\dot{\alpha}}\bar{\varphi}, & \mathcal{D}_\alpha\bar{F} = 2i\mathcal{D}_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} + B_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} - 4\lambda^i_\alpha\delta_i\bar{\varphi}. \end{array}$$

This implies indeed that the D_α-representation on chiral multiplets has QDS-structure off-shell [27, 28]. The result for the restricted BRST cohomology in presence of chiral matter multiplets (under the above-mentioned assumptions) is then easily obtained from section 4: the only difference is that in (4.29) $\mathcal{A}_\Delta$ can also depend on the $\bar{\varphi}$ whereas $\mathcal{B}_\Delta$ now also involves φ, χ, F, $\bar{\varphi}$, $\bar{\chi}$, $\bar{F}$ and their super-covariant derivatives, see [34] for details.

The results for the full BRST cohomology also change only slightly, provided one uses a standard action for the chiral multiplets. For instance, in the simplest case the equations of motion reduce at linearized level to $\Box\varphi \sim 0$, $\mathcal{D}^+_-\chi \sim 0$, $F \sim 0$. It is then easy to verify that the D_α-representation on the chiral multiplets has QDS-structure even on-shell. Indeed this representation decomposes into singlets given by the undifferentiated $\bar{\varphi}$'s and the following (D)-multiplets:

$$\{(D^+_+)^q\varphi, (D^+_+)^q\chi\},\ \{(D^+_+)^q\bar{\chi}, -2i(D^+_+)^{q+1}\bar{\varphi}\} \quad (q = 0, 1, \dots)$$

where a notation as in appendix C was used. Section 6 then provides straightforwardly the complete results for the BRST cohomology in presence of chiral matter fields: one must just count $(D^+_+)^q\varphi$, $(D^+_+)^q\bar{\varphi}$, $(D^+_+)^q\chi$ and $(D^+_+)^q\bar{\chi}$ among the $\hat{\mathcal{T}}$'s, add $\bar{\varphi}$ to the

arguments of $\mathcal{A}_\Delta$ in equation (6.19), and discard $\tilde{C}^*_{i_a}$ whenever there is a matter multiplet which transforms nontrivially under the i_ath abelian gauge symmetry.

The inclusion of linear multiplets is slightly more involved. Namely these multiplets will give rise to some additional cohomology as they contain 2-form gauge potentials. Nevertheless the results of this paper can be easily adapted to models containing linear multiplets because one can show that the D_α-representation on such multiplets has QDS-structure too (for the standard supersymmetry transformations and actions). As a consequence, the presence of linear multiplets effects the BRST cohomology in a manner very similar to the effect that the 2-form gauge potential has in new minimal supergravity.

The results also generalize straightforwardly to supergravity theories with more complicated actions (containing for instance higher powers in the curvatures), provided these actions can be viewed as a continuous deformation of the simple action with Lagrangian (6.1) in the sense of [7] and section 8. Namely then one can use standard arguments from spectral sequence techniques to relate the BRST cohomology in the more complicated theories to that in the simple ones considered here [8]. In particular the BRST cohomology can at most shrink but not grow when going to a more complicated theory by a deformation. Therefore there are for instance not more candidates for on-shell counterterms, nontrivial deformations or gauge anomalies than in the simple theories. The explicit form of the solutions to the cohomological problem may of course change.

11 Note on topological aspects

Let me finally briefly discuss a topic which was neglected so far and concerns 'topological' solutions to (1.1), i.e. solutions which are locally but not globally of the form $s\eta_4 + d\eta_3$. Such solutions correspond to local total forms which are locally but not globally $\tilde{s}$-exact and are called topological total forms in the following.

As mentioned already in section 4.1, the $\tilde{s}$-cohomology is locally trivial on total forms $\alpha(\mathcal{U},\mathcal{V})$. However, in general it is not globally trivial because the manifold of the $\mathcal{U}$'s and $\mathcal{V}$'s has a nontrivial De Rham cohomology which gives rise to topological total forms $\alpha(\mathcal{U},\mathcal{V})$. As we are considering *local* total forms, this De Rham cohomology boils down to the product of the De Rham cohomology of $GL(4)$ carried by the undifferentiated vierbein fields (see [26] for details) and of the usual De Rham cohomology of the spacetime manifold. The latter comes here into play because the x^μ and dx^μ count among the $\mathcal{U}$'s and $\mathcal{V}$'s. To give an example, the De Rham cohomology of $GL(4)$ gives rise to the following topological total form with total degree 3,

$$\alpha_{top}(\mathcal{U},\mathcal{V}) = Tr(\tilde{v}^3), \quad \tilde{v}_a{}^b = E_a{}^\mu \tilde{s} e_\mu{}^b \ .$$

The 3-form contained in α_{top} is a topological conservation law of first order in the terminology of section 7 as it is d-closed but in general only locally but not globally

d-exact. It is given by

$$j_{top} = Tr(v^3), \quad v_a{}^b = E_a{}^\mu de_\mu{}^b \ .$$

This topological conservation law is similar to (though different from) the one discussed already in [43].

Using now the Künneth formula (4.8), one can construct further topological forms involving α_{top} by multiplying it with nontrivially $\tilde{s}$-invariant total forms $\alpha(\mathcal{W})$. For instance, in new minimal supergravity a topological total form with total degree 4 is given by the product $\alpha_{top}\tilde{Q}^*$ with $\tilde{Q}^*$ as in section 6.2. The 4-form contained in $\alpha_{top}\tilde{Q}^*$ is thus a topological solution to (1.1) with ghost number 0 and has the form

$$\omega_4 = 4Tr(v^3)\, A^{(r)} + \dots \ , \quad A^{(r)} = dx^\mu A_\mu{}^{(r)} \ .$$

Another well-known source for topological solutions is of course the possible nontriviality of the fiber bundles associated with the gauge fields. Prominent examples for such solutions are the polynomials $Tr(F^2)$ in the curvature 2-forms F providing characteristic classes.

12 Conclusions

Let me finally summarize and comment the main results of the paper.

Gauge invariant actions: We have shown that in old minimal supergravity the most general local action, invariant under the standard gauge transformations, emerges from the density formula (5.1). This *proves* that in this case the most general gauge invariant local action can be constructed from superspace integrals à la [15]. In contrast, in new minimal supergravity there are a few additional terms, given in (5.12) and (5.13), which cannot be written as superspace integrals without further ado (one such term for each abelian gauge symmetry). The presence of these "exceptional" terms in new minimal supergravity is crucial because one of them (corresponding to the local R-symmetry) is just the supersymmetrized Einstein–Hilbert action, whereas the others contain Fayet–Iliopoulos terms relevant for spontaneous supersymmetry breaking [44]. As all these exceptional contributions contain Chern–Simons like terms involving the 2-form gauge potential and abelian gauge fields, the situation is somewhat similar to standard (non-supersymmetric) gravity. There the Chern–Simons contributions are the only exceptions to the rule that invariant actions are integrated scalar densities of the form "vielbein determinant $\times$ scalar function of the tensor fields" [23].

Consistent deformations: The gauge transformations are extremely stable under continuous deformations of the type described in the introduction. In old minimal supergravity the gauge transformations cannot be consistently deformed in a continuous and nontrivial manner whenever the Yang–Mills gauge group is semisimple[11]. As

[11] I stress again that in this form the statement applies to the formulation of the theory *with* auxiliary fields. The elimination of the auxiliary fields can of course modify the on-shell gauge

in ordinary (non-supersymmetric) gravity [26], there may be nontrivial deformations in presence of abelian gauge symmetries but it seems that all of them are well-known, gauging either global symmetries in the standard way or deforming abelian to standard nonabelian gauge multiplets. In new minimal supergravity the situation is slightly different. In particular there exists an unusual deformation which converts new into old minimal supergravity with gauged R-transformations and is reminiscent of a duality transformation, see [40] for details.

On-shell counterterms: We have obtained the complete lists of the possible counterterms that are gauge invariant on-shell for old and new minimal supergravity (these lists are actually overcomplete as they still contain terms which reduce on-shell to surface integrals). It turns out that apart from very few exceptions all these on-shell counterterms can actually be completed to off-shell invariants by means of the auxiliary fields. In old minimal supergravity exceptions occur only if there are (i) at least one nontrivial Noether current *and* one abelian gauge symmetry, or (ii) at least three abelian gauge symmetries. A similar result holds in new minimal supergravity.

Candidate gauge anomalies: We have classified the possible gauge anomalies completely (up to "topological anomalies"). The result indicates in particular that supersymmetry itself is not anomalous in minimal supergravity because all the candidate anomalies have counterparts in the corresponding non-supersymmetric theories. In other words, supersymmetry does not introduce new types of candidate anomalies. In old minimal supergravity the supersymmetrized version (9.2) of the familiar nonabelian chiral anomalies exhausts the candidate gauge anomalies whenever the Yang–Mills gauge group is semisimple. All other candidate gauge anomalies require thus the presence of at least one abelian gauge symmetry, as in standard Einstein–Yang–Mills theories [23, 26]. In new minimal supergravity there are special candidate anomalies due to the presence of the two-form gauge potential. They are given by (9.4) and (9.6). The former are somewhat similar to chiral anomalies and had been found already in [34]. The latter were previously unknown. They correspond one-to-one to the nontrivial Noether currents (if any) and involve the ghosts associated with the 2-form gauge potential.

Dynamical conservation laws: New minimal supergravity possesses precisely one dynamical conservation law of order 3, i.e. one dynamically conserved 1-form (up to trivial ones). Old minimal supergravity does not admit such a conservation law because the gauge transformations are irreducible in this case. The dynamical conservation laws of order 2 (i.e. the nontrivial conserved 2-forms) are exhausted by supersymmetric completions of the duals of abelian curvature forms, both in old

transformations. For instance, adding the cosmological contribution (5.6) to the standard supergravity Lagrangian (5.2) (without local R-symmetry, i.e. for $\lambda^{(r)} \equiv D^{(r)} \equiv 0$) provides a consistent deformation of old minimal supergravity with deformation parameter m which does not change the gauge transformations as long as one keeps the auxiliary fields. However, upon elimination of the auxiliary field M, this deformation modifies the supersymmetry transformation of ψ_μ on-shell by a term proportional to $i\bar{m}\bar{\xi}\bar{\sigma}_\mu$, due to the presence of M in the off-shell supersymmetry transformation of ψ_μ. This modification indicates of course spontaneous supersymmetry breaking related to the cosmological constant and gravitino mass term introduced by (5.6).

and new minimal supergravity. The results for the dynamical conservation laws of first order, i.e. for the Noether currents and the corresponding global symmetries are incomplete. Nevertheless we found both well-known global symmetries (global R-invariance of old minimal supergravity, invariance under global $SO(N)$-rotations of N abelian gauge multiplets), as well as further (possibly previously unknown) global symmetries of new minimal supergravity when it is coupled to abelian gauge multiplets. The latter global symmetries transform for instance the 2-form gauge potential among others into the dual of an abelian field strength. It is likely, though not proved, that these are all the inequivalent and nontrivial global symmetries of minimal supergravity in absence of matter multiplets.

Finally I stress again that these results hold thanks to the "QDS structure" [27, 28] of old and new minimal supergravity. This structure refers to the representation of the linearized supersymmetry algebra, or rather its subalgebra (C.1) on tensor fields and holds both off-shell and on-shell. The QDS structure remains intact even when chiral matter multiplets are included, at least for the standard actions. In this case the results remain therefore essentially unchanged, see section 10 for details. The inclusion of linear multiplets is also straightforward even though these multiplets will give rise to some additional cohomology, similar to the effect that the presence of the 2-form gauge potential has in new minimal supergravity.

It can be shown [27] that the QDS structure of minimal supergravity is somewhat related to the so-called constraints satisfied by the torsions and curvatures occurring in the covariant supergravity algebra (2.13). Other constraints, such as those realized in non-minimal supergravity [45], could destroy the QDS structure and therefore might lead to different cohomological results. In particular candidate anomalies for supersymmetry itself may be present in such a case. The arguments in [46] suggest that such candidate anomalies might occur in non-minimal supergravity, but this has not yet been confirmed by cohomological means (see however [34] for some results on the restricted BRST cohomology in non-minimal supergravity supporting this conjecture).

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A Conventions

The conventions concerning the BRST algebra (BRST operator, antibracket etc.) agree with those used in [20].

A.1 Lorentz algebra

Minkowski metric, ε-tensors:

$$\eta_{ab} = diag(1,-1,-1,-1), \quad \varepsilon^{abcd} = \varepsilon^{[abcd]}, \quad \varepsilon^{0123} = 1,$$
$$\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}, \quad \varepsilon^{\dot\alpha\dot\beta} = -\varepsilon^{\dot\beta\dot\alpha}, \quad \varepsilon^{12} = \varepsilon^{\dot 1\dot 2} = 1,$$
$$\varepsilon_{\alpha\gamma}\varepsilon^{\gamma\beta} = \delta_\alpha^\beta = diag(1,1), \quad \varepsilon_{\dot\alpha\dot\gamma}\varepsilon^{\dot\gamma\dot\beta} = \delta_{\dot\alpha}^{\dot\beta} = diag(1,1)$$

σ-matrices: $\sigma^a_{\alpha\dot\alpha}$ (α: row index, $\dot\alpha$: column index):

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\bar\sigma$-matrices:

$$\bar\sigma^{a\,\dot\alpha\alpha} = \varepsilon^{\dot\alpha\dot\beta}\varepsilon^{\alpha\beta}\sigma^a_{\beta\dot\beta}$$

$\sigma^{ab}, \bar\sigma^{ab}$-matrices:

$$\sigma^{ab} = \tfrac{1}{4}(\sigma^a\bar\sigma^b - \sigma^b\bar\sigma^a), \quad \bar\sigma^{ab} = \tfrac{1}{4}(\bar\sigma^a\sigma^b - \bar\sigma^b\sigma^a)$$

Lorentz ($SL(2,C)$) transformations:

$$l_{ab}V_c = -2\eta_{c[a}V_{b]}, \; l_{\alpha\beta}\psi_\gamma = -\varepsilon_{\gamma(\alpha}\psi_{\beta)}, \; l_{ab} = \sigma_{ab}{}^{\alpha\beta}l_{\alpha\beta} - \bar\sigma_{ab}{}^{\dot\alpha\dot\beta}\bar l_{\dot\alpha\dot\beta}. \tag{A.1}$$

A.2 Spinors, Grassmann parity and complex conjugation

We work with two-component Weyl spinors. Undotted and dotted spinor indices $\alpha, \dot\alpha$ distinguish the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations of $SL(2,C)$, related by complex conjugation.

Raising and lowering of spinor indices:

$$\psi_\alpha = \varepsilon_{\alpha\beta}\psi^\beta, \quad \psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta, \quad \bar\psi_{\dot\alpha} = \varepsilon_{\dot\alpha\dot\beta}\bar\psi^{\dot\beta}, \quad \bar\psi^{\dot\alpha} = \varepsilon^{\dot\alpha\dot\beta}\bar\psi_{\dot\beta}\ .$$

Contraction of spinor indices:

$$\psi\chi := \psi^\alpha\chi_\alpha, \quad \bar\psi\bar\chi := \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha}, \quad \psi^{\underline{\alpha}}\chi_{\underline{\alpha}} := \psi\chi + \bar\psi\bar\chi.$$

Lorentz vector indices in spinor notation:

$$V_{\alpha\dot\alpha} = \sigma^a_{\alpha\dot\alpha}V_a\ , \quad V^{\dot\alpha\alpha} = \bar\sigma^{\dot\alpha\alpha}_a V^a.$$

The Grassmann parity $\varepsilon(X)$ of a variable (field, antifield, differential or spacetime coordinate) or an operator is determined by the number of its spinor indices, its ghost number (gh) and its form degree (formdeg) according to

$$\varepsilon(X^{\dot\alpha_1\dots\dot\alpha_m}_{\alpha_1\dots\alpha_n}) = m + n + \mathrm{gh}(X) + \mathrm{formdeg}(X) \quad (mod\ 2).$$

The Grassmann parity of the variables, denoted collectively by Z^i, determines their statistics,

$$Z^i Z^j = (-)^{\varepsilon(Z^i)\,\varepsilon(Z^j)} Z^j Z^i.$$

Complex conjugation of a variable or an operator X is denoted by $\bar X$. Complex conjugation of products of variables and operators is defined by

$$\overline{XY} = (-)^{\varepsilon(X)\,\varepsilon(Y)} \bar X\ \bar Y.$$

In particular this implies

$$\overline{\partial/\partial Z} = (-)^{\varepsilon(Z)} \partial/\partial \bar Z\ .$$

B Gauge covariant algebra

This appendix spells out explicitly the realization of the gauge covariant algebra (2.13) in old and new minimal (Poincaré) supergravity. The super-covariant derivative $\mathcal{D}_a$ is defined according to

$$\mathcal{D}_a = E_a{}^\mu(\partial_\mu - A_\mu{}^I\delta_I - \psi_\mu{}^\alpha\mathcal{D}_\alpha - \bar\psi_{\mu\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}) \tag{B.1}$$

where

$$\{A_\mu{}^I\} = \{A_\mu{}^i\ ,\ \omega_\mu{}^{ab}\ :\ a > b\}$$

contains the Yang–Mills gauge fields $A_\mu{}^i$ and the spin connection

$$\begin{aligned} \omega_\mu{}^{ab} &= E^{a\nu}E^{b\rho}(\omega_{[\mu\nu]\rho} - \omega_{[\nu\rho]\mu} + \omega_{[\rho\mu]\nu}), \\ \omega_{[\mu\nu]\rho} &= e_{\rho a}\partial_{[\mu}e_{\nu]}{}^a - i\psi_\mu\sigma_\rho\bar\psi_\nu + i\psi_\nu\sigma_\rho\bar\psi_\mu\ . \end{aligned} \tag{B.2}$$

The nonvanishing $g_{IA}{}^B$ occurring in (2.13) are read off from

$$\begin{array}{lll} [l_{ab},\mathcal{D}_c] = 2\eta_{c[b}\mathcal{D}_{a]}, & [l_{ab},\mathcal{D}_\alpha] = -\sigma_{ab\,\alpha}{}^\beta\mathcal{D}_\beta, & [l_{ab},\bar{\mathcal{D}}_{\dot\alpha}] = \bar\sigma_{ab}{}^{\dot\beta}{}_{\dot\alpha}\bar{\mathcal{D}}_{\dot\beta}, \\ [\delta_{(r)},\mathcal{D}_a] = 0, & [\delta_{(r)},\mathcal{D}_\alpha] = -i\,\mathcal{D}_\alpha, & [\delta_{(r)},\bar{\mathcal{D}}_{\dot\alpha}] = i\,\bar{\mathcal{D}}_{\dot\alpha}. \end{array} \tag{B.3}$$

We use a formulation of old minimal supergravity with torsions $T_{AB}{}^C$ and curvatures $F_{AB}{}^I$ as in table B.1 where $T_{\mu\nu}{}^\alpha$ and $F_{\mu\nu}{}^I$ are given in terms of the gauge fields and the remaining torsions and curvatures according to

$$F_{\mu\nu}{}^I = 2(\partial_{[\mu}A_{\nu]}{}^I + \tfrac12 f_{JK}{}^I A_\mu{}^J A_\nu{}^K + e_{[\mu}{}^c\psi_{\nu]}{}^{\underline{\alpha}}F_{\underline{\alpha}c}{}^I + \tfrac12\psi_\mu{}^{\underline{\alpha}}\psi_\nu{}^{\underline{\beta}}F_{\underline{\alpha}\underline{\beta}}{}^I), \tag{B.4}$$

$$T_{\mu\nu}{}^\alpha = 2(\partial_{[\mu}\psi_{\nu]}{}^\alpha + e_{[\mu}{}^c\psi_{\nu]}{}^{\underline{\beta}}T_{\underline{\beta}c}{}^\alpha + \psi_{[\mu}{}^\beta A_{\nu]}{}^I g_{I\beta}{}^\alpha). \tag{B.5}$$

AB	ab	$\dot{\alpha}b$	$\dot{\alpha}\dot{\beta}$	$\alpha\dot{\beta}$
${T_{AB}}^c$	0	0	0	$2i\sigma^c_{\alpha\dot{\beta}}$
${T_{AB}}^\gamma$	${E_a}^\mu {E_b}^\nu {T_{\mu\nu}}^\gamma$	$\frac{i}{8}M\varepsilon^{\gamma\alpha}\sigma_{b\,\alpha\dot{\alpha}}$	0	0
${T_{AB}}^{\dot{\gamma}}$	$-{E_a}^\mu {E_b}^\nu {\bar{T}_{\mu\nu}}{}^{\dot{\gamma}}$	$i\,(\delta^{\dot{\gamma}}_{\dot{\alpha}}B_b + B^c{\bar{\sigma}_{cb}}{}^{\dot{\gamma}}{}_{\dot{\alpha}})$	0	0
${F_{AB}}^i$	${E_a}^\mu {E_b}^\nu {F_{\mu\nu}}^i$	$i\lambda^{i\,\alpha}\sigma_{b\,\alpha\dot{\alpha}}$	0	0
${F_{AB}}^{cd}$	${E_a}^\mu {E_b}^\nu {F_{\mu\nu}}^{cd}$	$iT^{cd\alpha}\sigma_{b\,\alpha\dot{\alpha}} - 2i\sigma^{[c}_{\alpha\dot{\alpha}}{T^{d]}}_b{}^\alpha$	$-M\bar{\sigma}^{cd}{}_{\dot{\alpha}\dot{\beta}}$	$2i\varepsilon^{abcd}\sigma_{a\alpha\dot{\beta}}B_b$

Table B.1: Torsions and curvatures in old minimal supergravity

The explicit realization of $\mathcal{D}_\alpha$ and $\bar{\mathcal{D}}_{\dot{\alpha}}$ on the tensor fields (2.14) can be obtained à la [47] from an analysis ("solution") of the Bianchi identities

$$0 = \sum_{ABC} (\mathcal{D}_A {T_{BC}}^D + {T_{AB}}^E {T_{EC}}^D + {F_{AB}}^I {g_{IC}}^D), \quad \text{(B.6)}$$

$$0 = \sum_{ABC} (\mathcal{D}_A {F_{BC}}^I + {T_{AB}}^D {F_{DC}}^I) \quad \text{(B.7)}$$

where

$$\sum_{ABC} X_{ABC} = (-)^{\varepsilon_A\varepsilon_C} X_{ABC} + (-)^{\varepsilon_B\varepsilon_A} X_{BCA} + (-)^{\varepsilon_C\varepsilon_B} X_{CAB}\ .$$

Using the notation of (3.1) and (3.2) one finds in particular

$$\mathcal{D}_\alpha M = \tfrac{16}{3}\,(S_\alpha - i\lambda^{(r)}_\alpha), \quad \text{(B.8)}$$

$$\mathcal{D}_\alpha \bar{M} = 0, \quad \text{(B.9)}$$

$$\mathcal{D}_\alpha B_{\beta\dot{\beta}} = \tfrac{1}{3}\varepsilon_{\beta\alpha}(\bar{S}_{\dot{\beta}} + 4i\bar{\lambda}^{(r)}_{\dot{\beta}}) - \bar{U}_{\alpha\beta\dot{\beta}}, \quad \text{(B.10)}$$

$$\mathcal{D}_\alpha \lambda^i_\beta = i\varepsilon_{\alpha\beta}D^i + {G_{\alpha\beta}}^i, \quad \text{(B.11)}$$

$$\mathcal{D}_\alpha \bar{\lambda}^i_{\dot{\alpha}} = 0, \quad \text{(B.12)}$$

$$\mathcal{D}_\alpha D^i = \mathcal{D}_{\alpha\dot{\alpha}}\bar{\lambda}^{i\dot{\alpha}} + \tfrac{3}{2}iB_{\alpha\dot{\alpha}}\bar{\lambda}^{i\dot{\alpha}}\ . \quad \text{(B.13)}$$

The $\mathcal{D}_\alpha$-transformations of ${T_{ab}}^\beta$, ${T_{ab}}^{\dot{\beta}}$ and ${F_{ab}}^I$ are easily obtained from the identities (B.6) and (B.7) with indices $(ABC) \equiv (\alpha ab)$ and are therefore not spelled out here (the linearized version of these transformations is given in appendix C).

The realization of the algebra (2.13) in new minimal supergravity can be obtained from the above formulae using the identifications

$$M \equiv 0, \quad B^a \equiv \tfrac{1}{6}\varepsilon^{abcd}H_{bcd}, \quad D^{(r)} \equiv -\tfrac{1}{4}\,(\mathcal{R} + H_{abc}H^{abc}), \quad \lambda^{(r)}_\alpha \equiv -iS_\alpha \quad \text{(B.14)}$$

with $H_{\mu\nu\rho}$ as in (2.7). H_{abc} satisfies

$$\varepsilon^{abcd}\mathcal{D}_a H_{bcd} = 0. \quad \text{(B.15)}$$

This can be verified using (B.1) and

$$\mathcal{D}_\alpha \tilde{H}_{\beta\dot{\beta}} = \varepsilon_{\alpha\beta}\bar{S}_{\dot{\beta}} - \bar{U}_{\alpha\beta\dot{\beta}}\ , \quad \tilde{H}^a = \tfrac{1}{6}\varepsilon^{abcd}H_{bcd} \quad \text{(B.16)}$$

which is consistent with (B.10) and (B.14).

C QDS-structure of minimal supergravity

C.1 Definition of QDS-structure

It will now be shown, both for old and for new minimal supergravity, that the off-shell and the on-shell representations of the linearized supersymmetry algebra (4.43) on tensor fields have 'QDS-stucture' in the terminology of [28]. Let me first repeat the definition of this structure. It refers to the representation of the subalgebra

$$\{D_\alpha, D_\beta\} = 0 \tag{C.1}$$

of (4.43) on the independent tensor fields in the theory. To analyze it, we use the same notation $\mathcal{T}_n^m$ as in section 3 for a Lorentz-irreducible multiplet of tensor fields whose components carry n undotted and m dotted spinor indices and are completely symmetric in them respectively. We now define operations D_+ and D_-

$$D_+\mathcal{T}_n^m \equiv \{D_{(\alpha_0}\mathcal{T}_{\alpha_1\cdots\alpha_n)}^{\dot\alpha_1\cdots\dot\alpha_m}\}, \quad D_-\mathcal{T}_n^m \equiv \{nD^{\alpha_n}\mathcal{T}_{\alpha_1\cdots\alpha_n}^{\dot\alpha_1\cdots\dot\alpha_m}\}. \tag{C.2}$$

The possible indecomposable representations of (C.1) ("D_α-multiplets") have been determined in [28]: there are singlets $(S) \equiv S_n^m$, 'quartet-representations' $(Q) \equiv \{Q^{(0)}{}_n^m, Q^{(-)}{}_{n-1}^m, Q^{(+)}{}_{n+1}^m, Q^{(+-)}{}_n^m\}$ which degenerate to triplets in the case $n = 0$, and 'zig-zag representations' $(Z) \equiv \{Z^{(0)}{}_n^m, \ldots, Z^{(k)}{}_{n+k}^m\}$ with an arbitrary number k of components. It should be remarked that the linearized supersymmetry algebra (4.43) itself does not rule out any of these representations. The representation of (C.1) is said to have QDS-structure if it decomposes completely into singlets S_0^m (i.e. singlets without undotted spinor index), (Q)-multiplets (of any kind) and very special (Z)-multiplets, called (D)-multiplets, of the form $\{D^{(0)}{}_n^m, D^{(+)}{}_{n+1}^m\}$. The properties of these multiplets are summarized in table C.1. I note that one has $Q^{(+-)} = \frac{1}{2}\, D^2\, Q^{(0)}$.

Multiplet	$\mathcal{T}$	$D_-\mathcal{T}$	$D_+\mathcal{T}$
(Q)	$Q^{(0)}{}_n^m$	$Q^{(-)}{}_{n-1}^m$	$Q^{(+)}{}_{n+1}^m$
	$Q^{(-)}{}_{n-1}^m$	0	$-n\, Q^{(+-)}{}_n^m$
	$Q^{(+)}{}_{n+1}^m$	$(n+2)\, Q^{(+-)}{}_n^m$	0
	$Q^{(+-)}{}_n^m$	0	0
(D)	$D^{(0)}{}_n^m$	0	$D^{(+)}{}_{n+1}^m$
	$D^{(+)}{}_{n+1}^m$	0	0
(S)	S_0^m	0	0

Table C.1: D_α-multiplets in QDS-theories

Remark: Note that the D_α-multiplets defined above are actually multiplets of multiplets because their components are complete Lorentz (resp. $SL(2,C)$) multiplets.

C.2 Off-shell D_α-multiplets

For old minimal supergravity, the D_α-transformations of all those tensor fields which do not carry super-covariant derivatives are listed in table C.2. The D_α-transformations of their super-covariant derivatives follow then from the algebra (4.43), using the linearized Bianchi identities (3.8)–(3.12). The D_α-transformations in new minimal supergravity are obtained from table C.2 using the identifications (B.14) and the additional Bianchi identity (B.15).

$\mathcal{T}$	$D_-\mathcal{T}$	$D_+\mathcal{T}$	$\mathcal{T}$	$D_-\mathcal{T}$	$D_+\mathcal{T}$
B	$-\frac{2}{3}(\bar{S}+4i\bar{\lambda}^{(r)})$	$-\bar{U}$			
M	0	$\frac{16}{3}(S-i\lambda^{(r)})$	$\bar{M}$	0	0
S	$\frac{1}{2}(\mathcal{R}+3iD_-^-B)$	$iG^{(r)}$	$\bar{S}$	0	$-\frac{3}{8}iD_+^+\bar{M}$
W	$6iD_+^-B-4iG^{(r)}$	$-\frac{1}{2}X$	$\bar{W}$	0	0
U	$2i(D_-^+B+\bar{G}^{(r)})$	$iD_+^+B-\frac{1}{2}Y$	$\bar{U}$	$\frac{3}{4}iD_+^+\bar{M}$	0
$\mathcal{R}$	0	$4iD_+^-\bar{S}$			
Y	$10iD_-^+\bar{U}+6iD_+^+\bar{S}$	$-2iD_+^+\bar{U}$			
X	$-20iD_+^-\bar{U}$	0	$\bar{X}$	0	$-8iD_+^+\bar{W}$
λ^i	$2iD^i$	G^i	$\bar{\lambda}^i$	0	0
G^i	$-6iD_+^-\bar{\lambda}^i$	0	$\bar{G}^i$	0	$2iD_+^+\bar{\lambda}^i$
D^i	0	$D_+^-\bar{\lambda}^i$			

Table C.2: Off-shell D_α-transformations in old minimal supergravity

It is now straightforward (though somewhat tedious) to verify the QDS-structure of old and new minimal supergravity. Indeed one finds that all the tensor fields are either D_α-singlets S_0^m or group into (D)- and (Q)-multiplets. More precisely, the singlets are exhausted by

$$(S):\quad \bar{M},\,\bar{W},\,\bar{\lambda}^i$$

and the (D)-multiplets are, using the notation $\{D^{(0)},\,D^{(+)}\}$,

$$(D):\quad \{(D_+^+)^q\,\bar{X},\,-8i(D_+^+)^{q+1}\bar{W}\},\ \{(D_+^+)^q\,\bar{G}^i,\,2i(D_+^+)^{q+1}\bar{\lambda}^i\}\quad (q=0,1,\ldots).$$

All other multiplets are (Q)-multiplets. Here I only list their lowest components $Q^{(0)}$ (the full multiplets are spelled out in [27]):

$$\begin{array}{llll}
(D_+^+)^qB, & \Box^p(D_+^+)^qD_-^-B, & \Box^p(D_+^+)^qM, & (D_+^+)^qW,\\
\Box^p(D_+^+)^qU, & \Box^p(D_+^+)^qD_-^+U, & \Box^p(D_+^+)^qD_+^-U, & \Box^p(D_+^+)^q(D_+^-)^2U,\\
\Box^p(D_+^+)^q\lambda^i, & \Box^p(D_+^+)^qD_-^+\lambda^i. & &
\end{array}$$

The higher components of the (Q)-multiplets are in general (multiplets of) linear combinations of the tensor fields $\mathcal{T}^r$. These linear combinations are linearly independent and form together with the singlets and the components of the (D)-multiplets

a basis for the tensor fields in the sense of section 3, equivalent to $\{\mathcal{T}^r\}$. This was shown explicitly in [27] and implies the QDS-structure of the off-shell representation of (C.1) in old and new minimal supergravity.

C.3 On-shell D_α-multiplets

Using table C.2 and the linearized equations of motion it is easy to verify that the D_α-transformations of the tensor fields (6.2) reduce on-shell to those given in table C.3.

$\hat{\mathcal{T}}$	$D_-\hat{\mathcal{T}}$	$D_+\hat{\mathcal{T}}$	$\hat{\mathcal{T}}$	$D_-\hat{\mathcal{T}}$	$D_+\hat{\mathcal{T}}$
W_q	0	$-\frac{1}{2}X_q$	$\bar{W}_q$	0	0
$\bar{X}_q$	0	$-8i\bar{W}_{q+1}$	X_q	0	0
λ^i_q	0	G^i_q	$\bar{\lambda}^i_q$	0	0
$\bar{G}^i_q$	0	$2i\bar{\lambda}^i_{q+1}$	G^i_q	0	0

Table C.3: On-shell D_α-transformations

The QDS-structure of the on-shell representation of (C.1) is evident from table C.3: the on-shell D_α-multiplets are singlets without undotted spinor index and (D)-multiplets (there are no (Q)-multiplets on-shell in this case!):

$$\begin{array}{ll} (S): & \bar{W}_0,\quad \bar{\lambda}^i_0 \\ (D): & \{W_q, -\tfrac{1}{2}X_q\},\ \{\bar{X}_q, -8i\bar{W}_{q+1}\},\ \{\lambda^i_q, G^i_q\},\ \{\bar{G}^i_q, 2i\bar{\lambda}^i_{q+1}\}\quad (q=0,1,\ldots) \end{array}$$

D Super-covariant Poincaré lemma

In this appendix it is proved that any $\tilde{s}$-exact and $\mathcal{G}$-invariant local total form $f(\tilde{\xi},\mathcal{T})$ is of the form $\tilde{s}h(\tilde{\xi},\mathcal{T})$ except for the abelian total curvature forms $\mathcal{F}^{i_a}$, the total curvature form $\mathcal{H}$ corresponding to $t_{\mu\nu}$ and the "total super-Chern 4-forms",

$$\begin{aligned} & f(\tilde{\xi},\mathcal{T}) = \tilde{s}\beta, \quad \delta_I f(\tilde{\xi},\mathcal{T}) = 0 \\ \Rightarrow\quad & f(\tilde{\xi},\mathcal{T}) = \tilde{s}h(\tilde{\xi},\mathcal{T}) + a_{i_a}\mathcal{F}^{i_a} + c\,\mathcal{H} + d_{IJ}\mathcal{F}^I\mathcal{F}^J \end{aligned} \tag{D.1}$$

where the a_{i_a} and c are arbitrary constants and the d_{IJ} are constant $\mathcal{G}$-invariant symmetric tensors. Furthermore, no nonvanishing linear combination of the $\mathcal{F}^{i_a}$, $\mathcal{H}$ and $d_{IJ}\mathcal{F}^I\mathcal{F}^J$ is $\tilde{s}$-exact in the space of local total forms $f(\tilde{\xi},\mathcal{T})$,

$$a_{i_a}\mathcal{F}^{i_a} + c\,\mathcal{H} + d_{IJ}\mathcal{F}^I\mathcal{F}^J = \tilde{s}h(\tilde{\xi},\mathcal{T}) \quad\Leftrightarrow\quad a_{i_a} = c = d_{IJ} = 0. \tag{D.2}$$

To prove (D.1) I first note that thanks to (4.12) we can assume without loss generality that β does not involve the $\mathcal{U}$'s and $\mathcal{V}$'s. Furthermore we can of course assume that f has a definite total degree G and thus consider

$$f(\tilde{\xi},\mathcal{T}) = \tilde{s}\beta(\mathcal{W}), \quad \delta_I f = 0, \quad \mathrm{totdeg}(f) = G. \tag{D.3}$$

For $G > 4$, the assertion follows immediately from (4.28). Indeed, (D.3) implies $\tilde{s}f = 0$ and (4.28) therefore ensures $f = \tilde{s}h(\tilde{\xi}, \mathcal{T})$ for some local h in the cases $G > 4$. Note that we can use (4.28) in the cases $G > 4$ because these results are derived independently of the above assertions, in contrast to the results for $G < 4$.

We are therefore left with the cases $G \leq 4$. Since β has total degree $(G-1)$, it vanishes for $G = 0$ (as it does not involve antifields and thus cannot have negative ghost number) and is necessarily of the form $h(\mathcal{T})$ in the case $G = 1$. We conclude

$$G = 0: \qquad f = 0, \tag{D.4}$$

$$G = 1: \qquad f(\tilde{\xi}, \mathcal{T}) = \tilde{s}h(\mathcal{T}) \tag{D.5}$$

which proves (D.1) for $G = 0, 1$.

The cases $G = 2, 3, 4$ are more involved. Using (4.33) they can be treated by adapting a method developed in appendix E of [26] to solve a similar problem in ordinary gravity. To that end we define δ_I on the $\tilde{C}^J$ and $\tilde{Q}$ according to

$$\delta_I \tilde{C}^J = -f_{IK}{}^J \tilde{C}^K, \quad \delta_I \tilde{Q} = 0 \tag{D.6}$$

i.e. $\tilde{C}^J$ transforms under δ_I according to the adjoint representation of $\mathcal{G}$, whereas $\tilde{Q}$ is $\mathcal{G}$-invariant. It is now easy to check that on local total forms $\alpha(\mathcal{W})$ one has

$$\delta_I = \{\tilde{s}, \partial_I\} \quad \Rightarrow \quad [\delta_I, \tilde{s}] = 0 \tag{D.7}$$

where ∂_I is the derivative w.r.t. $\tilde{C}^I$,

$$\partial_I = \frac{\partial}{\partial \tilde{C}^I} \ . \tag{D.8}$$

(D.7) implies that without loss of generality we can assume

$$\delta_I \beta = 0 \tag{D.9}$$

because $\delta_I f = 0$ implies $\tilde{s}(\delta_I \beta) = 0$, i.e. any $\mathcal{G}$-*noninvariant* contribution to β would have to be $\tilde{s}$-invariant and would thus not contribute to f in (D.3). Applying now ∂_I to (D.3) we get, thanks to (D.7),

$$\tilde{s}(\partial_I \beta) = 0 \tag{D.10}$$

since f does not depend on the $\tilde{C}^I$. Hence $\partial_I \beta$ is $\tilde{s}$-closed, has total degree $(G-2)$ and is thus $\tilde{s}$-exact for $G = 3, 4$ and constant for $G = 2$ by (4.33),

$$G = 2: \qquad \partial_I \beta = a_I = constant, \tag{D.11}$$

$$G = 3: \qquad \partial_I \beta = \tilde{s}h_I(\mathcal{T}), \tag{D.12}$$

$$G = 4: \qquad \partial_I \beta = \tilde{s}\beta_I(\mathcal{W}) \ , \tag{D.13}$$

where in (D.12) we used that h_I has vanishing total degree and thus depends only on the $\mathcal{T}$ (we also used (4.12) again).

Let us first consider the case $G=2$. Since in this case β has total degree 1, (D.11) implies evidently $\beta = h(\tilde{\xi},\mathcal{T}) + a_I\tilde{C}^I$. (D.9) then requires $a_I = 0$ unless I refers to an abelian element of $\mathcal{G}$. We conclude

$$\begin{aligned} G=2: \qquad & \beta = h(\tilde{\xi},\mathcal{T}) + a_{i_a}\tilde{C}^{i_a} \\ \Rightarrow \quad & f = \tilde{s}h(\tilde{\xi},\mathcal{T}) + a_{i_a}\mathcal{F}^{i_a} \ . \end{aligned} \tag{D.14}$$

Next we turn to the case $G=3$. In (D.12) we can assume with no loss of generality that the h_I transform under $\mathcal{G}$ according to the co-adjoint representation because this holds for $\partial_I\beta$ too (as a consequence of (D.9), due to $[\delta_I,\partial_J] = f_{IJ}{}^K\partial_K$) and because $\tilde{s}$ leaves the representation invariant due to (D.7). Applying ∂_J to (D.12) we thus conclude, using (D.7) again,

$$G=3: \quad \partial_J\partial_I\beta = f_{JI}{}^K h_K \ . \tag{D.15}$$

In the case $G=3$, β has total degree 2 and is thus at most quadratic in the $\tilde{C}^I$ and linear in $\tilde{Q}$. Hence, it is of the form

$$G=3: \quad \beta = \hat{h} + \tilde{Q}\hat{g} + \tilde{C}^I\hat{h}_I + \tfrac{1}{2}\tilde{C}^I\tilde{C}^J\hat{h}_{JI} \tag{D.16}$$

where $\hat{g}$ and the $\hat{h}$'s depend only on the $\tilde{\xi}$'s and $\mathcal{T}$'s and can be assumed to transform under $\mathcal{G}$ according to their indices due to (D.9). It is now easy to verify that (D.12) and (D.15) imply

$$\begin{aligned} G=3: \qquad & \hat{h}_{JI} = f_{JI}{}^K h_K, \quad \hat{h}_I = \tilde{s}_{susy}h_I \\ \Rightarrow \quad & \beta = \hat{h} + \tilde{Q}\hat{g} - \tilde{s}(\tilde{C}^I h_I) + \mathcal{F}^I h_I \\ \Rightarrow \quad & f = \tilde{s}(\hat{h} + \mathcal{F}^I h_I) + \mathcal{H}\hat{g} + \tilde{Q}(\tilde{s}\hat{g}). \end{aligned} \tag{D.17}$$

Now, since f does not depend on $\tilde{Q}$, differentiation of (D.17) w.r.t. $\tilde{Q}$ yields $\tilde{s}\hat{g} = 0$ which implies $\hat{g} = c = \mathit{constant}$ by (4.33) as $\hat{g}$ has vanishing total degree. This yields

$$G=3: \quad f = \tilde{s}h(\tilde{\xi},\mathcal{T}) + c\,\mathcal{H}, \tag{D.18}$$

with $h = \hat{h} + \mathcal{F}^I h_I$, and proves (D.1) in the case $G=3$.

The case $G=4$ can be treated similarly. Applying ∂_J to (D.13) yields

$$G=4: \qquad \partial_J\partial_I\beta = f_{JI}{}^K\beta_K - \tilde{s}\,\partial_J\beta_I \tag{D.19}$$

$$\Rightarrow \quad \tilde{s}\,\partial_{(J}\beta_{I)} = 0 \quad \Rightarrow \quad \partial_{(J}\beta_{I)} = d_{IJ} = \mathit{constant} \tag{D.20}$$

where we used that the ∂_I anticommute and that $\partial_{(J}\beta_{I)}$ has vanishing total degree. The d_{IJ} are thus symmetric $\mathcal{G}$-invariant constant tensors. As β_I has total degree 1, we conclude from (D.20):

$$\beta_I = h_I(\tilde{\xi},\mathcal{T}) + \tilde{C}^J(d_{JI} + h_{JI}(\mathcal{T})), \quad h_{JI} = -h_{IJ} \ . \tag{D.21}$$

Applying ∂_K to (D.19) yields then

$$\partial_K\partial_J\partial_I\beta = f_{JI}{}^L(h_{KL} + d_{KL}) + f_{KJ}{}^L h_{IL} + f_{IK}{}^L h_{JL} \ . \tag{D.22}$$

Using (D.13), (D.19), (D.21) and (D.22) it is now straightforward to determine first β and then f by a calculation similar to the one that led to (D.17). One finds

$$G=4:\quad f=\tilde{s}\hat{h}+\mathcal{H}\hat{g}+\tilde{Q}(\tilde{s}\hat{g})+d_{JI}\mathcal{F}^I\mathcal{F}^J \tag{D.23}$$

where $\hat{g}$ and $\hat{h}$ depend only on the $\tilde{\xi}$ and $\mathcal{T}$. Differentiating (D.23) w.r.t. $\tilde{Q}$ yields $\tilde{s}\hat{g}=0$ which implies $\hat{g}=\tilde{s}k(\mathcal{T})$ by (4.33) and (4.12). Using $\tilde{s}\mathcal{H}=0$ and defining $h=\hat{h}-\mathcal{H}k$, this finally results in

$$G=4:\quad f=\tilde{s}h(\tilde{\xi},\mathcal{T})+d_{JI}\mathcal{F}^I\mathcal{F}^J \tag{D.24}$$

and completes the proof of (D.1).

In the case $G=2$, (D.2) can be proved as follows. Assume

$$a_{i_a}\mathcal{F}^{i_a}=\tilde{s}h(\tilde{\xi},\mathcal{T})$$

holds for some h. Due to $\mathcal{F}^{i_a}=\tilde{s}\tilde{C}^{i_a}$ for abelian $\mathcal{F}$'s, this implies

$$\tilde{s}\left[a_{i_a}\tilde{C}^{i_a}-h(\tilde{\xi},\mathcal{T})\right]=0$$

and thus, by (4.33) and (4.12),

$$a_{i_a}\tilde{C}^{i_a}=h(\tilde{\xi},\mathcal{T})+\tilde{s}\,g(\mathcal{T}) \tag{D.25}$$

for some $g(\mathcal{T})$. However, as no $\tilde{s}\mathcal{T}$ contains a linear combination of the $\tilde{C}^{i_a}$ (with constant coefficients), the left and the right hand side of (D.25) must vanish separately which implies indeed $a_{i_a}=0$. Analogously one can treat the cases $G=3,4$ and complete the proof of (D.2).

E Linearized weak supersymmetry cohomology

In this appendix we will compute the weak cohomology of δ_{susy} in the space of $\mathcal{G}$-invariant local total forms $f(\tilde{\xi},\hat{\mathcal{T}})$ at total degrees ≥ 4 (cf. eq. (6.14)). It will be shown that this cohomology vanishes at all total degrees exceeding 4 and is at total degree 4 represented by $\mathcal{G}$-invariant local total forms

$$\hat{P}=\hat{D}_{\dot{\alpha}}\hat{D}^{\dot{\alpha}}\,\Xi\,\{\mathcal{A}(\bar{W},\bar{\lambda})+D^{\alpha}D_{\alpha}\mathcal{B}(\hat{\mathcal{T}})\}+c.c. \tag{E.1}$$

with Ξ as in (4.30) and $\hat{D}_{\dot{\alpha}}$ as in (4.46). Furthermore we will derive the results for total degree 3 presented in section 6.2.

Let us therefore consider

$$\delta_{susy}f(\tilde{\xi},\hat{\mathcal{T}})\sim 0,\quad \delta_I f=0,\quad \mathrm{totdeg}(f)=G\geq 3. \tag{E.2}$$

The problem will be analyzed along the lines of [28] to which I refer for details (see section 6 and appendix A of [28]). I note however that the case $G=3$ was not treated in [28] and deserves therefore some special attention.

We decompose both δ_{susy} and f according to the degree in the $\tilde{\xi}^a$. To this end we introduce the counting operator

$$\tilde{N} = \tilde{\xi}^a \frac{\partial}{\partial \tilde{\xi}^a} \ . \tag{E.3}$$

δ_{susy} decomposes into three pieces with $\tilde{N}$-degrees $1, 0, -1$ respectively,

$$\delta_{susy} = \delta_- + \delta_0 + \delta_+ \ , \quad [\tilde{N}, \delta_\pm] = \pm\delta_\pm \ , \quad [\tilde{N}, \delta_0] = 0. \tag{E.4}$$

These pieces are spelled out in table E.1 where b and $\bar{b}$ are the operators

$$b\,\hat{\mathcal{T}} = \tilde{\xi}^\alpha D_\alpha \hat{\mathcal{T}} \ , \quad \bar{b}\,\hat{\mathcal{T}} = \tilde{\xi}^{\dot\alpha} \bar{D}_{\dot\alpha} \hat{\mathcal{T}} \ , \quad b\,\tilde{\xi}^A = \bar{b}\,\tilde{\xi}^A = 0. \tag{E.5}$$

Here the linearized (weak) D_α-transformations given in table C.3 are to be used.

$\mathcal{W}$	$\delta_-\mathcal{W}$	$\delta_0\mathcal{W}$	$\delta_+\mathcal{W}$
$\tilde{\xi}^{\dot\alpha\alpha}$	$4i\tilde{\xi}^\alpha\tilde{\xi}^{\dot\alpha}$	0	0
$\tilde{\xi}^\alpha$	0	0	0
$\tilde{\xi}^{\dot\alpha}$	0	0	0
$\hat{\mathcal{T}}$	0	$(b+\bar{b})\,\hat{\mathcal{T}}$	$\tilde{\xi}^a D_a \hat{\mathcal{T}}$

Table E.1: Decomposition of δ_{susy}

$\delta_{susy} f \sim 0$ decomposes into

$$\begin{aligned}
0 &= \delta_- X_{\underline{p}}, && \text{(E.6)}\\
0 &\sim \delta_- X_{\underline{p}+1} + \delta_0 X_{\underline{p}}, && \text{(E.7)}\\
0 &\sim \delta_- X_{p+1} + \delta_0 X_p + \delta_+ X_{p-1} \quad \text{for } \underline{p} < p < \overline{p}, && \text{(E.8)}\\
0 &\sim \delta_0 X_{\overline{p}} + \delta_+ X_{\overline{p}-1}, && \text{(E.9)}\\
0 &\sim \delta_+ X_{\overline{p}} && \text{(E.10)}
\end{aligned}$$

where X_p is the part of $f(\tilde{\xi}, \mathcal{T})$ with degree p in the $\tilde{\xi}^a$,

$$f(\tilde{\xi}, \mathcal{T}) = \sum_{p=\underline{p}}^{\overline{p}} X_p \ , \quad \tilde{N} X_p = p X_p \ . \tag{E.11}$$

Note that in (E.6) we have used $=$ rather than $\sim$ as δ_- does not see the tensor fields at all. Since $f(\tilde{\xi}, \hat{\mathcal{T}})$ is defined only modulo weakly δ_{susy}-exact local total forms $\delta_{susy} h(\tilde{\xi}, \hat{\mathcal{T}})$, $X_{\underline{p}}$ can be assumed to represent a nontrivial cohomology class of the cohomology of δ_-. That cohomology has been determined in [27, 48] (see also [28, 34]). The result is that a δ_--closed function depends, up to δ_--exact pieces, on the $\tilde{\xi}^a$ only via the quantities

$$\vartheta^\alpha = \tilde{\xi}_{\dot\alpha} \tilde{\xi}^{\dot\alpha\alpha} \ , \quad \bar{\vartheta}^{\dot\alpha} = \tilde{\xi}^{\dot\alpha\alpha} \xi_\alpha \ , \quad \Theta = \tilde{\xi}_{\dot\alpha} \tilde{\xi}^{\dot\alpha\alpha} \tilde{\xi}_\alpha \ . \tag{E.12}$$

Moreover the dependence on these quantities is very restricted as one has

$$\delta_- f(\tilde{\xi}^A) = 0 \ \Leftrightarrow \ f = P(\bar{\vartheta}^{\dot{\alpha}}, \tilde{\xi}^{\alpha}) + P'(\vartheta^{\alpha}, \tilde{\xi}^{\dot{\alpha}}) + a\,\Theta + \delta_- h(\tilde{\xi}^A) \tag{E.13}$$

where a is constant. Furthermore no non-vanishing function $P(\bar{\vartheta}^{\dot{\alpha}}, \tilde{\xi}^{\alpha}) + P'(\vartheta^{\alpha}, \tilde{\xi}^{\dot{\alpha}}) + a\,\Theta$ is δ_--exact,

$$P(\tilde{\vartheta}^{\dot{\alpha}}, \tilde{\xi}^{\alpha}) + P'(\tilde{\vartheta}^{\alpha}, \tilde{\xi}^{\dot{\alpha}}) + a\,\Theta = \delta_- h(\tilde{\xi}^A) \ \Leftrightarrow \ P = -P' = \mathit{constant}, \ a = 0. \tag{E.14}$$

These results are very useful when analyzing (E.6)–(E.10). First we use (E.13) to conclude that $X_{\underline{p}}$ can be assumed to be of the form

$$G \geq 4: \qquad X_{\underline{p}} = P(\bar{\vartheta}^{\dot{\alpha}}, \tilde{\xi}^{\alpha}, \hat{\mathcal{T}}) + \bar{P}(\vartheta^{\alpha}, \tilde{\xi}^{\dot{\alpha}}, \hat{\mathcal{T}}), \tag{E.15}$$

$$G = 3: \qquad X_{\underline{p}} = P(\bar{\vartheta}^{\dot{\alpha}}, \tilde{\xi}^{\alpha}, \hat{\mathcal{T}}) + \bar{P}(\vartheta^{\alpha}, \tilde{\xi}^{\dot{\alpha}}, \hat{\mathcal{T}}) + 4\,\delta^1_{\underline{p}}\,\Theta\, R(\hat{\mathcal{T}}) \tag{E.16}$$

where the Kronecker symbol $\delta^1_{\underline{p}}$ occurs as Θ is linear in the $\tilde{\xi}^a$ and a factor 4 has been introduced for later convenience. Furthermore we can assume without loss of generality that P and $\bar{P}$ are related by complex conjugation and that R is real (cf. second remark at the end of section 4.1). Note that Θ has total degree 3 and can therefore occur only for $G = 3$ which complicates the analysis of this case as compared to the other ones. Note also that in fact we have $\underline{p} \in \{0, 1, 2\}$ as the ϑ's anticommute.

By inserting now (E.15) resp. (E.16) in (E.7) we obtain, using (E.14)

$$b\,P \sim 0 \tag{E.17}$$

with b as in (E.5). Furthermore we can assume without loss of generality

$$P \not\sim b\,Q \tag{E.18}$$

because otherwise P can be removed from f by subtracting a suitable δ_{susy}-exact piece from f without changing the form of $X_{\underline{p}}$, i.e. without reintroducing a δ_--exact piece in it (of course P, $\bar{P}$ or R may get redefined). Moreover P is required to be $\mathcal{G}$-invariant. Using $P = \bar{\vartheta}^{\dot{\alpha}_1} \cdots \bar{\vartheta}^{\dot{\alpha}_{\underline{p}}} \omega_{\dot{\alpha}_1 \cdots \dot{\alpha}_{\underline{p}}}(\tilde{\xi}^{\alpha}, \hat{\mathcal{T}})$ one concludes that $\omega_{\dot{\alpha}_1 \cdots \dot{\alpha}_{\underline{p}}}$ is determined by the weak cohomology of b on $l_{\alpha\beta}$-invariant functions $f(\tilde{\xi}^{\alpha}, \hat{\mathcal{T}})$ where $l_{\alpha\beta}$ generates Lorentz (resp. $SL(2, C)$) transformations of undotted spinor indices according to (A.1).

In order to compute the latter cohomology, we need the QDS-structure of the on-shell D_α-representation proved in appendix C.3. Thanks to this structure we can directly adopt the analysis and results of appendix A of [28] to conclude

$$bf(\tilde{\xi}^{\alpha}, \hat{\mathcal{T}}) \sim 0, \ l_{\alpha\beta} f = 0 \ \Leftrightarrow \ f \sim \mathcal{A}(\bar{W}, \bar{\lambda}) + D^2 \mathcal{B}(\hat{\mathcal{T}}) + bh(\tilde{\xi}^{\alpha}, \hat{\mathcal{T}}) \tag{E.19}$$

where $\mathcal{B}$ and h are $l_{\alpha\beta}$-invariant and we used $D^2 = D^\alpha D_\alpha$. This result is the key to the solution of (E.2) in the cases $G \geq 4$. Indeed, it implies in particular that $\omega_{\dot{\alpha}_1 \cdots \dot{\alpha}_{\underline{p}}}$ can be chosen so as not to depend on $\tilde{\xi}^{\alpha}$ at all, and thus that the total degree of P

equals two times its degree in the $\bar{\vartheta}$'s. As the latter are Grassmann odd and linear in the $\tilde{\xi}^a$, we conclude immediately that P can be assumed to vanish in all cases $G \geq 3$ except for the case $G = 4$, $\underline{p} = 2$ where (E.19) yields

$$G = 4,\ \underline{p} = 2: \quad P = -4i\, \bar{\vartheta}_{\dot{\alpha}} \bar{\vartheta}^{\dot{\alpha}} [\mathcal{A}(\bar{W}, \bar{\lambda}) + D^2 \mathcal{B}(\hat{\mathcal{T}})]. \tag{E.20}$$

In particular this implies that each solution to (E.2) with $G > 4$ is indeed trivial. Furthermore, using (E.13) again, it is easy to verify that the equations (E.8)–(E.10) do not impose further obstructions in the case $G = 4$ and lead to the solutions (E.1). This completes the investigation of (E.2) for $G \geq 4$.

We are thus left with the case $G = 3$, $\underline{p} = 1$ for which (E.16) reduces to

$$G = 3,\ \underline{p} = 1: \quad X_1 = 4\Theta\, R(\hat{\mathcal{T}}). \tag{E.21}$$

Here we used already that P can be assumed to vanish in this case by subtracting a trivial piece from f and redefining R suitably, if necessary. Recall that so far we have only used equations (E.6) and (E.7). We now have to analyze the restrictions on the function $R(\hat{\mathcal{T}})$ imposed by the remaining equations (E.8)–(E.10). To that end we need the explicit form of X_2 corresponding to (E.21). It is obtained from (E.7) and reads

$$G = 3,\ \underline{p} = 1: \quad X_2 = -i(\bar{\vartheta}_{\dot{\alpha}} \tilde{\xi}^{\dot{\alpha}\alpha} D_\alpha + \vartheta^\alpha \tilde{\xi}_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}})\, R(\hat{\mathcal{T}}) \tag{E.22}$$

up to a δ_--exact piece which can be neglected with no loss of generality. (E.8) now requires

$$\delta_+ X_1 + \delta_0 X_2 + \delta_- X_3 \sim 0 \tag{E.23}$$

for some X_3. Elementary algebra with spinor indices yields straightforwardly

$$\delta_0 X_2 = i(-\vartheta^\alpha \bar{\vartheta}^{\dot{\alpha}} [D_\alpha, \bar{D}_{\dot{\alpha}}] + \Theta \tilde{\xi}^{\dot{\alpha}\alpha} \{D_\alpha, \bar{D}_{\dot{\alpha}}\} + \tfrac{1}{2} \bar{\vartheta}\bar{\vartheta}\, D^2 + \tfrac{1}{2} \vartheta\vartheta\, \bar{D}^2)\, R(\hat{\mathcal{T}}). \tag{E.24}$$

Now, the first term in (E.24), involving the commutator $[D_\alpha, \bar{D}_{\dot{\alpha}}]$, is δ_--exact by (E.13) as it is δ_--closed and involves both ϑ and $\bar{\vartheta}$. The second term, involving the anticommutator $\{D_\alpha, \bar{D}_{\dot{\alpha}}\}$, cancels exactly with $\delta_+ X_1$ in (E.23) due to $\{D_\alpha, \bar{D}_{\dot{\alpha}}\} R \sim -2i D_{\alpha\dot{\alpha}} R$. The remaining two terms in (E.24), involving $D^2 R$ and $\bar{D}^2 R$ respectively, are δ_--closed but not δ_--exact, see (E.13) and (E.14). Hence, (E.23) requires

$$D^2 R(\hat{\mathcal{T}}) \sim 0. \tag{E.25}$$

Moreover, (E.23) now determines X_3 unambigously as X_4 vanishes in the case $G = 3$. One finds

$$X_3 = \tfrac{1}{12}\, \tilde{\xi}^{\dot{\alpha}\beta} \tilde{\xi}_{\beta\dot{\beta}} \tilde{\xi}^{\dot{\beta}\alpha} [D_\alpha, \bar{D}_{\dot{\alpha}}]\, R(\hat{\mathcal{T}}). \tag{E.26}$$

$X_1 + X_2 + X_3$ yields now indeed (6.15).

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