Neutrino Oscillations from Dirac and Majorana Masses

D. Ring*

Center for Theoretical Physics, Department of Physics, Texas A&M University, College Station, TX 77843-4242

Abstract

We present a scenario of neutrino masses and mixing angles. Each generation includes a sterile right handed neutrino in addition to the usual left handed one. We assume a hierarchy in their Dirac masses similar to, but much larger than the hierarchies in the quarks and charged leptons. In addition, we include a Majorana mass term for the sterile neutrinos only. These assumptions prove sufficient to accommodate scales of mass differences and mixing angles consistent with all existing neutrino oscillation data.

*Email address: Cduke@phys.tamu.edu
I. INTRODUCTION

While there is currently no direct experimental evidence for neutrino masses, there is growing indirect evidence in the form of neutrino oscillations, culminating in the recent observation of $\nu_\mu \rightarrow \nu_e$ at LSND [1]. The combined evidence suggests three independent mass splittings among the neutrinos participating in the oscillations. If each of these splittings is taken seriously, then a fourth neutrino is required to accommodate all the data [2]. Several such scenarios have been proposed [3]. In most cases small mass differences and small mixing angles are put in by hand, and it seems difficult to explain their origin without fine tuning. Furthermore, these scenarios treat generations on an unequal footing, mixing the extra sterile state with only the electron neutrino.

We examine here the viability of one sterile neutrino for each generation. Such models have not been considered previously due to the constraint $N_\nu \lesssim 4$ from big bang nucleosynthesis [4]. It is known, however, that this constraint can be avoided if the tau neutrinos have masses in the MeV range and decay rapidly into $\nu_e$ [5]. The three mass splittings then suggest a unique natural mass spectrum. When combined with various terrestrial experimental data, the solar neutrino deficit implies a neutrino almost degenerate with $\nu_e$, the atmospheric deficit implies a neutrino almost degenerate with $\nu_\mu$, and the LSND data implies the two pairs must be split by at least 0.1 eV. We therefore impose Dirac masses with a very large hierarchy and a CKM matrix analogous to the one in the quark sector. We then include a Majorana mass matrix on the right handed neutrinos only. The scale of the Majorana masses is $\mathcal{O}(10^{-2} \text{eV})$ and is appropriate for a seesaw mechanism [6] between the Grand Unification (GUT) scale and the electroweak scale. Neutrino mixings appropriate for the oscillation data will arise from the interplay between the Dirac and Majorana mass matrices.
II. FERMION MASS HIERARCHIES

Typically a model with a large mass hierarchy will have mass matrix elements whose scales obey

\[
    m_{ij} \lesssim \left( \begin{array}{ccc}
    \epsilon^4 & \epsilon^3 & \epsilon^2 \\
    \epsilon^3 & \epsilon^2 & \epsilon \\
    \epsilon^2 & \epsilon & 1
    \end{array} \right) \times m_0; \quad \epsilon \ll 1. \quad (1)
\]

Here \( m_0 \) is the scale of the largest mass eigenvalue. If the off diagonal elements are larger than these, the lighter masses will receive large seesaw contributions and the hierarchy will be destroyed. Note that these are only upper bounds on the scales of the matrix elements. Realistic models typically contain texture zeros \([7]\) or additional powers of \( \epsilon \) in their off diagonal elements. Such suppressions are in fact necessary to make contact with the Standard Model, as we will see below. In the Standard Model we have \( \epsilon_u \sim 1/14.3 \), \( \epsilon_d \sim 1/5.1 \), and \( \epsilon_e \sim 1/7.6 \) (see Figure 1). With \( m_{\nu} \) in the MeV range, the effects of the off diagonal elements of the neutrino Dirac mass matrix will be washed out by the charged lepton mixings.

The Dirac mass term in the neutrino Lagrangian is

\[
    \mathcal{L} = -\bar{\nu}_i m_{ij} \nu_j = -\bar{\nu}'_i m_{ij} \nu'_j + H.c. \quad (2)
\]

\( m_{ij} \) is diagonalized by

\[
    \bar{\nu}'_L = \bar{\nu}'_L U^{\nu'}_L \\
    \nu_R = U^{\nu'}_R \nu'_R \quad (3)
\]

and the charged leptons are diagonalized by

\[
    \bar{e}_L = \bar{e}'_L U^{e}_L \\
    e_R = U^{e'}_R e'_R, \quad (4)
\]

where we have suppressed generation indices. Thus for example, the neutrino mass term can be written
\[ \mathcal{L}_\nu = \mathcal{V}_L^\nu U_\nu^{\nu\dagger} L_i m_{ij} U_R^{\nu\dagger} \nu_R j + H.c. \]  

(5)

where \( U_\nu^{\nu\dagger} L_i m_{ij} U_R^{\nu\dagger} = \text{diag}(m_1, m_2, m_3) \).

If both the up-like and down-like members of an \( SU(2) \) multiplet have mass matrices like (1), then the down-like mass eigenstates will be rotated from the weak partners of the up-like mass states by the CKM matrix,

\[
V = U_\nu^{\nu\dagger} U_L^{\nu\dagger} \sim \begin{pmatrix}
1 & \epsilon & \epsilon^2 \\
\epsilon & 1 & \epsilon \\
\epsilon^2 & \epsilon & 1
\end{pmatrix},
\]

(6)

where \( \epsilon \) is the larger of \( \epsilon_u \) and \( \epsilon_d \). For example, the weak partner of the electron would be

\[ \nu_L = U_L^{\nu\dagger} \nu_L = V L_i \nu_L \]

(7)

and the CKM hierarchy parameter is \( \epsilon_e \). We assume the hierarchies of the \( \epsilon \) and \( \nu \) states are aligned with respect to each other. It is possible to consider two hierarchies which are related by a generic unitary transformation, but it is hard to imagine a mechanism which would generate such hierarchies naturally. In a generic basis, the matrix elements would appear to be fine tuned to \( \mathcal{O}(\epsilon^4) \). The alignment of the quark hierarchies is evident in the smallness of the off diagonal elements of their CKM matrix. The Cabibbo angle in particular is very close to its expected magnitude. Note, however, that the other off diagonal elements of \( V^q \) are smaller by an additional factor of \( \epsilon_d \) than would be expected on the basis of the quark hierarchies alone. Thus \( V^q \) is well described by the Wolfenstein parametrization [8],

\[
V^q = \begin{pmatrix}
1 - \epsilon^2/2 & \epsilon & \epsilon^3 A z \\
-\epsilon & 1 - \epsilon^2/2 & \epsilon^2 A \\
\epsilon^3 A (1 - z) & -\epsilon^2 A & 1
\end{pmatrix},
\]

(8)

where \( A \sim \mathcal{O}(1) \) is real, \( \epsilon \sim \epsilon_d \) is real, and \( z \) is a complex number with magnitude \( \mathcal{O}(1) \). \( A \text{ priori} \) there is no way to know whether \( V_L \) will follow this pattern or the pattern of (6) or some other texture. For definiteness we first use pattern (8) with \( \epsilon \sim \epsilon_e \) as an example and then discuss other possibilities.
In order to achieve mass splittings and mixing angles appropriate for the Atmospheric and Solar oscillation data, we include Majorana masses of $\mathcal{O}(10^{-2}\text{eV})$ for the right handed neutrinos. This is about the right scale to be generated by a seesaw between the GUT scale and the electroweak scale, although it is not obvious how sterile particles relate to the electroweak scale. It is interesting to note that with this interpretation, the electron neutrino will be at the bottom of a two stage seesaw. We write the Majorana mass matrix as

$$m_{ij} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix},$$

where $a$ priori each $a_{ij}$ is $\mathcal{O}(10^{-2}\text{eV})$.

We are now in a position to write the full $6 \times 6$ mass matrix. We write the right handed neutrinos in terms of their charge conjugates, $\nu_R = s^c$. We use (9), (5), and (7) with the pattern (8) to get

$$\mathcal{L}_\nu = \frac{1}{2} NCM'N + \text{H.c.}$$

where $C$ is the charge conjugation matrix, $N = (\nu_e, s_e, \nu_\mu, s_\mu, \nu_\tau, s_\tau)$, and

$$M' = \begin{pmatrix} 0 & m_1(1 - \epsilon^2/2) & 0 & m_2\epsilon & 0 & m_3\epsilon^3 A z \\ m_1(1 - \epsilon^2/2) & a_{11} & -m_1\epsilon & a_{12} & m_1\epsilon^3 A (1 - \bar{z}) & a_{13} \\ 0 & -m_1\epsilon & 0 & m_2(1 - \epsilon^2/2) & 0 & m_3\epsilon^2 A \\ m_2\epsilon & a_{12} & m_2(1 - \epsilon^2/2) & a_{22} & -m_2\epsilon^2 A & a_{23} \\ 0 & m_1\epsilon^3 A (1 - \bar{z}) & 0 & -m_2\epsilon^2 A & 0 & m_3 \\ m_3\epsilon^3 A z & a_{13} & m_3\epsilon^2 A & a_{23} & m_3 & a_{33} \end{pmatrix}.$$

The $m_i$ are defined by (5).

We take $\text{Im}(z) = 0$. We will not consider CP violation here. $M'$ is diagonalized by $M' = O P M P \bar{O}$, where $O$ is orthogonal and $P$ is a diagonal phase matrix. The weak
eigenstates are written in terms of the mass eigenstates by \( \nu_\alpha = U_{\alpha i} \nu_i = O_{\alpha i} P_i \nu_i \). We have

\[
\nu_1 = i(1 - \frac{e^2}{2}, -\frac{m_{11}}{m_{11}}, -\epsilon, \mathcal{O}(10^{-10}), -\epsilon^3 A(z - 1), \mathcal{O}(10^{-19}))
\]

\[
\nu_2 = \left( \frac{m_1}{m_{11}} - \frac{a_{12}}{m_2}, 1 - \frac{m_1^2}{2m_{11}}, -\frac{a_{12}}{m_2}, -\frac{m_1 a_{12}}{m_{11}}, \frac{e_2 A a_{12}}{m_2}, \mathcal{O}(10^{-15}) \right)
\]

\[
\nu_3 = \frac{i}{\sqrt{2}} (\epsilon, -\frac{a_{12}}{m_2}, 1 - \frac{e^2}{2} + \frac{a_{22}}{2m_2}, -1 + \frac{a_{22}}{4m_2}, -\epsilon^2 A, \epsilon^4 A a_{12}(z - 1/2))
\]

\[
\nu_4 = \frac{1}{\sqrt{2}} (\epsilon, \frac{a_{12}}{m_2}, 1 - \frac{e^2}{2} - \frac{a_{22}}{4m_2}, 1 + \frac{a_{22}}{4m_2}, -\epsilon^2 A, -\epsilon^4 A a_{12}(z - 1/2))
\]

\[
\nu_5 = \frac{i}{\sqrt{2}} (\epsilon^3 A z, \frac{a_{13}}{m_3}, \epsilon^2 A, \frac{a_{23}}{m_3}, (1 - \frac{\epsilon^4 A^2}{2}), -1)
\]

\[
\nu_6 = \frac{1}{\sqrt{2}} (\epsilon^3 A z, \frac{a_{13}}{m_3}, \epsilon^2 A, \frac{a_{23}}{m_3}, (1 - \frac{\epsilon^4 A^2}{2}), 1)
\]

and the masses are

\[
M_i = \left( \frac{m_1^2}{a_{11}}, a_{11}, m_2 - \frac{a_{22}}{2}, m_2 + \frac{a_{22}}{2}, m_3(1 + \frac{\epsilon^4 A^2}{2}), m_3(1 + \frac{\epsilon^4 A^2}{2}) \right),
\]

where we have dropped higher order terms.

**IV. EXPERIMENTS**

The probability that an initial \( \nu_\alpha \) of energy \( E \) will oscillate into \( \nu_\beta \) after a distance \( L \) is

\[
P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 2 \sum_{i \neq j} \text{Re}[U_{\alpha i} U_{\beta j}^* U_{\beta i}^* U_{\beta j}] \sin^2 \frac{\Delta_{ij}}{2}
\]

\[
\Delta_{ij} = \frac{L (m_i^2 - m_j^2)}{2E}.
\]

For the LSND experiment, the relevant terms are

\[
P_{\mu e} = \epsilon^2 (2 \sin^2 \frac{\Delta_{13}}{2} + 2 \sin^2 \frac{\Delta_{14}}{2} - \sin^2 \frac{\Delta_{34}}{2}).
\]

The last term vanishes since its wavelength is too long for the 30m LSND baseline. The other terms are effectively equal, and together have the same effect as a two flavor \( \nu_\mu - \nu_e \) oscillation with \( \delta M^2 = m_3^2 \) and \( \sin^2 2\theta = 4\epsilon^2 \). We may therefore appeal to published two flavor analyses.
The LSND experiment was designed to be most sensitive at the mass splitting preferred by the CHDM model of cosmological structure formation [9], \( \delta M^2 \sim 6\text{eV}^2 \). The allowed regions of \( \sin^2 2\theta \) at that \( \delta M^2 \) depend on how the data is analyzed. The 99% likelihood region is \( 0.002 < \sin^2 2\theta < 0.01 \), while the 80% confidence level band is \( 0.0012 < \sin^2 2\theta < 0.005 \). The confidence band uses only the number of events, while the likelihood region uses all the information about the events including neutrino energy and distance from source to detection point so it is the best way to determine favored regions of \( \delta M^2 \) and \( \sin^2 2\theta \) [10]. The difference is important because \( 0.002 < \sin^2 2\theta \) is excluded at 90% confidence level by the BNL E776 experiment [11]. Thus if we compare similar types of bounds, there is a marginally allowed region consistent with the cosmologically preferred \( \delta M^2 \).

On the other hand, there is a large region at lower \( \delta M^2 \) allowed by all the data (including limits from r-process nucleosynthesis [12]). The 90% likelihood region of LSND, combined with the 90% confidence limits from E776 and the Bugey reactor experiment [13] allow \( 0.25 < \delta M^2 < 2.3\text{eV}^2 \) and \( 0.002 < \sin^2 2\theta < 0.04 \), giving \( 0.5 < m_2 < 1.5\text{eV} \) and \( \frac{1}{45} < \epsilon < \frac{1}{10} \). The upper end of the range of \( \epsilon \) is reasonably close to our expectation of \( \epsilon \sim \frac{1}{10} \). If \( \sin^2 2\theta \) is found to be in the lower end of this range, then the mixing of the charged leptons must be supressed from its expected hierarchical value for almost any conceivable neutrino mass scenario.

For the atmospheric deficit, the relevant probability is

\[
P_{\mu\mu} = 1 - \sin^2 \frac{\Delta_{34}}{2}.
\]

This has the same effect as a two flavor \( \nu_\mu - \nu_\tau \) oscillation with \( \delta M^2 = 2m_2 a_{22} \) and \( \sin^2 2\theta = 1 \). Maximal mixing is allowed by the combined Frejus, NUSEX, IMB, Kamioka sub-GeV, and Kamioka multi-GeV zenith angle dependent data for \( 4 \times 10^{-4} < \delta M^2 < 0.01\text{eV}^2 \) [14], giving \( 0.13 < a_{22} < 10\text{meV (milli-eV)} \). There is a small probability (\( \sim 2\% \)) of oscillation into \( \nu_\tau \). Matter effects are insignificant.

The vacuum disappearance probability for \( \nu_\epsilon \) is

\[
P_{ee}^{\text{vac}} = 1 - 2\epsilon^2 \sin^2 \frac{\Delta_{13}}{2} - 2\epsilon^2 \sin^2 \frac{\Delta_{14}}{2}.
\]
FIG. 1. Fermion mass hierarchies. The up and down quark and charged lepton hierarchies are shown. We include the proposed Dirac neutrino masses. The bars indicate the range of masses consistent with experiments.

This is too small an effect for the solar neutrino deficit, so we assume a $\nu_e - \nu_x$ small angle MSW mechanism. The allowed parameters are $0.003 < \sin^2 2\theta < 0.012$ and $4 < \delta M^2 < 11\text{meV}^2$. We have $\delta M^2 = a_{11}^2$ and $\sin \theta = \frac{m_1}{a_{11}}$, giving $2 < a_{11} < 3.3\text{meV}$ and $0.05 < m_1 < 0.2\text{eV}$. Note the similarity in the scales of $a_{11}$ and $a_{22}$.

The Dirac neutrino masses are plotted in Figure 1. We note in passing the possibility of extending the hierarchies to a fourth generation. Three of the particles would have suggestively similar masses. It turns out that to be consistent with weak neutral current data [15], the fourth up-type quark would have to have a similar mass to the other three
particles, in conflict with the existing hierarchy.

The probabilities for $\nu_e$ appearance experiments are

$$P_{\mu\tau} = \epsilon^4 A^2 \left( -\sin^2 \frac{\Delta_{23}}{2} + \sin^2 \frac{\Delta_{34}}{2} + \sin^2 \frac{\Delta_{45}}{2} + \sin^2 \frac{\Delta_{56}}{2} - \sin^2 \frac{\Delta_{61}}{2} \right)$$

$$P_{e\tau} = \epsilon^6 A^2 \left( -2(z-1)(\sin^2 \frac{\Delta_{23}}{2} + \sin^2 \frac{\Delta_{34}}{2}) - 2z(z-1)(\sin^2 \frac{\Delta_{34}}{2} + \sin^2 \frac{\Delta_{45}}{2}) \right)$$

$$+ z (\sin^2 \frac{\Delta_{23}}{2} + \sin^2 \frac{\Delta_{34}}{2} + \sin^2 \frac{\Delta_{45}}{2} + \sin^2 \frac{\Delta_{56}}{2}) - \sin^2 \frac{\Delta_{61}}{2} + z^2 \sin^2 \frac{\Delta_{61}}{2}\right)$$

(19)

Each expression contains terms where $\Delta_{ij}$ will be large for any conceivable experiment. For those terms, $\sin^2 \frac{\Delta_{ij}}{2}$ will average to 1/2 over the finite $E$ and $L$ resolution of an experiment. Thus we may estimate the probabilities as $P_{\mu\tau} \sim \epsilon^4$ and $P_{e\tau} \sim \epsilon^6$. Currently the best limits for large $\delta M^2$ are $P_{\mu\tau} \lesssim 0.002$ and $P_{e\tau} \lesssim 0.073$ from the E531 experiment at Fermilab [16].

$P_{\mu\tau}$ gets its scale from $V_{23}^L \sim \epsilon^2$. Thus, while the parametrization (8) is viable, maximal hierarchical mixing with $V_{23}^L \sim \epsilon$ is ruled out for any scenario with $\delta M^2_{\mu\tau} \gtrsim 10\text{eV}^2$. Maximal hierarchical mixing is not constrained in the $e-\tau$ channel.

Upcoming experiments may be able to distinguish this scenario from the other possibilities in the next few years. The prediction that the atmospheric deficit is caused by $\mu - s$ oscillations with maximal mixing is unique to this scenario. The mixing angle could be pinned down with further atmospheric neutrino data. Chooz [17] and San Onofre [18] can eliminate the possibility that the atmospheric deficit is $\mu-e$ by directly measuring $\nu_e$ disappearance probabilities. And ICARUS [19] and MINOS [20] might be sufficient to rule out $\mu-\tau$. This would leave $\mu-s$ as the only alternative. An MeV $\nu_e$ is then almost inevitable to save BBN since there would be at least four active neutrino flavors at the time of nucleosynthesis. Observations of the solar neutrino spectrum can firmly establish the mass splitting and mixing angle for the neutrinos responsible, and KARMEN [21] can confirm the LSND result, which would eliminate $\mu-e$ as a possibility for the solar deficit since the mass splitting would be too large. An observation of $\mu-\tau$ oscillations at CHORUS [22], NOMAD [23], or COSMOS [24] would then firmly establish $\mu-s$ for the solar channel. While CHORUS and NOMAD themselves have a chance of observing $\mu-\tau$, COSMOS is very likely to observe this channel, but very unlikely to see $e-\tau$. It is clear that the next few years will be very exciting for neutrino physics.
V. CONCLUSIONS

We have shown how neutrino masses appropriate for the various oscillation data can be fit into a hierarchical mass scenario analogous to the hierarchies in the quark and charged lepton sectors. Small mass splittings and small mixing angles result from the interplay of Dirac and Majorana mass terms. The scenario satisfies all experimental and astrophysical constraints. It is unique among proposed solutions in that the atmospheric oscillations are $\mu$s with maximal mixing, a prediction which could be tested experimentally in upcoming experiments.
REFERENCES


