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MONTE CARLO STUDY OF LIGHT TRANSMISSION THROUGH A CYLINDRICAL TUBE

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1. Introduction

Since light guides entered the field of physics about 15 years ago, their transmission properties have been studied, both experimentally and theoretically¹⁻⁴⁾. In particular the transmissivity for light through fibres with cylindrical cross section has been investigated for specular walls³⁾ as well as for light guides, where the photon propagation may be described by the diffusion equation²⁾.

The transmission of light through tubes of circular cross section is, however, not only of interest in the case of optical fibres where a high transmissivity is desired: it is also important in vacuum technology for example, where a cylindrical tube may be applied as a thermal radiation screen. Whereas in the case of a light guide the transmission function is determined by the optical properties of the fibre material, the reflectivity at the fibre boundary and the length-to-diameter ratio, it is governed by the reflectivity of the inner wall and by the length-to-diameter ratio only, provided that the space enclosed by the cylinder is the vacuum.

Let us for example, consider a cryopump designed to operate at extreme ultrahigh vacuum conditions. The basic construction is drawn schematically in Figure 1. The vacuum system is connected with a cryopump, the main components of which are a cryosurface (at liquid helium temperature in order to pump hydrogen) and a tubular screen against thermal radiation (at liquid nitrogen temperature in order to reduce the emission of thermal radiation by the screen itself). The vacuum system emits molecules as well as thermal photons into the cryopump. Some molecules are trapped by the cryosurface. Others, in particular hydrogen molecules, which are already sticking to it, may be released by thermal photons. This causes an equilibrium pressure which may be considerably higher than is predicted by the vapour pressure function of those gases which are pumped by the cryosurface⁵⁾.

The efficiency of the cryopump is thus determined by the transmission probabilities for both molecules and photons. As far as molecules are considered, this problem is common to vacuum physics and it has been solved long ago. For the transmission of photons, however, only complicated and

non-elementary expressions have so far been found, and even these were only approximations^{2,3)}.

The Monte Carlo method⁶⁾ offers a means of solving many complicated problems in physics for which an exact analytical solution is either very complicated or non-elementary or where it is even impossible to find one at all. It is in fact often much easier to write a Monte Carlo program instead of finding an analytical solution, especially if that solution must finally be evaluated by a computer since in many cases it is non-elementary.

The transmission probability for molecules passing through a cylindrical tube has successfully been calculated by Smith and Lewin⁷⁾ using the Monte Carlo method. This method has also been applied in this work to obtain the transmission function for photons.

2. The Monte Carlo Program

The following assumptions were made in order to solve the problem :

- a) the light which enters the tube is of either isotropic angular distribution or it is distributed according to Lambert's law⁸⁾ (see Figure 3); both cases were treated individually;
- b) the radiation density is uniform at the entrance aperture;
- c) the trajectory of each individual photon depends on its initial position and its direction at the entrance aperture only, trajectories of different photons are independent from each other;
- d) reflections at the cylinder walls are specular, i.e. the walls are ideally smooth. Fresnel reflection is not taken into account;
- e) the dimensions of the tube are sufficiently large for the application of geometrical optics.

Since the history of two different photons is independent from each other, the transmission probability is estimated to be the ratio of the number of photons leaving the exit aperture to the number of photons entering the tube, for a large number of photons, computing the single trajectories of single photons one after another. The error due to this estimation is varying inversely to the square root of the total number of computed trajectories. A relative error of less than 1% is obtained calculating up to 10^5 trajectories for each set of individual parameters in this work.

The Monte Carlo program is not discussed here in detail, since the somewhat simplified program flow diagram is shown in Figure 2. The problem of the photon's initial distribution is, however, not trivial. As one of the most general cases, consider the light being emitted from the walls of a very big vacuum chamber. Hence the light source may be assumed to be a perfect diffuser (i.e. cosine angular distribution, see Figure 3) and of uniform radiation density situated at the entrance plane of the tube. In a second conception, the light source is assumed to be an isotropic emitter (i.e. uniform angular distribution, see Figure 3) and of uniform emission density at the entrance plane. It represents the case where the origins of each photon are spread out uniformly in space, as for example in a gas discharge. Both the cases are of purely academic nature, however, since many of the real situations may be considered to be a mixture of both of them, they will be treated separately.

For the calculations of the photon trajectories, it is assumed that the law of specular reflection applies. Since Fresnel reflection is estimated to account for about 5% of the transmissivity at $L < 3D$ only, and since it may be neglected for longer tubes³⁾, it will not be considered.

A real tube wall never being perfectly smooth, the reflection of light must always be partly diffuse²⁾. In the most extreme case, the wall may become a perfect diffuser. Smith and Lewin⁷⁾ have already treated this problem using the Monte Carlo method for molecules, we ought only to replace "molecules" by "photons" and the sticking coefficient S by $1-R$, where R is the reflectivity, in order to apply their results to the transmission of photons through a tube, of which the inner wall is a perfect diffuser.

The initial conditions for each photon, i.e. its position and direction, were determined from a set of random numbers $\{a_1 \dots a_4\}$ with $0 \leq a_n \leq 1$, created by a random number program. The initial position is then obtained from

$$\underline{r} = (a_1; a_2; 0) \quad (1)$$

where a_1 and a_2 must both be replaced by a new set $\{a'_1; a'_2\}$ if $|\underline{r}| > 1/4$, i.e. if the initial position is not within the entrance plane of the tube, the diameter of which is arbitrarily set to $D = 1$. The initial direction is given by the vector :

$$\underline{u} = (p \cos \phi; p \sin \phi; q) \quad (2)$$

where

$$p = (1 - a_3^2)^{\frac{1}{2}}; \quad q = a_3; \quad \phi = 2\pi a_4 \quad (3a)$$

for an initial uniform distribution, and

$$p = a_3^{\frac{1}{2}}; \quad q = (1 - a_3)^{\frac{1}{2}}; \quad \phi = 2\pi a_4 \quad (3b)$$

for an initial cosine distribution. The derivation of equations (2), (3a) and (3b) is given in more detail in the Appendix.

After the coordinates of a photon's collision point on the tube wall have been calculated, another random number a_5 with $0 \leq a_5 \leq 1$ is generated. This number determines whether the photon is reflected or absorbed by the wall whose reflectivity is R . If $a_5 > R$, the photon is absorbed and the program continues with a new photon, otherwise the trajectory of the reflected photon is calculated (see Figure 3).

3. Results

The results are shown in Figure 4 for uniform initial distribution and for cosine initial distribution in Figure 5, respectively; the transmission probability is plotted as a function of the length-to-diameter ratio L/D of the tube with reflectivities varying from $R = 0$ up to $R = 0.99$ as the parameter.

In both cases, the transmission probability for long tubes is given by a set of parallel straight lines in the log-log representation. In order to be precise, we define a tube to be long, if

$$L/D \geq \frac{4}{\ln(1/R)} \quad (4)$$

in accordance with Daich et al.³⁾.

The transmission function T decreases with a power of -2 for long tubes, and with a power of -1 to -2 for short tubes. Taking Figures 4 and 5, one may write for long tubes :

$$T = \frac{f(R)}{c \cdot (L/D)^2} \quad (5)$$

where $c = 8$ for uniform initial distribution and $c = 4$ for cosine initial distribution. The function $f(R)$ only depends on the reflectivity. The transmission for $R = 0$ is in fact given by :

$$T_d = 1 - \frac{2L}{D} \left[\{(L/D)^2 + 1\}^{\frac{1}{2}} - L/D \right] \quad (6)$$

for a cosine initial distribution (diffuse emitter) which behaves like $\frac{1}{4(L/D)^2}$ for long tubes⁷⁾, and by

$$T_i = 1 - \left\{ 1 + \frac{1}{4(L/D)^2} \right\}^{-\frac{1}{2}} \quad (7)$$

for uniform initial distribution (isotropic emitter) for $R = 0$ and for $L/D \geq 2$, behaving like $\frac{1}{8(L/D)^2}$ for long tubes.

The function $f(R)$ from Equ. (5) may be re-written in the form

$$f(R) = \exp \{g(R) \cdot R\} \quad (8)$$

Using the transmission probabilities T as obtained from the Monte Carlo calculations (see Figures 4 and 5), $g(R)$ has been evaluated from Eqs. (5) and (8) and has been plotted as a function of the reflectivity. The result is given in Figure 6. Two facts may be seen from this figure:

- $g(R)$ and hence $f(R)$ are independant from the type of initial distribution;
- $g(R)$ is constant in the range $R = 0.3$ to 0.7 , whereas it is only slightly higher for $R \leq 0.2$; for reflectivities exceeding 0.7 $g(R)$ rises steeply with R .

According to these results, the transmission probability is given by the approximation

$$T = \frac{\exp \{5.05 R\}}{c \cdot (L/D)^2} \quad (9)$$

with $c = 4$ for cosine and $c = 8$ for uniform initial distribution, respectively, providing that $R \leq 0.6$ and that the tube is long, as defined in Equ. (4). The relative error resulting from Equ. (9) is less than 10%. For $R > 0.6$, as well as in the case of short tubes, the transmission probability must be read directly from Figures 4 and 5.

The results as obtained in this work may be compared with analytical approximations for long tubes given by Daich et al.³⁾ in a previous publication for an isotropic emitter

$$T_i = \frac{[5(1 - R + 2.5R^2) + (2/\ln R)^2]}{4(L/D)^2} \quad (10)$$

and for a diffuse emitter

$$T_d = \frac{[1 + 8R + 16R^2 + 8R^3 \{(1/\ln R)^2 - 3/\ln R - 9\}]}{4(L/D)^2} \quad (11)$$

Some transmission probabilities, as calculated from Eqs. (10) and (11), are listed together with the corresponding results from this work in Table I for $L/D = 25$ and $R \leq 0.8$ (long tube).

It becomes evident from this table, that

- there is a poor correspondence between the transmissivities for an isotropic emitter T_{i1} and T_{i2} for $R < 0.5$. For higher reflectivities Equ. (10) yields values which are up to twice as high as those from the Monte Carlo calculation, and that
- the correspondence is good within a factor of two, in the case of diffuse initial distribution.

The values obtained from Equ. (10) are generally higher than those resulting from Equ. (11), using the same parameters. Thus, Equ. (10) is certainly not a good approximation at low reflectivities since the obtained transmissivities should be lower, due to the more beam-like character of the diffuse emitter to which Equ. (11) applies. Furthermore, the transmission probability given by Equ. (10) diverges as the reflectivity goes to zero, whereas it should become $1/8(L/D)^2$.

A check as in Table I has been carried out over the whole range from $L/D = 8$ to $L/D = 500$. From a comparison of the results, as obtained from the approximation given in Equ. (9) with those as found directly by the Monte Carlo calculation, it may be concluded, that

Equ. (9) is generally a good approximation with a relative error of less than 10% for long tubes and $R \leq 0.6$, and that it may replace the expressions given by Daich et al (3).

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5. References

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- (8) Lambert's law :the radiation flux reflected per unit solid angle in a given direction is proportional to the cosine of the angle between the direction of emission and the normal to the surface. Such a surface is said to be a perfect diffuser.

Appendix

The probability $d\omega$ that a photon is emitted into a unit solid angle $d\Omega$ is

$$d\omega = \frac{1}{2\pi} d\Omega = \frac{1}{2\pi} \sin \theta d\theta d\phi \quad (1a)$$

for an isotropic emitter, and

$$d\omega = \frac{1}{\pi} \cos \theta d\Omega = \frac{1}{\pi} \sin \theta \cos \theta d\theta d\phi \quad (1b)$$

for a diffuse emitter, where θ and ϕ are the azimuthal and polar angles with respect to the normal to the emitting surface. Since the two angles are independent from each other, they may be obtained as functions of two random numbers r_1 and r_2 with $0 \leq r_1 \leq 1$ and $0 \leq r_2 \leq 1$:

$$\theta = \theta(r_1) \text{ and } \phi = \phi(r_2)$$

Hence from Equ. (1a)

$$\frac{d\omega}{dr_1 dr_2} = \sin \theta \frac{d\theta}{dr_1} \cdot \frac{d\phi}{2\pi dr_2} \quad (2a)$$

and from Equ. (1b)

$$\frac{d\omega}{dr_1 dr_2} = 2 \cdot \sin \theta \frac{d \sin \theta}{dr_1} \cdot \frac{d\phi}{2\pi dr_2} \quad (2b)$$

Since $d\omega = dr_1 \cdot dr_2$ and because of the complete independence of r_1 and r_2 from each other, we get from Equ.(2a)

$$r_1 = -\cos \theta \text{ and } r_2 = \phi/2\pi \quad (3a)$$

and from Equ.(2b)

$$r_1 = \sin^2 \theta \text{ and } r_2 = \phi/2\pi \quad (3b)$$

The initial direction is then given by

$$\underline{u} = (p \cos \phi; p \sin \phi; q) \quad (4)$$

where $p = (1 - r_1^2)^{\frac{1}{2}}$; $q = r_1$; $\phi = 2\pi r_2$

or $p = r_1^{\frac{1}{2}}$; $q = (1 - r_1)^{\frac{1}{2}}$; $\phi = 2\pi r_2$

for uniform and cosine distribution respectively.

Table I

Comparison of transmissivities calculated from expressions given by Daich et al.³⁾ together with the results of this work for a cylindrical tube with $L = 25D$.

R	$T_{i1} \times 10^3$	$T_{i2} \times 10^3$	$T_{i3} \times 10^3$	$T_{d1} \times 10^3$	$T_{d2} \times 10^3$	$T_{d3} \times 10^3$
0.1	2.2	0.37	0.33	0.78	0.72	0.67
0.2	2.4	0.60	0.55	1.1	1.2	1.1
0.3	3.0	0.95	0.92	1.4	1.9	1.9
0.4	3.9	1.6	1.5	1.8	3.1	3.1
0.5	5.6	2.6	2.6	2.6	5.1	5.1
0.6	8.7	4.7	4.3	5.1	9.2	8.5
0.7	16	9.4	---	14	18	---
0.8	36	22	---	47	44	---

T_{i1} and T_{d1} are the transmissivities obtained from Equ. (10) and Equ. (11) for uniform and cosine initial distribution, respectively, given by Daich et al.³⁾.

T_{i2} and T_{d2} are the corresponding correct transmissivities obtained from the Monte Carlo calculation of this work.

T_{i3} and T_{d3} are the corresponding transmissivities obtained from the approximation given in this work, see Equ. (9).

Figure 1

Figure captions

- Fig. 1 Cut-away sketch of a cryopump with a disk-shaped cryosurface at the end of a cylindrical radiation shield. This is in order to demonstrate the application of a radiation screen rather than a design of an efficient cryopump.
- Fig. 2 Monte Carlo flow diagram. The diameter is arbitrarily set equal to 1; N is the number of photons entering the tube, I is the number of photons leaving the exit aperture and N_{MAX} is the upper limit of N , which in most cases was equal to 100,000.
- Fig. 3 Illustration of a cosine (diffuse emitter) and of a uniform angular distribution (isotropic emitter), respectively. The length of the arrows is proportional to the probability for a photon to be emitted with the angle ϕ to the normal of the surface.
- Fig. 4 Transmission of light with uniform initial distribution through a cylindrical tube as a function of its length-to-diameter-ratio with the reflectivity of its inner surface as the parameter.
- Fig. 5 Transmission of light with cosine initial distribution through a cylindrical tube as a function of its length-to-diameter-ratio with the reflectivity of its inner surface as the parameter.
- Fig. 6 The function $g(R)$ as defined in Equ. (8).

Cosine Distribution

Uniform Distribution

Figure 3

Figure 2

Figure 5

Figure 4

Figure 6