PERFORMANCE STUDY ON
PROTON–PROTON STORAGE RINGS AT SEVERAL HUNDRED GEV/C

SUMMARY REPORT OF THE LATTICE WORKING GROUP

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1. Introduction

The questions asked of the Lattice Working Group fall into two categories - the normal cell and the overall lattice. Questions in the first set are, by and large, rather specific, having to do with the various devices to be accommodated in the normal cell - e.g., main magnets, vacuum stations, beam position monitors, correction elements and so on. Within a two-week study period, it is possible to make progress of a more definite sort in this area than on the global problems associated with the overall lattice. This circumstance is reflected in the discussion below - a relatively long section on the normal cell is followed by a relatively short section on the lattice as a whole.

A complete trial lattice for the LSR had been developed prior to the study. It presumes the use of conventional magnets. The group used this lattice and its associated parameters as a starting point, and the remarks below are all within the context of conventional magnets.

2. The Normal Cell

In the trial lattice, the normal half cell contains three dipoles, each 7.2 m in magnetic length, a quadrupole with gradient length of 3.3 m, an intermagnet gap of length 2.5 m on one side of the quadrupole, and three intermagnet gaps of 0.4 m each. The half cell length is therefore 28.6 m. For 400 GeV, the dipole field is 1.8 T and the quadrupole gradient is 20.9 T/m. The free aperture for the beam is a circle with a 60 mm diameter.

In the various subsections to follow, we will first comment on the main magnets. Conventional dipole magnets, 7.2 m in length, for d.c. operation at 1.8 T without manifestly unbelievable power requirements are massive objects. We have somewhat arbitrarily chosen a power dissipation of 60 MW for the dipoles of each ring. At this power level, two rings will just fit in a tunnel having the same dimensions in cross-section as the SPS enclosure.

Next, we note that though the 0.4 m intermagnet gap is adequate for coil ends and end protection, no space has been allowed for intermagnet
vacuum stations and clearing electrodes. There is no apparent reason
to expect that the vacuum requirements in the LSR will be less stringent
than those of the ISR; therefore, we outline a pumping system suitable
for an average pressure that does not exceed $10^{-11}$ torr.

In the several following subsections, space requirements for addi-
tional devices are estimated. The net result of all the space argu-
ments is an increase in the half cell length to 31 m.

2.1 Main Magnets

For a vertical vacuum chamber aperture of 60 mm, the magnet gap
height must be at least 78 mm (i.e. chamber thickness 1 mm, bakeout strip
2 mm, thermal insulation 5 mm, clearance and tolerance 1 mm).

The SPS dipoles $B_2$, with gap height of 52 mm and 6.26 m long, have
a loss at peak current (4900 A), corresponding to 1.8 T, of
$2.5 \times 42.3 = 106$ kW per dipole.

Scaling the gap height by a factor 1.5 to reach 78 mm, and the coil
cross-sectional area by a factor $1.5^2$, the dissipation per unit length
stays constant. Increasing the length from 6.26 m to 7.2 m, the dissi-
pation goes up to 122 kW per dipole.

Taking advantage of the more modest gap width requirement in LSR
than in SPS, one can still multiply the coil cross-sectional area by
$4/3$, without exceeding an overall steel width of $1.5 \times$ SPS. This move
reduces the dissipation per dipole to $3/4 \times 122 = 91$ kW.

The resulting dipoles have a coil cross-sectional area equal to
three times that of the SPS $B_2$ dipoles. The overall dimensions of the
core are $0.78 \times 1.25$ m. A sketch of the dipole cross section is shown
in Figure 1. The 7.2 m long dipole's weight is 56 ton, of which
6.5 ton are coils. The dissipation in $6 \times 108$ dipoles is $60$ MW per ring.

The aperture of the SPS quadrupoles (inscribed diameter 88 mm) is
adequate for the circular vacuum chamber with heating elements and
thermal insulation. The maximum gradient is 20 T/m.
For a magnetic length of 3.3 m, we estimate the overall length to be 3.6 m.

The dissipation is 26 kW at peak current, which gives
\[ 2 \times 108 \times 26 \times 10^{-3} \text{ per ring.} \]

At 400 GeV, the power dissipated in the main magnets of the normal cells of the LSR would be a total of 131 MW.

2.2 Vacuum Stability and Pumping System

The stability limit against pressure run-away can be obtained by comparison with the ISR, using for scaling the criterion as developed in Reference 3. This criterion yields for the proposed structure of 7.2 m distance between two pumps and a circular aperture of 6 cm diameter a limit of \((I_n)_{\text{crit}} = 7 \text{ A}\), while it yields for the outer arcs of the ISR \((I_n)_{\text{crit}} = 100 \text{ A}\). \(I\) is the current in A and \(n\) the net gas desorption coefficient under ion bombardment. At present, the pressure runs away at several places in the ISR at 20 to 25 A. Hence, \(n\) must be 4 to 5 at these places. The surface is stainless steel, baked several times for 24 h at 300° C. The same \(n\) would limit the current in the ISR to 1.4 or 1.8 A.

Instead of trying to reduce \(n\) everywhere on the many kilometres of chamber length to \(\leq 1\), it is recommended that a titanium sublimation pump be installed in the middle of all bending magnets. This would require that each magnet be split into two blocks with a slot of about 200 m in between, but would increase the stability limit by a factor of 4.

We recommend as main pumping station between the magnets a sputter-ion pump of 40 l/s and a titanium sublimation pump, the conductance of which is only limited by the conductance of the connecting pipe. This combination has a base pressure of about \(3 \times 10^{-12} \text{ torr}\).

The additional mid-magnet pump would reduce the pressure difference between pump and mid-pump position from \(1.8 \times 10^{-11} \text{ torr}\) (assuming \(q = 1.5 \times 10^{-13} \text{ torr l of H}_2/\text{s cm}^2\) as for the ISR) to \(4.5 \times 10^{-12} \text{ torr}\). If one makes further use of the 200 mm gap for the installation of
clearing electrodes, the clearing, as measured in terms of residual neutralization, would be improved by a total factor of 8.

The average pressure in the proposed pumping system would be $10^{-3}$ torr or less.

We estimate that a main pumping station will require a space allocation of 400 mm (flange and bellows included) not presently provided in the trial lattice. Clearing electrodes will be accommodated within this space, hence we do not make an additional space allowance for this function.

2.3 Pick-up Stations and Correction Dipoles

Our intention is to estimate the peak to peak distortions of the closed orbits expected in both the H and V planes, starting from the LSR machine parameters as given in the preliminary study report 2). We also estimate the number of pick-ups and correcting dipoles we need in the normal lattice for reducing the orbit of the bare machine. Finally, it is possible to give a limit for the strength of the dipoles.

2.3.1 Number of Pick-up Stations and Correcting Dipoles Needed in the Normal Lattice

Figure 2 gives the correction efficiency $\eta = \gamma/\gamma_c$ as a function of the parameter $(n-1)/Q$ where $n$ is the number of correction dipoles, which we have taken to be equal to the number of pick-up measurements 4).

To obtain a factor of 10 in the correction effect, we can see from this curve (where $\delta = 0$) that we need the following condition:

$$\frac{n-1}{Q} = 4$$

As $Q$, in the lattice only, is roughly 27, we have:

$$n = 109$$

i.e. one pick-up and one dipole per cell.

We can try to take account of the measurement errors and of the errors in the correction application. Defining the following
parameter $\delta$,

$$
\delta = \sqrt{5(\frac{d^2}{\hat{y}^2} + \frac{\delta k^2}{\hat{k}^2})}
$$

where

$\hat{y}$ = measurement error of the pick-up

$\delta k$ = error in the correction kick

$\hat{k}$ = amplitude of the correction kick

we can trace the curves of $\eta$ for different values of the error $\delta$ (see Figure 2). ISR experience would place us in the region of $\delta = 0.1$. In this case, if we want an efficiency at least equal to 8, we have to choose:

$$
\frac{n - 1}{Q} = 8
$$

$$
n = 217
$$

i.e. two pick-ups and two dipoles per cell.

2.3.2 Estimation of the Orbit Amplitudes in the Normal Lattice Part Only

Using the statistical formulae\textsuperscript{5}) giving the rms amplitude and the maximum (at 98%) amplitude of the orbit, we get the following results:

a) H plane

$$
<\Delta x>_{\text{rms}} = \sqrt{64 \times 10^6 \left(\frac{\Delta B}{B_m}\right)_{\text{rms}}^2 + 533.6 \left<\Delta y\right>_{\text{rms}}^2}
$$

where $\left(\frac{\Delta B}{B_m}\right)_{\text{rms}}$ is the field fluctuation between the dipoles

$\left<\Delta y\right>_{\text{rms}}$ is the displacement of the quadrupoles in mm

taking $Q_{\text{lattice}} = 27$ and $\Delta n_h = 50$

Assuming: $\left(\frac{\Delta B}{B_m}\right)_{\text{rms}} = 10^{-4}$ $<\Delta y>_{\text{rms}} = 0.1 \text{ mm}$
we have: \[ \langle x \rangle_{\text{rms}} = 2.4 \text{ mm} \]

and \[ \tilde{x}_{\text{p-to-p}} (98\%) = 24 \text{ mm} \]

b) \( V \) plane

\[ \langle z \rangle_{\text{rms}} = \sqrt{\frac{\delta_{\text{rms}}^2}{505_{\text{rms}}^2 + 533.6 \langle \Delta y \rangle_{\text{rms}}^2}} \]

where \( \delta_{\text{rms}} \) is the dipole tilt in mrad

\( \langle \Delta y \rangle_{\text{rms}} \) is the displacement of the quads in mm.

Assuming: \( \delta_{\text{rms}} = 0.1 \text{ mrad} \)

\( \langle \Delta y \rangle_{\text{rms}} = 0.1 \text{ mm} \),

we have: \[ \langle z \rangle_{\text{rms}} = 2.4 \text{ mm} \]

and \[ \tilde{x}_{\text{p-to-p}} (98\%) = 24 \text{ mm} \]

Then, taking a correction efficiency \( \eta \) of 8, we presume that we will be able with two pick-ups and dipoles by cell to reduce the peak-to-peak to 3 mm.

2.3.3 Estimation of the Orbit Amplitudes in the Total Machine with Insertions

Taking the same formulae, we have:

a) \( H \) plane

\[ \langle x \rangle_{\text{rms}} = 7.5 \times 10^3 \sqrt{\frac{3.5 (\Delta B)}{B_{\text{rms}}}^2 + 30 \langle \Delta y \rangle_{\text{rms}}^2 + 97 \langle \Delta y_i \rangle_{\text{rms}}^2} \]

where \( \frac{(\Delta B)}{B_{\text{rms}}} \) is the field fluctuation
\[ \langle \Delta y \rangle_{\text{rms}} \] is the quadrupole displacement in the lattice in m

\[ \langle \Delta y_i \rangle_{\text{rms}} \] is the quadrupole displacement in the insertions in m

Assuming: \[ \left( \frac{\Delta B}{B_m} \right)_{\text{rms}} = 10^{-4} \quad \langle \Delta y \rangle_{\text{rms}} = 10^{-4} \text{ m} = \langle \Delta y_i \rangle_{\text{rms}} \]

we have: \[ \langle x \rangle_{\text{rms}} = 8.5 \text{ mm} \]

and \[ \hat{z}_{\text{p-to-p}} (98\%) = 59 \text{ mm} \]

If we assume that the tolerances can be twice as restrictive on the quads in the straight insertion (\[ \langle \Delta y_i \rangle_{\text{rms}} = 0.5 \cdot 10^{-4} \text{ m} \]) we can reduce this peak-to-peak to 40 mm.

b) V plane

\[ \langle z \rangle_{\text{rms}} = 5.7 \cdot 10^3 \sqrt{\frac{2}{2.86_{\text{rms}} + 30 \langle \Delta y \rangle_{\text{rms}}^2 + 80 \langle \Delta y_i \rangle_{\text{rms}}^2}} \]

where \[ \delta_{\text{rms}} \] is the dipole tilt in rad

Assuming: \[ \delta_{\text{rms}} = 10^{-4} \quad \langle \Delta y \rangle_{\text{rms}} = 10^{-4} \text{ m} = \langle \Delta y_i \rangle_{\text{rms}} \]

we have: \[ \langle z \rangle_{\text{rms}} = 6 \text{ mm} \]

and \[ \hat{z}_{\text{p-to-p}} (98\%) = 46 \text{ mm} \]

In the same way, that can be reduced to 31 mm.
2.3.4 Required Intensity for the Correcting Dipoles

Assuming a peak-to-peak distortion of \( \approx 20 \) mm in the lattice and trying to correct with one dipole only, we find

\[ \text{BL} = 0.5 \text{ Tm} \]

As the correction is distributed, that can give a limit for the correcting dipole design.

2.4 Skew Quadrupoles

The skew quadrupoles are intended for controlling the excitation of the \( Q_h = Q_v \) coupling resonance. This is of particular importance in machines like the existing CERN ISR where working lines are placed very close to the diagonal and the beam has a large emittance ratio. However, the proposed LSR has equal emittances in the two planes. Also, the optics of the new machine are designed to give ample luminosity which may remove the need for modifying the emittance ratio, by shaving for example, as in the ISR, or for sitting very close to the diagonal to have the maximum resonance-free region.

It is still felt, however, to be appropriate to include skew quadrupoles in the machine lattice. First, the emittance ratio may be changed. Second, coupling does have certain undesirable side effects for injection optimization systems and \( Q \)-measuring systems which detect coherent oscillations.

Some general conclusions can be drawn from a report written by H.G. Hereward, E. Keil and J.D. Young, ISR-TH/66-21 "On the Effects of Skew Quadrupoles and Tilted Magnets\(^6\) and from the experience gained with the CERN ISR.

- Providing the skew quadrupoles are reasonably uniformly distributed, the effect on \( \alpha_p, \text{vert} \) is not serious. (In the ISR this is 28 quadrupoles among 192 main magnet units.) This is illustrated in the above report by the calculation that the maximum
permissible systematic tilt in the magnet units for \( \alpha_p, \text{vert} \)
is ten times greater than that for coupling.

- ISR experience indicates that the skew quadrupoles will be amply
  strong if they can compensate a systematic tilt of 0.5 mrad in
  all lattice quadrupoles.

- A setting tolerance of say five times better than that which
  coupling measurement can resolve should not cause any problems.

The above has led to the following proposal:

- 54 skew quadrupoles distributed in alternate periods
- maximum integrated strength 0.26 T
- current tolerance \( \pm 3 \times 10^{-3} \) of max. (In fact, far better should
  be easily achievable.)

These quadrupoles would probably be about 500 mm in overall length
with a 300 mm steel length.

Criteria for power supply connections are less well defined. This
choice depends to a certain extent on the coupling measurement system.
In the ISR an overall coupling coefficient is obtained by measuring the
period of interchange between the planes. This leads to a coupling
coefficient,

\[
C_q = \left( \frac{R^2}{Q} \right) \left( \frac{1}{B_0} \right) \left( \frac{\partial B_r}{\partial r} \right)_\text{average}
\]

where \( R \) is machine radius, \( Q \) is the tune, \( B_0 \) is the magnetic rigidity
and \( \left( \frac{\partial B_r}{\partial r} \right)_\text{average} \) is the uniform skew quadrupole field around the whole
machine which would give the observed effects. The \( \left( \frac{\partial B_r}{\partial r} \right)_\text{average} \)
is closely related to the zero harmonic in the actual distribution.
Since the zero order coupling resonance is of interest and only the one
parameter \( C_q \) can be measured anyway, the series operation of all quad-
ruoples seems reasonable. However in Reference 6, it was shown that
the oscillations could be decoupled at a symmetry plane by two indepen-
dent parameters. At a non-symmetry plane, it appears that four para-
meters may be sufficient. This problem requires further study also with reference to any special effects arising in the insertions.

2.5 Chromatic Aberration Compensation

Comment on the compensation of the various effects of quadrupole chromatic aberration, such as chromaticity, will be deferred until Section 3, since the entire lattice is intimately involved. Here, we need only note that the primary compensating elements are sextupoles and that provision for them must be made in each half cell. We have reserved a space of 0.6 m for this purpose.

The question will arise as to why we have not assumed that the bulk of the chromaticity compensation will be done by shaping of the bending magnet pole profile, as one might choose to do in a low field magnet. This approach is less appealing here, for the matching of conductor and steel contributions to the field pattern over the wide excitation range is considerably more difficult. In any event, even if more extensive design effort on the bending magnets reveals that pole shaping is advantageous, there will still be a need for additional sextupoles for chromaticity adjustment under various operating conditions.

2.6 Other Elements

In addition to the devices discussed above, space will be needed for other magnets (e.g. octupoles for shaping of the working line, corrections of resonances), sector valves, and an additional pumping station in the straight section. Insofar as the magnets are concerned, time has not permitted us to analyze their strengths and distribution; the lengths listed in the next subsection are simply space allowances.

We have made no attempt to assess possible requirements for special magnets or other devices in normal cells adjacent to special purpose insertions, such as those for injection or beam dump.
2.7 Summary of the Normal Cell

2.7.1 Space Summary

Bending magnets \( (\ell_B = 7.2 \text{ m}) \)

Quadrupole \( (\ell_Q = 3.3 \text{ m}) \)

Vacuum stations (with clearing)

Straight section including:

Vacuum station (without clearing)

Pick-up station

Correction dipole

Sextupole

Stator magnet (x³ and higher)

Space for resonance correction

Sector valves, skew quadrupoles

\[
3 \times 7.6 \text{ m} = 22.8 \text{ m}
\]

\[
3.6
\]

\[
4 \times 0.4 \text{ m} = 1.6
\]

0.3 m

0.3 m

0.3 m

0.6 m

0.5 m

0.5 m

3.0 m

TOTAL LENGTH OF HALF CELL . . . .

\[
R_o = \frac{2 \times 31 \times 10^8}{2\pi} = \frac{1065}{m}
\]

as compared to 983 m in trial lattice.

2.7.2 General Parameters

The design procedure for proton storage rings outlined by Keil\(^7\) utilizes a number of adjustable parameters which reflect the constraints imposed by the hardware. We tabulate below their values for the normal cells of the trial lattice and of this report:

<table>
<thead>
<tr>
<th></th>
<th>Trial Lattice</th>
<th>This Report</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B_n)</td>
<td>1.80 T</td>
<td>1.80 T</td>
</tr>
<tr>
<td>(B_Q)</td>
<td>0.63 T</td>
<td>0.58 T</td>
</tr>
<tr>
<td>(C_Q)</td>
<td>0.47</td>
<td>0.35</td>
</tr>
<tr>
<td>(R/\rho)</td>
<td>1.33</td>
<td>1.44</td>
</tr>
</tbody>
</table>
2.8 Arrangement of the Rings in the Tunnel

A vertical placement of the rings - i.e. one on top of the other - appears to be preferable. With this arrangement, as sketched in Figure 3, the rings will fit just barely in a tunnel of the same size as that for the SPS, with enough space remaining for a magnet transport vehicle. This placement allows injection into the two rings from the outside, independent of the cross-over layout.

Two arguments in favour of horizontal placement were presented. First, that the vertical scheme will introduce a vertical $\alpha_p$ as the beams are brought together at the interaction regions and this dispersion will require compensation. Second, the experimenters have expressed a desire for as much access around each beam as possible, even into the normal lattice; the more widely separated rings inherent to a horizontal layout are compatible with this position. Removal of the vertical dispersion is straightforward and likely to be far less costly than the increase in tunnel cross-section necessary to permit a horizontal layout. With regard to the second argument, we are not able to assess the additional utility of the LSR that would accrue by increasing access to the neighbourhood of each ring in the normal lattice.

3. The Overall Lattice

We are confident that it will come as no surprise to the organizers of the study that answers to the questions asked concerning the overall lattice are not immediately forthcoming. One need only scan the printout from the "AGS" program for the trial machine to appreciate the complexity of the LSR lattice with its three varieties of interaction regions. Analytical methods offer guidance as to the sorts of calculations that should be performed concerning chromatic aberration compensation, non-linear driving terms and so on. The numerical work must be done with the aid of the computer, and this is a slow process.

So our activities under this heading may be characterized as an effort to clarify the calculations that should be done in certain areas and as an exhortation that such calculations be subsequently performed.
We divide our summary below into two subsections - some remarks on superperiodicity and comments specifically related to chromatic aberration compensation.

3.1 Comments on Superperiodicity

The potential evils associated with low symmetry in synchrotrons are well known, and we need not catalogue them here. In current proton storage ring designs, with their extreme variations in amplitude function, $\beta$, the effects of low symmetry become quantitatively worse. However, in contrast to the synchrotron, the impetus for devising methods of compensating these effects is more compelling, for a storage ring with a variety of interaction region types is inherently a device having low symmetry insofar as the dynamics is concerned, regardless of what its apparent symmetry may be on a site plan. Then the LSR, with two high beta, two low beta and two general purpose insertions, has at most a two-fold periodicity no matter how the interaction regions are distributed around its perimeter.

Just to illustrate the sort of problems that two-fold periodicity brings, we plot, in Figure 4, $Q_h$ and $Q_v$ versus $\delta p/p$ as taken from the "AGS" printout. The insertions and normal cells are matched at $\delta p/p = 0$. No chromaticity compensation has been introduced, and the high $\beta$'s in the insertions have increased the slopes of $Q_h$ and $Q_v$ by about a factor of two above that which would be the case for a ring containing normal cells only.

For two-fold periodicity, systematic gradient errors will introduce a stopband at every integral value of tune. But, for insertions matched at some particular momentum, the chromatic aberration of the quadrupoles will constitute a systematic gradient error at other values of the momentum. A stopband should appear at $Q = 35$, for example, and this stopband is shown as the shaded error in the $Q_h$ curve for $0.25% < \delta p/p < 0.45%$. The $Q_v = 35$ stopband is not shown - it "starts" at $\delta p/p \approx 0.45%$. No stopband appears at $Q = 35.5$, since it would not arise from a systematic gradient "error" in the two-fold periodic lattice.
In a machine of higher symmetry, chromaticity might be the only chromatic aberration effect that one need worry about. As the figure suggests, however, here it is also necessary to compensate the gradient "errors". Since both functions require the introduction of sextupoles, it is also necessary that they be configured so as not to provide driving terms for third-integral structure resonances, such as at

\[ Q = 35 + 1/3 = (2 \times 53/3). \]

3.2 Chromatic Aberration Compensation

Two methods were proposed, which in a sense represent extremes in approach. One is to compensate the chromatic aberration "where it happens" by requiring that each gradient, \( B' \), have an associated sextupole moment, \( B'' \), related by \( B' = B'' \alpha_p \). This method is conceptually direct. It brings with it the restriction that \( \alpha_p \) must be non-zero in the insertions. Further, since building a sextupole moment into a conventional quadrupole is quite unattractive, additional magnets would appear in the insertions where space is at a premium. The magnitude of the third-integral driving terms is not known as yet.

The other method proposed compensation by putting sextupoles at the proper harmonics into the normal cells. The use of \( \pi/2 \) phase advance per cell in the normal lattice makes it easy to suggest a prescription. Sextupoles, equal in strength, at corresponding points in four successive normal cells, introduce a zeroth harmonic term but neither \( 2\psi \) nor \( 3\psi \) terms. Two sextupole strings, one at \( F \) quads and one at \( D \) quads, covering multiples of four cells, would adjust the chromaticity in both planes. Similarly, sextupoles, equal in strength but alternating in sign, at corresponding points in four successive normal cells provide a \( 2\psi \) term but neither zeroth nor \( 3\psi \) terms. Two strings would compensate for gradient stopbands in both planes independent of the chromaticity and without exciting the third-integer resonances. (If the sextupoles are individually powered, the four strings could, of course, become two.) This approach gets the sextupoles out of the insertions, so that \( \alpha_p \) can be made small or zero in them, and tries to eliminate third-integer effects from the outset. But further calculation is needed to determine its feasibility, for it makes no statement about \( (\delta p/p) \) effects.
References


5) C. Bovet et al., CERN-MPS-51/Int.DL/68-3.


8) For a more extensive discussion of the consequences of chromatic aberration in two-fold periodicity lattices, see M. Month, Particle Accelerators, 3, 183 (1972).
Fig. 1  Dipole for 400 GeV LSR with a dissipation of 60 MW per ring
Fig. 2 Orbit correction efficiency
Fig. 3  Superposed Storage Rings: space occupation in the tunnel
SOME NOTES ON CORRECTION OF CHROMATICITY IN LARGE STORAGE RINGS

M. Donald

1. Introduction

In this note we do not intend to calculate a sextupole correction scheme for the LSR, nor do we suggest a definite foolproof strategy for doing so. The intention is more to understand the nature of chromatic aberrations and the pitfalls waiting for those who try to correct for them. In modern machines the precision correction of chromaticity is necessary for satisfactory operation. The advent of highly irregular machines with low beta insertions and low superperiodicity aggravates the problem, as does the large size of the new generation of storage rings.

In the time available for the study no detailed calculations can be done, nor is it productive to do other than rough rule-of-thumb estimates.

2. General Remarks on 2nd Integer Resonances

We shall use the notation 2nd integer resonance rather than 2nd order resonance so as to avoid confusion with terms like 2nd order effect (2nd order perturbation theory).

(a) Linear Chromaticity

The linear chromaticity of a machine comes about from the zero order harmonic of the focusing parameter $K$ in combination with the $\beta$ function.

$$K_h = \frac{1}{B_p} \frac{\partial \beta_Z}{\partial x}$$

$$K_v = -\frac{1}{B_p} \frac{\partial \beta_Z}{\partial x}$$

$$\xi_h = \frac{dQ_h}{Q_h} \frac{dp}{p} = \frac{1}{4\pi Q_h} \int_c \beta_h \Delta K_h \, ds \quad , \quad \frac{\Delta K}{K} = -\frac{dp}{p}$$
For simple periodic machines \( \xi_h \approx \xi_v \approx -1.2 \) to \(-1.4\).

For large electron machines with low beta insertions \( \xi \approx -2 \) to \(-5\) depending on whether in injection or collision tune.

For the LSR \( \xi \approx -2 \).

The matching of the insertions is done for the design momentum but an off-momentum particle sees an effective \( \Delta k \) error in all the quadru-
poles. The zero order harmonic of this error gives rise to the linear part of the chromaticity, which is higher than that for a simple machine because of the high value of the beta functions in the high beta quad-
ru poles.

This \( \Delta k \) error also has a strong harmonic content at multiples of the superperiod number, and gives rise to 2nd integer resonances when

\[
2\eta_h = m N_{\text{sup}} \\
2\eta_v = m N_{\text{sup}}
\]

Due to the superperiodicity of 2 in the LSR these resonances will occur at every integer value of \( Q \). The chromaticity curves may now take the shape:
Figure 2

The worst consequence of this resonance is that if correction is only done for the zero order harmonic of the error the chromaticity curve will not be straight.

Figure 3

A method of correction proposed by Autin and favoured at the ISR is to correct the $\Delta K$ at source in each quadrupole, or as near to every
quadrupole as possible. This method relies on having a reasonable value of dispersion function $\alpha_p$ at every quadrupole.

Setting \[ \frac{\Delta K}{K} = - \frac{\Delta p}{p} = \alpha_p \frac{dp}{p} \frac{k'}{k} \]
gives \( k' = \frac{K}{\alpha_p} \) as the correction needed in each quadrupole.

This method of correction exactly counteracts the linear part of the chromaticity and all harmonics of the 2nd integer resonance. This leaves only 3rd integer correction to be done and higher order effects to be considered.

The advantage of this form of correction is illustrated by the phase advance characteristic near to the high beta quadrupoles.

For correction of a 2nd integer resonance we wish to place effective quadrupoles near to $\mu = \pm n\pi/2$ from the disturbance. We also wish to place these effective quadrupoles as near to the disturbance as possible so as to localize the effect. We also want the correction for the horizontal resonance to be at a place with high $\beta_h$ and low $\beta_v$, and vice versa for the vertical effect. We find for the LSR:

<table>
<thead>
<tr>
<th>Position</th>
<th>$\beta_h$</th>
<th>$\mu / 2\pi$</th>
<th>$\beta_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>High $\beta$ quadrupole (h)</td>
<td>768</td>
<td>3.076</td>
<td>445</td>
</tr>
<tr>
<td>$\pi/2$ away</td>
<td>46</td>
<td>2.576</td>
<td>46</td>
</tr>
<tr>
<td>$\pi$ away</td>
<td>30.7</td>
<td>2.076</td>
<td>59.8</td>
</tr>
<tr>
<td>High $\beta$ quadrupole (v)</td>
<td>207.2</td>
<td>2.358</td>
<td>809</td>
</tr>
<tr>
<td>$\pi/2$ away</td>
<td>86.4</td>
<td>1.858</td>
<td>18.9</td>
</tr>
<tr>
<td>$\pi$ away</td>
<td></td>
<td>1.385</td>
<td>G.P. insertion region</td>
</tr>
</tbody>
</table>

Similar results were found for the EPIC lattice, and it is probably true to say that it is a general property of such insertions that at distances $\mu = \pm n\pi/2$ from the high beta quadrupoles, the beta functions are unsuitable for correction. The consequence of this is that strong
sextupoles have to be used for correction in the insertions since they tend to back each other out.

Imperfect 2nd integer correction due to chromatic effects leads to a beating of the beta functions in the normal lattice cells. This may not be too important in the LSR since the aperture is determined by considerations of coasting beam stability and vacuum requirements. For an electron machine, however, it is more serious as lack of aperture leads to short quantum lifetimes. However, if the LSR is beam-beam limited, the 2nd integer resonance leads to a variation of $\beta^*$ (the beta value at the interaction point) with momentum and subsequent over-reaching of the limit by either the $+$ or $-$ momentum particles. If the beam-beam $\Delta q$ is large as in electron machines the linear beam-beam tune shift causes such a 2nd integer resonance.

3. **3rd Integer Resonances**

By placing sextupoles in the lattice 3rd integer resonances may be excited. The resonances of most concern to the LSR with working line around $Q \approx 35.2$ are

\[
3Q_h = 104, 106
\]

\[
Q_h + 2Q_v = 104, 106
\]

\[
2Q_v - Q_h = 34, 36
\]

These are excited by sextupoles with normal orientation. Sextupole errors in the dipole field are one source of error, but with well designed magnets and lack of eddy current effects, the effect of these on resonances will be small compared to the effects due to the sextupoles introduced for 2nd integer correction.

It is at this point that the correction scheme proposed by Autin first begins to have difficulties. For 2nd integer aberrations to be corrected at the quadrupoles there must be a reasonable $\alpha_p$ function throughout the machine. To correct for the 3rd integer resonances excited by the correction scheme more sextupoles must be introduced into regions of finite $\alpha_p$, which in turn upset the 2nd integer correction.
Lee Teng (this study) has suggested another means of sextupole correction which as we shall see later has considerable advantages.

In this scheme sextupoles are arranged in the main lattice in groups of 4. The main lattice is arranged so that there is exactly 90° of phase shift per period in both the horizontal and vertical betatron motion. Special matching and tuning sections have to be included at each end of the main lattice since the phase shift across the insertion is always unequal in the two planes.

For correction in the horizontal plane, 4 sextupoles are arranged close to 4 F quadrupoles and powered together.

For the zero order harmonic the effect of each sextupole is added and the linear chromaticity is corrected.

For 2nd integer resonances the sextupoles are 180° out of phase with each other and the net effect is zero

\[ \uparrow \quad \downarrow \quad \uparrow \quad \downarrow \]

For 3rd integer resonances the sextupoles are 270° out of phase, but the effects still cancel.

\[ \uparrow \quad \downarrow \quad \rightarrow \quad \Xi \quad \leftarrow \quad \uparrow \]

For the difference resonance \( 2Q_v - Q_h \) and for the 1st integral sextupole effect the net result is also zero.

\[ \uparrow \quad \rightarrow \quad \Xi \quad \leftarrow \quad \uparrow \]

To create a 2nd integer correcting term to balance the mismatch due to the insertions he introduces 2 pairs of sextupoles at the same positions or close to them. The polarities of these 4 sextupoles are

\[ + \quad - \quad + \quad - \]
The effect on $2Q_v - Q_h$ and $Q_h$ resonances is zero.

\[ \uparrow \quad \rightarrow \quad \equiv \quad \leftarrow \quad \downarrow \]

The effect on 2nd integer resonances $2Q_h$ and $2Q_v$ is positive and generation of harmonic correction terms is possible.

\[ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \]

The effect on 3rd integer resonances is zero

\[ \uparrow \quad \rightarrow \quad \leftarrow \quad \equiv \quad \leftarrow \quad \downarrow \]

To get this arrangement 4 sextupoles and 2 power supplies are necessary for both horizontal and vertical planes.

For off-momentum particles the lattice periodicity is now changed so that every 2 periods are the same instead of every period. There will be a beating in beta function amplitude (and $\alpha_p$) every 2 cells but this will be regular and probably not very large. The beta functions at the ends of the lattice are effectively moved to match the perturbed beta functions of the insertions.

From the experience gained from the EPIC sextupole study and from discussions with P. Morton about the SPEAR and PEP schemes, it appears that for these machines it is better to do almost all the sextupole correction in the normal lattice and use the insertion only for touching up the correction. EPIC in particular is suitable for this sort of correction since it has long straight sections where the $\alpha_p$ function is zero. It is in these regions that 3rd integer touch-up correction
can be done without affecting the 2nd integer effects.

When the next two sections are taken into consideration, Lee Teng's scheme seems (without detailed calculation) to have decided advantages over Autin's.

4. Non-Linearity of $\alpha_p$ Function

Month has pointed out (this study) that the non-linearity of the $\alpha_p$ function is important, and is very dependent on the placing of sextupoles.

The correction of 2nd integer effects is dependent on knowing the $\alpha_p$ function and using it to get a $\Delta K$ variation with momentum by using sextupoles. If the sextupoles themselves perturb the $\alpha_p$ function, then calculation is difficult, the correction is non-linear with momentum and in extreme cases the process could be divergent.

Just as the 2nd integer resonances arise from a momentum error in the quadrupoles, so the $\alpha_p$ function arises from a momentum error in the dipoles

$$\frac{\Delta B}{B} = \frac{\Delta \alpha_p}{\alpha_p}$$

and zero order harmonic, period harmonic and super period harmonic effects are present.

When sextupoles are added to the lattice, an off-momentum particle sees a dipole field as well as sextupole and quadrupole fields.

$$B = \frac{1}{2} a^2 B''$$

Expand $x$ about closed orbit $x_0 = \alpha_p \frac{dp}{p}$

$$x = x_0 + a = \alpha_p \frac{dp}{p} + a$$

$$\Delta B = \frac{1}{4} \left[ a^2 + 2 \alpha_p \frac{dp}{p} + (\alpha_p \frac{dp}{p})^2 \right]$$
The driving term for a 1st integer resonance is proportional to \( \sqrt{p} \cdot \Delta B \).

1st integer driving term \( = a_p \sqrt{p} \frac{dp}{p} \)

Sextupoles should therefore be kept out of regions of high \( a_p \) which are not properly compensated for 1st integer resonance (Lee Teng's lattice is compensated over 2 periods). Sextupole compensation at the high beta quadrupoles in the LSR lattice is bad.

5. Fourth Integer Resonances

Second order perturbation theory has been about for a long time, but seems largely to have been forgotten since the effects for small machines of simple geometry were either small or relatively small compared to first order effects.

Hagedorn calculated the Hamiltonian driving terms for 4th integer resonances excited by sextupoles and these effects are seen in the EPIC tracking calculations along with resonances of higher order.

The non-linear motion due to the sextupoles, 1st and 3rd integer resonances, when put back into the sextupoles causes 4th integer effects proportional to the square of the 1st and 3rd integer ones. If the perturbation is small the 1st order theory terms dominate, but as the sextupoles get stronger (or the beta functions larger) the 4th integer effects get very strong.

By tuning out as many 1st and 3rd integer effects as possible including difference resonances the situation improves but since the terms involve an infinite sum of resonances only limited improvement is possible.

A typical 4th integer term is the driving term for the coupling resonance

\[ 2Q_h - 2Q_v = 0 \]

\[
\delta_{2002}(0) = \sum_{q} \left\{ \frac{h_{1100} h_{1002}(-q)}{Q_{x} - q} - \frac{3h_{0100} h_{3000}(-q)}{Q_{x} + 2Q_{y} + q} + \frac{2h_{0110} h_{1002}(-q)}{Q_{x} - q} \right\}
\]

+ other terms generated by skew sextupoles.
For working lines such as in the ISR where one operates close to the coupling resonance it is advisable to keep the $2Q_h - 2Q_v = p$ resonance as small as possible by not exciting the driving term.

To see why it is important to keep sextupoles out of the high beta region one can look at the following equations.

Chromaticity correction $\propto \int a_p \beta K' \, ds$

For a given chromaticity therefore

$K' \, ds = \frac{\xi}{a_p \beta}$

3rd integer effects

$\propto \int \beta^{3/2} K' \, ds \propto \frac{\xi}{a_p} \cdot \sqrt{\beta}$

4th integer effects

$\propto \left[ \int \beta^{3/2} K' \, ds \right]^2 \propto \left[ \frac{\xi}{a_p} \right]^2 \cdot \beta$

6. Work in Progress at the Rutherford Laboratory

M.R. Harold has been responsible for sextupole studies at the Rutherford Laboratory and a brief description of these studies will serve to illustrate some of the points in this note.

Studies started with an early lattice with important parameters

$\beta_{x^*} = 0.4 \, m \quad \text{(Horizontal beta function at the interaction point)}$

$\beta_{y^*} = 0.1 \, m \quad \text{(Vertical " " " " " " ")}$

and $\xi = \frac{\Delta Q}{Q} / \frac{dp}{p} \approx -5$

The chromaticity was corrected with normal cell sextupoles only, but this resulted in unacceptable increases in $\beta$ for off-momentum particles ($\Delta p/p = \pm 5 \cdot 10^{-3}$). With the aid of 4 additional sextupoles in the $a_p$ matching section the $\beta$ values could be properly matched, but very large sextupole values resulted and the consequent stable region was far too small (about $\pm 15 \, \text{mm}$ in the normal cells at $\beta_h = 40$).
The next stage was as follows.

(i) The normal cell $\xi$ was corrected with normal cell sextupoles only.

(ii) Five matching cell sextupoles were used to correct the insertion $\xi$ and to match properly the $\beta$ values to the normal cell $\beta$ values.

(iii) The residual 3rd integer resonance forcing terms were cancelled using five sextupoles in the $\alpha_p = 0$ region. The resonances involved were

$$3Q_h = 56, 60$$
$$Q_h + 2Q_v = 56, 60$$
$$2Q_v - Q_h = 20$$

Again very small stable regions resulted, even though the $\xi$'s were reduced by raising $\beta_x^*$ to 2.5 m and $\beta_y^*$ to 0.25 m.

On returning to correction using normal cell sextupoles only, the reduced $\xi$'s were found to have resulted in a tolerable $\beta$ variation in the normal cells (~10%) and a much larger stable region of ±70-80 mm for horizontal motion. For coupled motion the stable region was ±60-65 mm.

Similar figures were found for $\beta_x = 2.0$ m, $\beta_y = 0.2$ m, and work is proceeding on a lattice with $\beta_x = 0.15$ m. It is hoped that further improvements will result from partial compensation of the resonance forcing terms by further small sextupoles in the $\alpha_p = 0$ region or in the $\alpha_p$ matching cell. Sextupoles which oppose the chromaticity correction, however, will not be considered.

The linear beam-beam $Q$-shift at the interaction point has also been considered and affects the values of $\beta$ in the lattice. In the lattice

$\beta_{\text{max}}$ changes from 42 to 70 for $\frac{\delta \beta_p}{\beta_p} = +5 \cdot 10^{-3}$

$\beta_{\text{max}}$ remains unchanged at 42 for $\frac{\delta \beta_p}{\beta_p} = -5 \cdot 10^{-3}$
These results fit in very well with what might be expected from the other contents of this note except for the fact that the chromaticity is quite flat when a 2nd order resonance is shown to be present by the heating of the beta functions. (This flatness, however, has not yet been studied in detail). With the latest arrangement of sextupoles the $a_p$ function is perturbed very little as might be expected from minimizing the sextupole strengths and keeping them out of regions of high $\beta$ and $a_p$.

7. Scaling Laws for Large Machines

The $Q$-shift of the off-momentum particles caused by the high beta quadrupoles is dependent only on the interaction region itself.

$$\Delta Q = \int_\beta \Delta K \, ds = \int_\beta K \frac{dp}{p} \, ds$$

$$= \int_\beta K \, ds$$

Since $\beta_{max} = \ell^2/\beta^*$ where $\ell$ is the length of the interaction region, and the focal strength of the lens $K \, ds$ is given by

$$K \, ds = \frac{1}{\ell}$$

and $\Delta Q = \frac{\ell^2}{\ell \beta^*} = \ell/\beta^*$

For very fine chromaticity correction as needed in proton storage rings the $\Delta Q$ caused by the mismatch is important since it leads to irregularities in the working line.

From this point of view the insertion is dominant and machine size is unimportant.

As the machine increases in size the relative chromaticity due to the insertions decreases (unless the number of interaction points is increased). For very large machines the insertions are of smaller importance. The normalized chromaticity

$$\frac{\Delta Q}{\frac{dp}{p}}$$
decreases to nearer that of a regular machine, and the limit of feasibility is reached by AQ considerations similar to those that would be used for a normal machine.

8. Conclusions

Sextupole correction in large storage rings is not a trivial matter and deserves detailed study. For proton storage rings the exact control of the working line is imperative since the radiation damping inherent in electron-positron rings is absent. To see the order of difficulty we compare the LSR and EPIC.

Q-shift for off-momentum particles caused by high β quadrupoles

\[ ΔQ = \int β \cdot K \frac{dp}{p} \, ds \]

<table>
<thead>
<tr>
<th></th>
<th>β</th>
<th>K</th>
<th>ds</th>
<th>β · K · ds</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPIC</td>
<td>360 m</td>
<td>0.2 m⁻²</td>
<td>1.4 m</td>
<td>100</td>
</tr>
<tr>
<td>LSR</td>
<td>700</td>
<td>0.016</td>
<td>6.6</td>
<td>73</td>
</tr>
</tbody>
</table>

The Q-shift caused by the insertions is very similar, but in the LSR the situation is complicated by having three insertions in a row. Since the Q of the normal lattice in the LSR is double that of EPIC the contribution of the insertions to the normalized chromaticity is less, but we feel that the total chromaticity is probably more important than normalized chromaticity.

For similar Δp/p the mismatch due to 2nd integer resonance should be comparable for the same distance from the resonance.

For the LSR the Δp/p figure does not appear to be resolved, but it is unlikely to be much less than needed in EPIC. In favour of LSR is the fact that the machine is not aperture limited and there is plenty of space for β variation. Against LSR is the fact that the superperiodicity is only 2 and it is impossible to get far from a systematic 2nd integer resonance.
For 3rd and higher integer resonances the LSR is in much better shape; the emittances are very small compared with EPIC and only a very small part of the vacuum vessel is taken up by particle emittance. The fixed points for 3rd integer resonances may be able to come well inside the vacuum vessel without bad effects.

It is difficult without doing detailed calculation for a particular lattice to give rules of thumb for sextupole correction schemes, but the EPIC experiences may be applicable.

1. Keep sextupoles out of high $\beta$ regions.
2. Keep sextupole strengths as small as possible by
   (a) correcting as close to quadrupoles as possible,
   (b) avoiding sextupoles cancelling each other (in good effects, that is).
3. Keep sextupoles out of high $\alpha_p$ regions (no practical experience here).

References