PERFORMANCE STUDY ON
PROTON–PROTON STORAGE RINGS AT SEVERAL HUNDRED GEV/C

INSERTIONS WORKING GROUP REPORT

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INTRODUCTION

Close interaction and continued dialogue between the storage ring designers and users are needed for the design of the beam-crossing insertions, the experimental halls, and other related features of the machine. This discussion and exchange have been carried on in the past and will certainly continue indefinitely in the future. Based on results of interchanges in the past, preliminary designs were made for three types of beam-crossing insertions — low-β (high luminosity), high-β, and general-purpose insertions. The Autumn Study provided an opportunity for a further intensive interaction between the users and the designers. The present designs of the three insertions were re-examined in relation to experimental requirements and were modified to better match the experiments to which they are applied. The tuning ranges of these insertions were investigated. The configurations and dimensions of the experimental halls surrounding these insertions were worked out based on designs of experimental apparatus. Other features of the storage rings desirable for experimental use were examined and whenever possible incorporated in the design. Future re-examinations may well lead to conclusions different from those arrived at here. But the reasonings leading to the present conclusions, right or wrong, will add to the understanding of the experimental needs and the capability of meeting these needs in the design of the machine.

EXPERIMENTAL REQUIREMENTS

1. Unequal energy operation

The capability of operating the two rings at unequal energies has several rather important advantages.

(a) Without moving the detector one can vary the centre-of-mass angle of the emitted (detected) particle by changing the ratio of the energies of the two beams. For example, a detector located at 90° can detect particles emitted at 53° in the centre-of-mass for a 4 to 1 energy ratio. This is especially important if the detector is very large or very long so that it is impossible or impractical to move.
(b) Consider the diffraction dissociation experiment\textsuperscript{1})

\[ p + p \rightarrow p + X \]

where the outgoing proton is strongly peaked forward (\( \approx \) mrad) and has a momentum very close to that of the incident proton. By varying the energy of the target proton (directed opposite to the outgoing proton) one can measure the s-dependence of the differential cross-section \( \frac{d^2\sigma}{dMdt} \) (\( M = \) mass of \( X \)) at fixed values of \( M \) and \( t \) without moving the outgoing proton spectrometer. More than a mere convenience, doing the experiment this way will yield higher precision than moving the spectrometers while varying the equal energy of the two beams.

(c) For future options in which the two rings must be stored with particles of different types (ep or pp) the same features necessary for particles of different energies are required.

The present designs of the high-\( \beta \) and general-purpose insertions already allow for unequal energy operation, but because of the use of common dipoles for both beams the low-\( \beta \) insertion must be modified.

2. Large angle experiments

Because of the very low cross-sections at large angles these experiments require high luminosity, hence the low-\( \beta \) insertion.

(a) All large angle experiments will require the highest luminosity which it is possible to obtain.

(b) Because of the rapid fall-off of the cross-section with angle these experiments must be designed to have a tight selection of angle. This is easier to obtain with a smaller (shorter) source. For a longer source (beam-collision diamond) to avoid saturating the detectors by particles copiously produced at smaller angles from either end of the source the detectors must be longer and placed farther away from the source. Therefore the cost of the detector assembly will rise roughly as the square of the source length while the event rate increases only linearly\textsuperscript{2}). This gives a desired source length of \( \sqrt{1} \) m meaning that
more than 99% of the luminosity (preferably 99.9%) should be contained within the 1 m length.

(c) With a source length of 1 m a clear drift space of 3 m beyond each end of the source is required for the detector assembly. We will, therefore, take the minimum drift length to be 10 m or ±5 m from the beam crossing.

3. Small angle experiments

For small angle elastic scattering experiments the outgoing protons will go through the beam quadrupoles, hence follow the beam optics. At 400 GeV × 400 GeV the angle at which the Coulomb scattering amplitude equals the nuclear scattering amplitude is ±90 µrad. We should be able to measure angles somewhat smaller than this (where Coulomb scattering dominates) with a reasonable resolution of, say ±10%.

(a) Even at 90 µrad to obtain a resolution of ±10% the beam divergence at the crossing should be less than ±9 µrad. For a normalized beam emittance of 30π mm-mrad this requires a high-β value of β* = 867 m which is rather difficult to attain. If the beam emittance is reduced by shaving, the required β* may be decreased. With the emittance shaved down to ½ × 30π mm-mrad and a β* of 400 m the beam divergence is then ±6.6 µrad.

(b) The detector resolution will have to be folded in. The detector should be placed 90° in betatron-oscillation phase from the beam crossing. With drift chamber as detector one can get a spatial resolution of ±0.1 mm. This corresponds to an angular resolution at the beam crossing of

\[
\frac{5 \text{ µrad}}{\sqrt{\beta_d \text{(in m)}}}
\]

where β_d is that at the detector and where we have taken a β* of 400 m. In order that this is small compared to the beam-divergence resolution of ±6.6 µrad we should have β_d < 10 m. For detectors with 1 mm resolution, the required β* becomes uncomfortably large.
(c) The values of \( \beta^* \) and \( \beta_d \) do not have to be identical in the horizontal and the vertical planes. For example \( \beta_{h}^* = 300 \) m and \( \beta_{v}^* = 400 \) m would be quite all right.

(d) The spatial dispersion of the beam at the detector should be small compared to the detector resolution of \( \pm 0.1 \) mm which means that \( \alpha_p \) must be essentially zero at the detector.

(e) Although high luminosity is not critical for these experiments it is still desirable to maximize the luminosity.

4. Horizontal vs. vertical beam crossing

There are some weak arguments preferring horizontally separated rings with horizontal beam crossings. In such an arrangement the orbits are planar, and one has only the horizontal dispersion to worry about. It is easier to arrange for a sufficiently large horizontal separation between the rings so that detectors can be placed next to an outgoing beam on all sides. Moreover, for conventional magnets the vertical dimension is smaller, hence one can come closer to the beam from the top and bottom sides which are more accessible in the horizontal arrangement.

5. Energy dependence of luminosity

Because of the rising cross-sections with energy it is desirable to have higher (or at least, comparable) luminosity at lower energies. On the other hand, the \( \gamma^2 \) dependence of luminosity limited by the beam-beam tune shift is rather fundamental. So far no practical idea has been suggested to overcome this \( \gamma^2 \) law.

INSERTIONS

The performances of the three insertions which have so far been designed were examined and evaluated in relation to the experimental requirements. A number of modifications were proposed for each insertion to improve its utility. Some of these modifications were worked out quantitatively during the study.
The tuning range of some insertions was investigated. The performance limitations given by the beam-beam tune shift are at best only a rough guess at present. On the other hand to push for higher performance the higher $\beta_{\text{max}}$ values encountered in these insertions make alignment tolerances and stopband corrections more stringent. The actual operation will certainly be a progression of improvements in performance starting from low performance and low $\beta_{\text{max}}$, hence relaxed tolerances and corrections, progressing to high performance which may well go beyond that specified in the design. It is therefore important to know the tuning ranges of these insertions.

A great deal of work remains to complete the list of modifications and investigations proposed.

1. Low-$\beta$ (high luminosity) insertion

(a) Source (beam-collision diamond) length

The present design with a beam crossing angle of $\alpha = 0.9$ mrad and a horizontal $\beta_h$ value of $\beta_h^* = 5$ m gives a source length of ~2.7 m. To reduce this length to 1 m one has to increase $\alpha$ at a loss of luminosity. For various values of $\beta_h^*$, a vertical $\beta_v^* = 1.3$ m, and for uniform and Gaussian beam distributions one gets $^{4,5}$

<table>
<thead>
<tr>
<th>$\beta_h^*$ (m)</th>
<th>Uniform (100%)</th>
<th>Gaussian (99.9%)</th>
<th>Gaussian (99%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ (mrad)</td>
<td>$L(10^{32}\text{cm}^{-2}\text{sec}^{-1})$</td>
<td>$\alpha$</td>
<td>$L$</td>
</tr>
<tr>
<td>1</td>
<td>1.18</td>
<td>9.1</td>
<td>1.40</td>
</tr>
<tr>
<td>2</td>
<td>1.54</td>
<td>7.0</td>
<td>1.80</td>
</tr>
<tr>
<td>3</td>
<td>1.86</td>
<td>5.8</td>
<td>2.16</td>
</tr>
<tr>
<td>4</td>
<td>2.13</td>
<td>5.1</td>
<td>2.48</td>
</tr>
<tr>
<td>5</td>
<td>2.38</td>
<td>4.5</td>
<td>2.76</td>
</tr>
<tr>
<td>6</td>
<td>2.60</td>
<td>4.1</td>
<td>3.01</td>
</tr>
</tbody>
</table>

$^+$ updated by B. Zotter
where the two columns under the Gaussian distribution correspond to having 99.9% and 99% of the luminosity contained in the 1 m length. This table shows that to maximize luminosity one should minimize \( \beta^* \) hence \( \alpha \). It is easy to see that with fixed source length the luminosity is proportional to \( (\beta^* \beta^*)^{-\frac{1}{2}} \) hence low-\( \beta^* \) and low-\( \beta^* \) are equally important. However, due to limitations on \( \beta_{\text{max}} \) \( \beta^* \) has to be larger in the plane in which the first lens is defocussing.

After having arrived at some lowest practical values of \( \beta^*_h \) and \( \beta^*_v \) one can gain back some luminosity by increasing the beam current, provided the beam-beam limit is not exceeded.

(b) Drift space on either side of the beam crossing

Decreasing the distance between the first quadrupoles on either side of the beam crossing helps to reduce \( \beta^* \) and/or \( \beta_{\text{max}} \) in the insertion. The present beam-crossing drift space of \( \pm 10 \) m can be reduced to \( \pm 5 \) m. In addition, using specially designed narrow quadrupoles, we can reduce the separation between the first quadrupoles and the 15 m long beam-separating dipoles from 7 m to 2 m. Thus the distance from the beam crossing to the first quadrupoles can be reduced by 10 m from 32 m to 22 m.

(c) Unequal energy operation

With the beam-separating dipoles on either side of the crossing used for both beams, beams of unequal energy must be tilted in the crossing drift space. The tilt angle \( \theta \) and the displacement \( w \) at the common dipoles are given below\(^5\) for \( p_1 = 400 \) GeV/c and various values of \( p_2 \).

<table>
<thead>
<tr>
<th>( p_2 ) (GeV/c)</th>
<th>( \theta ) (mrad)</th>
<th>( w ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>1.93</td>
<td>30.0</td>
</tr>
<tr>
<td>200</td>
<td>4.51</td>
<td>70.0</td>
</tr>
<tr>
<td>150</td>
<td>6.16</td>
<td>95.4</td>
</tr>
<tr>
<td>100</td>
<td>8.13</td>
<td>125.9</td>
</tr>
</tbody>
</table>

In addition to needing very wide dipoles (\( \pm 126 \) mm) the two high-\( \beta \) quadrupoles would have to be displaced in a transverse direction - and
with very stringent tolerances. Three other solutions were proposed which avoid this problem:

(i) In the present design, without reducing the crossing drift space, let us take $\alpha = 2.5$ mrad at $\beta_h^* = 5$ m. The beam separation at the dipole is, then, $2.5$ mrad $\times 10$ m $= 25$ mm which is adequate to clear a current septum dividing the dipole into two independent halves. To obtain $18$ kG on one side and, say, $4.5$ kG on the other, the septum has to carry about $11$ kA per cm of magnet gap. If the thickness of the septum is $1$ cm the current density will be $11$ kA/cm$^2$ which is attainable with cooling. However, if the $5-10$ A of beam ever hits the septum the consequence is rather unpleasant. Also, with this large $\beta_h^*$, hence large $\alpha$, the luminosity is rather low ($\sim 4 \times 10^{32}$ cm$^{-2}$sec$^{-1}$).

(ii) With the same beam crossing configuration, even without the beam-separating dipole, the beam separation at the first quadrupole is $2.5$ mrad $\times 32$ m $= 80$ mm which is ample to clear the septum of an iron-septum quadrupole. Indeed in this scheme one can reduce to some extent the $32$ m drift length to the first quadrupole thereby somewhat increasing the luminosity by reducing $\beta_h^*$ and $\alpha$. The consequence of beam hitting iron septum is certainly less traumatic than that of beam hitting a current septum. The septum magnets for both solutions are shown in Fig. 1.

(iii) A third solution uses a common separating magnet, and two pairs of magnets to adjust the angle in each channel individually.

2. High-$\beta$ insertion

(a) The possibility of having unequal high $\beta^*$ values in the horizontal and the vertical planes, e.g. $\beta_h^* = 300$ m and $\beta_v^* = 400$ m should be exploited to reduce the $\beta_{\text{max}}$ in the insertion.

(b) In the present design $|\alpha_p|$ is rather large ($5.45$ m) at the beam crossing. One alternative is to rearrange the dipoles at the ends of the insertion so that $\alpha_p = \alpha'_p = 0$ throughout the $\sim 120$ m central region where the beams are straight. Since the detector locations ($\frac{\pi}{2}$ phase from the crossing point) are inside this central straight
region such a modification will make \( \alpha_p = 0 \) at both the source (beam crossing region) and the detectors as desired.

(c) The tuning range of the modified insertion should, then, be investigated.

3. **General purpose insertion**

The overall design of this insertion is satisfactory. Possible improvements and desirable additional studies are:

(a) Introducing dipoles to make the crossing angle, hence the luminosity, adjustable. This should be done in such a way that the capability of running the two beams at unequal energies is not impaired.

(b) The tuning range should be investigated.

4. **High-\( \alpha_p \) insertion**

An insertion in which the total incident energy of a collision can be resolved to within a pion mass might be useful. This would require high \( \alpha_p \) and low \( \beta_h \) at the beam crossing. For example, a \( \beta_h = 10 \text{ m} \) gives a full beam width of 1.7 mm. Together with an \( \alpha_p = 5 \text{ m} \) this gives a momentum resolution of \( \frac{\Delta p}{p} = \frac{1.7 \text{ mm}}{5 \text{ m}} = 3.4 \times 10^{-4} \) or \( \Delta p = 136 \text{ MeV/c at } p = 400 \text{ GeV/c} \). Higher \( \alpha_p \) or lower \( \beta_h \) will bring the resolution down below a pion mass. Such an insertion should have the beams crossing in the vertical plane.

**LATTICE COMPENSATION**

The insertions are matched only for one given momentum. Because of the chromatic aberration of the quadrupoles, for an off-momentum orbit there exist gradient "errors" which, if not compensated, will open up ¼-integer (gradient) stopbands. The contribution to this gradient "error" comes mainly from the insertions and, more specifically, from quadrupoles at high-\( \beta \) locations. Moreover, to manipulate the working line in the tune diagram we must be able to adjust the chromaticity. Again, a large contribution to the chromaticity comes from the insertions and mostly from quadrupoles at high-\( \beta \) locations. To zero'th order one
wants to reduce the rather high chromaticity to zero. A straightforward way to compensate both chromaticity and off-momentum gradient "error" (mismatch) is to superimpose a sextupole field in each quadrupole so that \( B^n \equiv \frac{B_1}{\alpha_p} \). The change in the quadrupole strength with momentum is then compensated by a change in gradient produced by the sextupole through orbit dispersion. However, this can only be done if the dispersion is non-zero (\( \alpha_p \neq 0 \)) at all quadrupoles, whereas in most insertions \( \alpha_p \approx 0 \) or small. In addition, the sextupoles introduced in this manner will generally produce large excitation terms for \( \frac{1}{3} \)-integer resonances.

It is clearly more desirable to locate the sextupoles only in the normal-cell part of the lattice. Care must be taken to arrange these sextupoles in such a way that \( \frac{1}{3} \)-integer resonances are not excited. Normally this is done by carefully tailoring the placement of the sextupoles. If the betatron phase advance of a normal cell is \( \frac{\pi}{2} \), there exist simple arrangements of sextupoles in the normal cells which produce the necessary compensations without exciting \( \frac{1}{3} \)-integer resonances \(^8\). In any case, we can assume that compensation of chromaticity and off-momentum mismatch is carried out by placing sextupoles only in the normal cells and that \( \alpha_p \) can be made zero in the insertions as desired. Of course, with every change of insertion the compensating sextupoles must be retuned.

**EXPERIMENTAL HALLS**

The discussion on the layout and size of the experimental halls was based on a recent study by CERN high-energy physicists under the chairmanship of L. Di Lella, in which the physics interest of 400 GeV proton storage rings was considered and a number of possible experiments designed \(^9\). In the present discussions consideration was also given to the fact that the storage rings will probably be deep underground. Cost considerations will therefore limit the size of the halls rather severely. Since the design of the insertions is such that they are in principle interchangeable it may be necessary to use this fact to
match physics experiments, insertions and experimental halls by exchanging insertions occasionally.

1. The General Purpose Insertion

Experiments using the general purpose insertions can be expected to have detectors covering all angles. Spectrometers in the very forward direction will have to analyse particles with momenta up to 400 GeV/c and will therefore be long. They will certainly need all of the 80 m free space around the downstream beam pipes. Intermediate and large angle spectrometers will also be needed but scaling from the ISR suggests that analysis of particles up to about 10 GeV/c will be adequate. An experimental hall with sufficient space for such a spectrometer installation is shown in Fig. 2. A central part of 30 m diameter with a 6 m deep pit would accommodate one or more rotating spectrometers to cover medium and large angles and the tunnel diameter would be enlarged to 10 m over the whole length of the downstream arms for forward spectrometers.

2. The High-β Insertion

The high-β insertion has been designed for experiments at very small angles extending well into the Coulomb interference region. In principle such experiments require only very small detectors close to the downstream beams. However, it would seem reasonable to allow for at least a limited installation around the interaction region and also take into account the possibility of conversion into a general purpose insertion. The proposal is to have simply an enlarged tunnel of 10 m diameter for ±80 m from the interaction point.

3. The Low-β Insertion

The low-β insertion has a free space of only ±5 m around the crossing point; with the additional problem of very high particle fluxes small angle experiments will be impossible. The very high luminosity is in any case intended for low cross-section experiments in the large
angle region. In the study\textsuperscript{9} mentioned above a large detector to study μ-mesons at large angles was designed and also a large solenoid. Either experiment could be comfortably accommodated in a circular hall of 20 m diameter with 6 m above and below the beam. The hall shown in Fig. 3 has in addition two tunnels at 90°, one of 10 m diameter and 15 m long, the other 6 m diameter but 25 m long, which would be available for spectrometers which may have to analyse particles above 10 GeV/c. With two low-β insertions one or both of these side tunnels might be suppressed for the second region, though they are probably also useful for access and assembly. Although necessarily fixed at 90° in the laboratory system the angular range of these spectrometers in the centre-of-mass system might be varied by using unequal beam energies.

4. General Facilities

There was no real discussion on the many other details such as crane facilities, access and counting rooms. It was felt that this belonged to a later stage when considering the exact siting of the machine. For a machine which is deep underground the physics experiments will have to choose between very expensive counting rooms specially dug as close as possible to the intersections or experiments controlled remotely from counting rooms on the surface with all fast electronics inaccessible in the machine itself.
Figure 1 - Septum magnets for low-$\beta$ insertion
Figure 2 - Experimental halls for low-$B$ insertion
Figure 3 - Experimental halls for general purpose insertion
OPERATION AT UNEQUAL ENERGIES

L. Di Lella

The only quasi-two-body process which can be studied at the super ISR is diffraction dissociation

$$p + p \rightarrow p + X$$ (1)

where X is a system of mass $M_X$ with baryon number + 1, which decays into hadrons. The interest of studying this process at very high energies is the fact that states of very high mass can be coherently excited in the collision. The maximum value of $M_X$ is

$$\left( \frac{M_X}{s} \right)_{\text{max}} \approx \frac{\sqrt{s}}{4} = 200 \text{ GeV at } \sqrt{s} = 800 \text{ GeV.}$$

The protons associated to the state X in (1) are emitted at very small angles, with momentum $p'$ given by

$$p' = \frac{\sqrt{s}}{2} - \frac{1}{2} \frac{M_X^2}{\sqrt{s}}$$

This value is very close to that of the elastic protons, $p = \sqrt{s}/2$.

Experimentally, one detects protons which are emitted at very small angles with momentum very close to that of the circulating beam. The results are expressed in terms of a double differential cross-section,

$$\frac{d^2 \sigma}{dM_X dt},$$

where $t = -p_p' s^2$ is the square of the invariant four-momentum transfer from the incident proton to the outgoing proton.

It is interesting to measure the s-dependence of the cross-section at fixed values of $M_X$ and $t$. The minimum value of $t$ for given values of $p$ and $p'$ is determined by the minimum detectable angle $\theta_{\text{min}}$, which is determined by the geometry of the machine and the detector. Since the
cross-section falls rapidly with increasing $t$ for given $M_x$, it is most convenient to measure it at the smallest value of $|t|$, which is obtained at $p = 100$ GeV/c. Under these conditions, one can keep $t$ fixed and vary $s$, by varying the energy of the other beam (the one directed away from the spectrometer). For example, for $\theta_{\text{min}} = 1$ mr, $p = 100$ GeV/c, $p' = 98$ GeV/c, $|t| = 0.0098 \text{(GeV/c)}^2$. It is possible to study the $s$ dependence of the cross-section at this small value of $|t|$ by varying the momentum of the other beam from 100 to 400 GeV/c. This corresponds to centre-of-mass energies between 200 and 400 GeV/c. Furthermore, there is no change of angle and momentum for the detected proton.

If the operation with unequal energies is abandoned, the $s$ dependence can be studied only at much larger values of $t$. For example, if $\sqrt{s} = 400$ GeV is obtained with equal momenta, the same value of $M_x$ given above would correspond to $|t| = 0.0392 \text{(GeV/c)}^2$, which is four times larger than in the case where $\sqrt{s} = 400$ GeV is obtained colliding a 100 GeV/c beam with a 400 GeV/c one. Moreover, in this case, the angle and momentum of the detected proton will change with energy.

The conclusion is that operation at unequal energies, although not essential, increases considerably the range of kinematical variables which can be measured in the study of diffraction dissociation phenomena and simplifies the detection problems.

More generally, operation at unequal energies offers the possibility of changing the centre-of-mass angle without changing the position of the detector. For example, a detector located at 90° could detect particles emitted at 53° in the centre-of-mass, in the case of a 400 GeV/c beam colliding with a 100 GeV/c one. This practical advantage to the experimentalist may become important if the sizes of the hall and the detector do not allow easy changes of the angle.
IMPLICATIONS OF A LONG INTERACTION DIAMOND FOR PHYSICS EXPERIMENTS

K. Potter

A point source is in general preferable to a finite distribution, but also unobtainable. The following notes consider the particular problems which can be expected at the SISR where the interaction diamond will be one or more metres long.

The greatest difficulty will probably be for large angle experiments which because of the rapid fall-off of particle production with angle are by definition studying rare processes. These experiments require the highest luminosity and inevitably feature a high degree of selectivity. Real event rates are in the one per hour range while luminosities of $10^{33}\text{cm}^{-2}\text{s}^{-1}$ give $10^8$ interactions/second a rejection of $10^{11}:1$. The most efficient place to make this selection is in the trigger, which is of course never achieved but always the ultimate aim.

Precisely because of the rapid fall-off with angle a very valuable event selection depends on the angle of the detected particles. A tight selection of angle is easy with a small source, but difficult with a large one as illustrated by the following example. At 400/400 GeV the flux of secondaries can be expected to rise by more than two orders of magnitude between 100 and 200 mrad. Taking an arbitrary limit at 200 mrad, the closest allowed approach of a large angle detector to a 1 m long interaction diamond is 20 cm, assuming it to be of the same length as the diamond and has zero acceptance below 200 mrdians. Clearly this distance increases proportionally to the diamond length. While this retreat from the source may be possible for some detectors a linear increase in size results if the acceptance is to stay the same. In fact the cost of a detector will increase as the square of the diamond length assuming cost is proportional to surface area. A large solid angle device designed for a 2 m diamond can be expected to cost between twice and four times that for a 1 m diamond.

For small acceptance spectrometers the disadvantages of a long
source are less acute and probably completely unimportant at small angles. At larger angles cost considerations of analysing magnets and detectors will again come in although perhaps not as steeply as above. A spectrometer which does not accept the whole source will have normalisation problems which are likely to be important in spectrometer type experiments.

Conclusion

Large angle experiments studying rare processes require the highest possible luminosity; however, it is clear that their dimensions must rise proportionally to the length of the source and therefore the cost will rise more nearly as the square. This will probably be too high a price to pay but it will be hard to use only the centre of the source, thus wasting luminosity, without incurring a heavy penalty of 'background' from the unused luminosity.

It would seem to be better to accept a loss of luminosity by reducing the size of the diamond in the first place, particularly since the only experiments untroubled by a long source, small angle spectrometers, have no need of a high luminosity.
A VARIABLE LOW-BETA INSERTION FOR THE LSR

K. Steffen and B. Zotter

1. For many reasons it seems desirable to be able to adjust the values of the betatron functions at the intersection - and incidentally the maxima of the beta functions in the insertion - after the beam has been injected and stacked. Preferably this should be done by varying the strengths of the matching quadrupoles only - possibly with a concomitant adjustment of the lattice quadrupoles in order to keep the total phase-shift (Q-value) constant.

2. Care must be taken to find a set of solutions that belong to the same family, i.e. where all quadrupole strengths change continuously, or the beam may be lost during the tuning. Existing computer routines such as AGS are not designed for this task but rather try to find the best match for each new condition. This problem has been overcome by advancing in small steps, and using the last match as input for the next step. Nevertheless, the solutions have sometimes changed to another family, as can be seen in Figure 1, where the values of the horizontal betatron function change suddenly between solutions 2 and 3 at the lattice end of the insertion. An improved version now gives solutions that stay in the neighbourhood of curves 3 and 4 also for betatron values corresponding to curves 1 and 2, and seem to yield smooth curves for the variation of all quadrupole strengths (Figure 4).

3. The solutions illustrated in Figures 1 to 3 are labelled according to the value of the vertical betatron function at the intersection. The maxima of both betatron functions have been reduced from the values given in Ref. 1 by decreasing both the free length between separating magnets to 2 × 8 m, and the distance from the separating magnet to the first quadrupole to 2 m. The first reduction appears possible as all experimental set-ups designed so far seem to fit into a space of 2 × 4 m, while the second reduction is possible when we assume that quadrupoles
can be designed which have only about 35 cm for one transverse dimension.

4. Further discussion on the requirements of unequal energy operation and the possibility of using the LSR for proton-antiproton collisions indicate that a simple separating magnet may be undesirable, but could be replaced either by a series of septum magnets \(^2\) or by several magnets which permit adjustment of the bending angle \(^3\). However, unequal energy operation also appears possible with the present scheme if we use steering magnets \(^4\) and accept a transverse displacement of the first pair of quadrupoles in each beam line. The principle of tunable low-beta insertions, however, will be useful for all proposed solutions except those which use common quadrupoles, where only discrete operating conditions can be found for a given pair of beam momenta. For this reason common quadrupoles have not been investigated, although they may lead to lower maxima than solutions which have bending magnets next to the intersection. Also the increase of the crossing angle to large enough values to avoid bending magnets altogether - as in the ISR - seems not realistic as it leads to a very large reduction in luminosity.
OPTIMIZATION OF THE OVERLAP LENGTH
FOR COLLISIONS IN LOW-Ø INSERTIONS

M. Month

For coasting beams crossing, say horizontally, at a non-zero crossing angle, $a$, the luminosity is essentially independent of $\beta_h^*$ (the value of $\beta_h$ at the crossing point). This fact is utilized in the design of low $\beta$ insertions. By choosing $\beta_h^* > \beta_v^*$, and by choosing the first focusing element away from the collision point to be vertical, the maximum rise of $\beta$ in both planes is kept to a minimum.

However, the choice of $\beta_h^*$, although not affecting the luminosity, does determine the overlap region for collisions, sketched in Fig. 1.

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Fig. 1 - Overlap Region. $2l$ is the overlap length. $a$ is the $\frac{1}{2}$ beam size. $\alpha$ is the crossing angle. $c$ is the collision point. If $a$ corresponds to $\sqrt{2}$ rms, then 95% of all collisions take place in the overlap diamond if the beams are Gaussian.
We want to optimize the interaction region by choosing $\beta_n^*$ and $a$ so that a given fraction of all nuclear collisions takes place within an overlap diamond length $2\lambda$. The presumption is that collisions taking place outside this overlap region are not experimentally useful and may in fact contribute to interfering background.

We consider the problem in terms of the beam size within the overlap region. We ask: given a beam size, $a$, and an interaction diamond length $2\lambda$, can we choose $a$ and $\beta_n^*$ in an optimum way. The optimum is clearly that we want (1) the smallest crossing angle, since requiring a large one will decrease the available luminosity, and (2) the largest $\beta_n^*$, since $\beta_n^*$ will not appreciably influence the luminosity, but having a large value for $\beta_n^*$ will allow a smaller value for $\beta_{\text{max}}$ along the low $\beta$ insertion length in both vertical and horizontal planes.

Define the beam size, $a$, by

$$a = \frac{1}{\sqrt{\pi}} \frac{r}{\sqrt{\beta_n \varepsilon}} = \frac{r}{\sqrt{\pi \gamma}} \sqrt{\beta_n} \frac{E}{r}$$

where $\varepsilon$ is the horizontal emittance at energy $\gamma$,

$E$ is the normalized emittance,

$\beta_n$ is the value of $\beta$ at some point along the collision path,

and $r$ is a factor which relates the beam size $a$ to the number of rms widths in the beam.

Since the emittance is defined corresponding to $\sqrt{\pi}$ rms widths for a Gaussian beam distribution, $r$ is simply the number of rms widths assumed for the beam.

Our problem consists of determining the distance of the end point of the overlap diamond from the collision point in terms of $a$ and $a$, and setting this expression equal to $\lambda$. We have simply

$$\lambda = \frac{2a}{\alpha}$$
Note that $a$ is a function of $\ell$ since the $\beta$ function is rising from the collision point to the diamond end. Recalling that

$$\beta_n = \beta_n^* + \frac{g^2}{\beta_n^*} \tag{3}$$

(2) can be put in the form

$$\frac{\pi \gamma \alpha^2}{4r^2 E} = \frac{\beta_n^*}{\ell^2} + \frac{1}{\beta_n^*} \tag{4}$$

Thus, for given $\gamma, r, E$, the crossing angle has a minimum at

$$\beta_n^* = \ell \tag{5}$$

Thus, for a given $\ell$, the optimum choice is to fix $\beta_n^*$, and in fact to fix it to $\ell$, the half length of the required overlap diamond size. We write

$$c^2 = \frac{\pi \gamma}{4r^2 E} \tag{6}$$

with (4) becoming

$$c^2 \frac{\alpha^2}{\ell^2} = \frac{\beta_n^*}{\ell^2} + \frac{1}{\beta_n^*} \tag{7}$$

and we sketch (7) in Fig. 2.

![Graph](image)

**Fig. 2** - For a given $\beta_n^*$, the crossing angle required to achieve an interaction diamond of length $2\ell$. 

The minimum value of $\alpha$, from (7), is given by

$$\alpha = \sqrt{\beta_h^* \frac{E}{\pi \gamma}}$$

(8)

The source of this minimum in $\alpha$ can be seen physically in the following way. There are two competing processes, the crossing angle of the two beams tending to separate the beams and the increasing beam size (arising from the rising $\beta$) tending to keep the beams together. For $\beta_h^* > \lambda$, the $\beta$ function is changing slowly enough so that the crossing angle must simply overcome the essentially constant beam size. The larger the $\beta_h^*$, the larger the crossing angle required, as seen in Fig. 2. However, as $\beta_h^*$ decreases below $\lambda$, the required crossing angle does not continue to diminish. This is because for "small" $\beta_h^*$, the size begins to increase so rapidly, i.e. the $\beta$ function is not uniform, but quadratic in length along the interaction diamond, that the required crossing angle again starts to increase. Although not strictly pertinent to this discussion, it is interesting to sketch the overlap diamond length as a function of $\beta_h^*$ for fixed crossing angle. This is done in Fig. 3. Note the minimum interaction length when $\beta_h^* = \lambda$ and the rapid increase of $\lambda$ to infinity as $\beta_h^*$ approaches the critical value

$$\left(\beta_h^*\right)_{\text{critical}} = \frac{1}{c^2} \alpha^2$$

(9)

This corresponds to the condition that the beams (with size $r$ rms widths) cannot separate at all with the given crossing angle $\alpha$.

We therefore have the following procedure. We are given $\gamma$ and $E$. We are given $r$ from the constraint that a given fraction of the nuclear collisions must take place in the interaction diamond of given length $\lambda$. For example, if 95% of the nuclear collisions are to take place in the diamond, then $r = \sqrt{0.95}$, assuming Gaussian beams. We then must determine the values for $\beta_h^*$ and $\alpha$ which satisfy this constraint.
Fig. 3 - Interaction diamond length as a function of $\beta_h^*$. At the minimum, $\beta_h^* = \ell$.

The optimum choice for $\beta_h^*$ and $a$ are given in (5) and (8). If other conditions (i.e. limit of $\beta_{\text{max}}$) necessitate a larger value of $\beta_h^*$, then, in order to satisfy the interaction length constraint, we must choose an $a$ larger than optimum. This can be obtained from (7) given the required $\beta_h^*$.

Let us consider a severe example, where a large fraction (> 99%) of the nuclear interactions must be contained in the diamond of length $2\ell$.

Take $r = 4$ rms widths. For an interaction length, $\ell = 0.5 \text{ m}$, and with $E = 30 \times 10^{-6} \text{ rad m}, \gamma = 400$, we obtain for the optimum values of $\beta_h^*$ and $a$, $\beta_h^* = 0.5 \text{ m}$, and $a = 2.47 \text{ mrad}$. For lower energy, since the optimum $a$ goes like $1/\sqrt{\gamma}$, we have for $\gamma = 100$, $a = 4.94 \text{ mrad}$.

However, if $\beta_h^* = 0.5 \text{ m}$ cannot be achieved, then $a$ must be increased. For example, if we take $\beta_h^* = 5 \text{ m}$, which is roughly what is presently
contemplated for low $\beta$ insertions, the $\alpha$ required at $\gamma = 400$, from (7), is $\alpha = 5.56$ mrad. At $\gamma = 100$, the required $\alpha$ is, $\alpha = 11.12$ mrad. Since $\alpha$ is proportional to $r$, these required crossing angles can be somewhat relaxed. For example, if we maintain $\beta_h^* = 5.0$ m, and take $r = 2$, corresponding to 95% beam containment in a 1 m diamond, the crossing angle required at $\gamma = 400$ is $\alpha = 2.78$ mrad, while at $\gamma = 100$, $\alpha$ must be, $\alpha = 5.56$ mrad. For non-Gaussian beams, these results must be appropriately modified.

It thus appears that the overlap diamond constraint could be luminosity limiting in low $\beta$ collisions, and should therefore be a consideration in the design performance goals.
ADDENDUM

Optimization of the Overlap Length for Collisions in Low-β Insertions

Instead of using the overlap length as a measure of luminosity containment, we can compute the per cent containment directly from expressions for the luminosity. The useful luminosity is given by \(2\)

\[
L = L(\epsilon) = \frac{\pi^2 \gamma}{2e^2 \sqrt{c E_h E_v} \beta_h^* \beta_v^*} \int_{-\infty}^{\infty} \frac{\tau \, ds}{(1 + \frac{s^2}{\beta_h^*})(1 + \frac{s^2}{\beta_v^*})} e^{-\frac{\pi \alpha^2 s^2}{4E_h \beta_h^*(1+\epsilon^2/\beta_h^*)}}
\]  
(10)

and the per cent containment

\[
P = \frac{L(\epsilon)}{L(\infty)}
\]  
(11)

where \(L(\infty)\) is the same as \(L(\epsilon)\) except integrated to infinity, I is the average current in each beam, and the other symbols are as previously defined. The subscripts h and v stand for horizontal and vertical respectively. \((11)\) can be solved for \(\alpha(\beta_h^*)\) with fixed P. The result, together with a plot of the corresponding luminosity, is given in Fig. 4. It can be seen that the results with luminosity containment constraint agree qualitatively with those for the beam containment calculations.

Acknowledgement

Much thanks to Mrs. Y. Marti for performing the necessary numerical computations.

Reference

2) E. Keil; CERN/ISR-TH/72-33 (1972).
Fig. 4 - Xing angle and luminosity versus $\beta_h^*$ for different percent luminosity containment in a fixed overlap length. $(2\alpha) = 1.0 \text{ m}, \beta_v^* = 1.3 \text{ m}, E_h = E_v = 30 \times 10^{-6} \text{ rad.m}, \gamma = 400$. $L$ is evaluated over the useful overlap length $(2\alpha)$. 
APPENDIX 5

VARIABLE CROSSING-ANGLE AND UNEQUAL-ENERGY OPERATION
OF THE LSR LOW-BETA INSERTION

B.W. Montague and B. Zotter

1. In order to adjust the crossing-angle in the low-beta insertion with common bending magnets, steering magnets can be placed in each of the four arms of the insertion. If we accept a small tilt of the beams at the intersection, the steering magnets can also be used for unequal energy operation.

2. The formalism for the geometry shown in Figure 1 has been derived by one of the authors (BWM) for a low-beta, small-angle insertion in the ISR.1

---

Figure 1
For a momentum ratio $\rho = \frac{p_1}{p_2}$ and a desired crossing angle $2\phi$ we get

$$\theta_1 = \frac{\alpha_0 k_0}{k_0 + k_1} \frac{\rho - 1}{\rho + 1} + \frac{2\phi}{1 + \rho} \frac{k_0}{k_1}$$

$$\theta_2 = 2\phi \frac{k_0}{k_1} - \theta_1$$

At equal energies in both rings ($\rho = 1$) these expressions simplify to

$$\theta_1 = \theta_2 = \phi \frac{k_0}{k_1}$$

For unequal energies, but zero crossing angle ($\phi = 0$), we get on the other hand

$$\theta_1 = -\theta_2 = \frac{\alpha_0 k_0}{k_0 + k_1} \frac{\rho - 1}{\rho + 1}$$

3. As a numerical example, we take the following parameters for the LSR:

$$k_0 = 15.5 \text{ m}$$
$$k_1 = 32.5 \text{ m}$$
$$\alpha_0 = 20 \text{ mrad}$$

For $\rho = 1$ we find $\theta_1 = \theta_2 = 0.477\phi$. For a crossing angle of $4 \text{ mrad}$ ($= 2\phi$), the steering magnets need to provide less than $1 \text{ mrad}$, and the transverse displacement of the beams is about $\pm 3 \text{ cm}$.

4. For unequal energies, but zero crossing angle, we find with $p_1 = 400 \text{ GeV/c}$ the following values:

<table>
<thead>
<tr>
<th>$p_2$ (GeV/c)</th>
<th>$\rho$</th>
<th>$\theta_1 = -\theta_2$ (mrad)</th>
<th>$l_{\text{trans}}$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>1.33</td>
<td>1.0</td>
<td>3.0</td>
</tr>
<tr>
<td>200</td>
<td>2.00</td>
<td>2.2</td>
<td>7.2</td>
</tr>
<tr>
<td>150</td>
<td>2.67</td>
<td>3.0</td>
<td>9.8</td>
</tr>
<tr>
<td>100</td>
<td>4.00</td>
<td>3.9</td>
<td>12.7</td>
</tr>
</tbody>
</table>
We see that the required displacement is much larger — especially if we add the requirement for finite crossing angle. The separating magnet would normally require a gap of some 30 cm width to accommodate the separating beams. This requirement will be approximately doubled if we foresee lower energy beams in either ring. The preceding quadrupole doublet would have to be moved transversely — and with high precision — because of the high beta-values there. Both these requirements are somewhat restrictive, but could probably be met.

Reference

1) B.W. Montague, B. Zetter; CERN/ISR-TH/73-47.
LOW-BETA INSERTIONS WITHOUT COMMON ELEMENTS

L. Teng and B. Zotter

1. Our previous designs for LSR had common separating magnets in the low-beta insertion in order to reach the small crossing angles of less than one milliradian required for optimum luminosity. Although a scheme has been studied that permits operation at unequal energies by horizontal steering magnets, this has the disadvantage that the first quadrupoles — where both beta functions are large — have to be moved radially. In addition, it appears that most experiments will require shorter diamond lengths, and cannot make use of the luminosity due to particles colliding outside about ±0.5 m — actually, these collisions are undesirable background.

2. There are two obvious remedies to this situation: the first is to reduce the horizontal beam size by making $\beta_n$ smaller at the intersection. This will lead to an increase of $\beta_{max}$, or — if we keep the maximum values of the betatron function fixed — to a decrease of luminosity.

   The other possibility is to increase the crossing angle. This leads again to a decrease in the luminosity, unless $\beta_{vo}$ is also reduced, or a bigger current and/or smaller emittance are assumed. However, with a larger crossing angle we can use septum magnets instead of a single separating magnet, thereby easing operation at unequal energies, and even use the intersection for proton-antiproton collisions.

3. The septum magnet would have approximately the shape shown in Figure 1, where a septum thickness of about 20 mm would permit operation of the 100 × 400 GeV/c, if we assume a gap height of some 6 cm, and a current density in the septum of 6000 A/cm² (including cooling channels). The main field would then be set to 1.1 T, while the septum provides about 0.7 T which adds on one side, and subtracts on the other. However, this is not sufficient for pp operation at full energy, which requires ±1.8 T on the two sides of the gap.
4. Since the beam separation continues to grow, we can have thicker septa a few metres further, and increase the capability of handling also antiprotons. This will mean only a slight increase in the total magnet length.

For a crossing angle of 4 mrad, and 10 m distance from the intersection to the first septum magnet, we obtain a half-width of 10 mm for the vacuum chamber. This appears sufficient, as the beam halfwidth is only about 2 mm for $\theta_h = 5$ m at the intersection, and $\alpha' = 20$ mrad. After 4 m, the beam separation has grown to 64 mm (assuming 2 mrad bend for 0.7 T maximum) and the septum thickness can be doubled. After another 4 m, the septum can be large enough to carry the field to 1.8 T on both sides, of which another 10 m are needed if we want the same separation angle of 19.2 mrad as before. This does not appear necessary, however, as the beam separation is then 40 cm at the exit, and quadrupoles could be as thin as 30 cm (requiring about 6 m of final magnet).

5. A completely different approach would be to use ultra-thin quadrupoles of the Panofsky type before the bending magnet starts. This might be the best way to limit the maxima of the beta-functions in both planes, but could be rather expensive in power consumption. It may be of interest to find out whether a superconducting version of these quadrupoles has ever been tried. Further study of this solution seems to be indicated.
6. In conclusion, we summarize that septum magnets appear to be a possible solution to the problem of operating the LSR at unequal energies, if we have to increase the crossing angle in order to decrease the diamond length. However, the septa may be very vulnerable to accidental damage by the proton beam, and the alternate solution of the problem with ultra-thin quadrupoles may be more attractive.

References

1) R. Chasman et al; Preliminary Study of a CERN 400 GeV Storage Ring Facility. CERN/ISR-GS/TH/74-45.

2) L. Teng, E. Zotter; Low-beta insertions without common elements (this study).

3) A. Carren; private communication.

4) B. Montague, E. Zotter; Variable crossing-angle and unequal-energy operation of the LSR low-beta insertion (this study).
A MODIFIED LOW-BETA INSERTION FOR OPERATION OF THE LSR
AT UNEQUAL ENERGIES

A. Garren

It would be desirable for some experiments to be able to take measurements with different energy values of one beam compared to the other. To this end some modification of the low-beta insertion previously suggested 1) is required, since there the two beams pass through common dipole magnets.

One possibility, referring to Fig. 2 of Ref. 1 is to mount the quadrupole triplet of one beam between the two dipoles on a moveable arm, and to follow them with additional dipoles to bend the beam back to the proper displacement and angle. The moveable elements are the most objectionable feature of this solution. It is doubtful that such motion could be done while the beam is circulating during possible deceleration, and even if only done between successive runs, a time-consuming realignment might be needed. The chief merit of this approach is that the path length distance from the crossing point to the nearest quadrupole is not increased. This is important in order not to aggravate the difficult problem of chromaticity correction.

An alternative solution without moveable elements is proposed in this note, involving the use of septum magnets. This scheme in its turn has drawbacks and advantages.

The idea is illustrated in the figure. B1 is a common dipole; the following three dipoles are separate magnets controlling the two beams independently. The solid lines represent both beams at 400 GeV, while the dotted line below represents the inner beam at 100 GeV colliding with the outer beam at 400 GeV. In the example all the dipoles are 6 m long, and the fields are as follows:

<table>
<thead>
<tr>
<th></th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>B5</th>
<th>B6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer beam, 400 GeV</td>
<td>1.8</td>
<td>0.9</td>
<td>1.8</td>
<td>-0.9</td>
<td>-1.8</td>
<td>-1.8</td>
</tr>
<tr>
<td>Inner beam, 100 GeV</td>
<td>1.8</td>
<td>-0.9</td>
<td>-1.8</td>
<td>1.8</td>
<td>-0.45</td>
<td>-0.45</td>
</tr>
</tbody>
</table>
At the entrance to B2 the beam centres are separated by 8 cm*, which should be sufficient to have a coil thickness of 1.5 cm on each B2 magnet along the centre, or a total coil thickness of 3 cm. This leaves 3 cm between the inside edge of the beam and the coil. Unfortunately each B2 magnet would have to carry about 50 kA of current, so considerable power is required. Some iron can probably be introduced in B3 and B4 to carry some of the flux beside the centre coils, so they may be less formidable.

The distance from the crossing point to the nearest quadrupole is 34 m, with 8 m to the common dipole, and the overall insertion half-length is 130 m. The corresponding distances in Ref. 1 are respectively 31 m, 10 m and 125 m. Hence, apart from the power in the septum magnets, there seems to be no qualitative drawback in this scheme compared to the original design of Ref. 1. The use of the method suggested here may be even more attractive in a superconducting LSR.

The writer wishes to thank B.W. Zotter for his helpful suggestions on the design and J.-C. Schnuriger for making the above estimates concerning the septum magnets.

Reference


* For zero crossing angle the separation is 8 cm. The latest parameters include a 2.4 mrad angle, which increases the separation to 12 cm.
In a storage ring lattice with normal cells and insertions the insertions are matched only for one given momentum. For an off-momentum orbit the mismatch caused by the chromatic aberration of the quadrupoles produces a gradient "error" with the insertion periodicity, which can be as low as 1. This gradient "error" opens up 1/2-integer stopbands and limits the momentum aperture of the lattice. In addition, the chromaticity of the lattice must be controlled to adjust properly the working line in the betatron tune-diagram. For both functions, sextupole fields must (in effect) be added to quadrupoles to modify their chromatic aberration by virtue of the orbit dispersion. Clearly the sextupoles should be arranged in such a way that 1/3-integer resonances are not excited.

An obvious way of accomplishing this is to compensate the chromatic aberration of each quadrupole by adding to it a sextupole field given by

\[ B'' = \frac{B'}{\alpha_p} \]

where \( \alpha_p \) is the dispersion function. However, it is usually desirable to have \( \alpha_p = 0 \) in the insertion quadrupoles, thereby making this scheme of direct compensation unworkable. We propose here a simple arrangement of sextupoles in only the normal cell sections of the lattice. This serves the same purposes and satisfies all the criteria provided the betatron phase advances are \( \frac{\pi}{2} \) per cell in both planes.

1. Chromaticity control

We approximate the quadrupoles and sextupoles by lumped elements.

For chromaticity control we need to introduce
\[ A = \sum_n \beta_n \alpha_n S_n \text{ where } S = \frac{B_n}{B_0} = \text{sextupole strength} \]

in such a manner that

\[ B = \sum_n \beta_n \alpha_n S_n e^{2i\phi_n} = 0 \text{ where } \phi = \text{betatron phase} \]

(so that the off-momentum \( \frac{1}{2} \)-integer stopbands are not affected) and

\[ C = \sum_n \beta_n^2 \alpha_n S_n e^{3i\phi_n} = 0 \]

(so that the \( \frac{1}{4} \)-integer resonances are not excited). There is a total of six such relations, three each for the horizontal and the vertical planes. (S has opposite signs in the two planes.) For FODO normal cells with \( \frac{\pi}{2} \) phase advances in both planes a simple arrangement is to place sextupoles of equal strength \( S_F \) at groups of four successive F quadrupoles and sextupoles of strength \( S_D \) at the associated D quadrupoles. For each group of four cells, we have

\[ A_H = 4(\tilde{\beta}_p S_F + \tilde{\beta}_y S_D) \]
\[ A_V = 4(\tilde{\beta}_p S_F + \tilde{\beta}_y S_D) \]

where \( \tilde{\beta} \) and \( \tilde{\gamma} \) denote respectively the maximum and the minimum values in the normal cell. Of course, \( \tilde{\beta}_h \) and \( \tilde{\beta}_v \) need not be exactly the same, nor need \( \tilde{\gamma}_h \) and \( \tilde{\gamma}_v \) be the same: this approximation only simplifies the equations without affecting the validity of the conclusions.

To first order in \( \frac{\Delta p}{p} \), \( A_H \) and \( A_V \) are used to adjust the chromaticity in the two planes. For either a group of four F-sextupoles or D-sextupoles we have
\[ B = \beta \alpha_p S e^{2i\phi} (1 + e^{i\pi} + e^{2i\pi} + e^{3i\pi}) = 0 \]
\[ C = \beta^2 e^{3i\phi} (1 + e^{i\frac{3\pi}{2}} + e^{i\frac{5\pi}{2}} + e^{i\frac{9\pi}{2}}) = 0 \]

Thus, one can vary \( S_F \) and \( S_D \) to adjust the chromaticity in the horizontal and the vertical planes without affecting the off-momentum \( \frac{1}{3} \)-integer stopbands or exciting the \( \frac{1}{3} \)-integer resonances.

2. **Closing the off-momentum \( \frac{1}{3} \)-integer stopbands**

For this we want to introduce \( B \) in such a manner that \( A = C = 0 \) (so that the chromaticity is not affected and \( \frac{1}{3} \)-integer resonances are not excited). To do this, the simplest arrangement is to put sextupoles of strengths \( +S_F, -S_F, +S_F, -S_F \) at the \( F \) quadrupoles and a similar set of strengths \( \pm S_D \) at the \( D \) quadrupoles. Then for each group of four cells

\[ B_H = (\hat{\delta} \alpha_p S_F + \hat{\delta} \alpha_p S_D) e^{2i\phi} (1 - e^{i\pi} + e^{2i\pi} - e^{3i\pi}) \]
\[ = 4(\hat{\delta} \alpha_p S_F + \hat{\delta} \alpha_p S_D) e^{2i\phi} \]
\[ B_V = -4(\hat{\delta} \alpha_p S_F + \hat{\delta} \alpha_p S_D) e^{i(2\phi + \frac{\pi}{2})} \]

and for either a group of four \( F \)-sextupoles or \( D \)-sextupoles

\[ A = 8 \alpha_p S (1 - 1 + 1 - 1) = 0 \]
\[ C = \beta^{3/2} e^{3i\phi} (1 - e^{i\frac{3\pi}{2}} + e^{i\frac{6\pi}{2}} - e^{i\frac{9\pi}{2}}) = 0 \]

To compensate totally the off-momentum gradient "error" we need another \( B \) term with a different phase. In the racetrack lattice, one possibility is to use the sextupole strings in the two curved sections at both ends of the racetrack. Presumably the phase advances in the straight sections are generally not integer multiples of \( \frac{\pi}{2} \), hence the phase difference between the two strings will not be integer multiples of \( \pi \).
The two strings can thus be tuned to give any phase and amplitude to compensate for the off-momentum gradient "error". With each change of insertions in the straight sections the four strings of sextupoles must be retuned to compensate for the changed off-momentum gradient "error".

3. Several features of this scheme should be mentioned

(a) The chromaticity control sextupoles and the off-momentum gradient "error" compensating sextupoles were discussed separately for clarity. In practice, they should, of course, be combined and their strengths set to the superposed values.

(b) To minimize the required strength of individual sextupoles one should occupy as many groups of four successive normal cells as is available.

(c) With sextupoles located only in the normal cells we can have zero dispersion ($\alpha_p = \alpha_p' = 0$) in the insertions as desired.

(d) In general, for a group of four F-sextupoles or D-sextupoles we have

$$\sum \beta_n^k \alpha_p \phi_n e^{i(p\phi_n + q\phi_n)} = 0$$

for

$$p + q = 1, 2, 3, 5, 6, 7, \ldots \; (+++ \text{ series})$$
$$p + q = 0, 1, 3, 4, 5, \ldots \ldots \; (+-- \text{ series}),$$

and whatever $k$ and $l$. Hence this scheme avoids affecting or exciting many error, coupling, and higher order effects or resonances which we shall not identify and enumerate here.

(e) The scheme proposed depends on having $\pi/2$ phase advance per normal cell. This means that tune adjusting insertions should be installed in the straight sections to adjust the tunes without changing the phase advance in normal cells. Every time the crossing insertions are changed the tune-adjusting insertions must be retuned as well as the various
strings of sextupoles in the normal-cell sections of the lattice.

The possibility that the $\frac{5}{2}$ normal-cell phase advance may be useful for chromaticity adjustment without $\frac{1}{3}$-integer resonance excitation was suggested to one of the authors (LCT) by Dr S. Ohnuma.

Reference

REPORT OF STUDY GROUP ON PHYSICS WITH PROTON STORAGE RINGS
IN THE REGION OF SEVERAL HUNDRED GEV

(Introduction by L. Di Lella, chairman)

A study group to review the physics interest of proton storage rings in the region of several hundred GeV, was set up in March, 1974, with the purpose to help the Study Group on Long Term Plans in considering possible future long range developments of experimental facilities at CERN.


Most of the subjects of current interest in the physics with colliding proton beams received attention. These include elastic scattering, measurements of the total cross-section, multiple production and production of high transverse momentum leptons and hadrons. In all these cases, the feasibility of the relevant experiments was studied by attempting a practical design of an experimental set-up.

Since the experiments are closely connected with the machine in the case of storage rings, it was necessary for most of the cases to take into account a list of storage rings parameters. A machine consisting of two 400 GeV proton rings, with a maximum luminosity of $10^{33}$ cm$^{-2}$ s$^{-1}$ was used in this study. Such a machine corresponds to one of the models which are at present under study in the ISR Department, and its preliminary specifications are described in Report 1. It was necessary to work in close contact with the machine experts, in order to try to resolve the sometimes conflicting requirements between machine design and physics experiments, and between physics experiments themselves.
The problems related to the measurement of the machine luminosity were also studied (see reports 3 and 4). It was found that the luminosity can be determined with a precision comparable to that achieved at the present ISR.

Reports 5 through 9 contain the results of the studies performed on the physics subjects mentioned above. However, it is worthwhile to stress here the main conclusions reached so far:

1) Proton storage rings with a total centre-of-mass energy of $\sim$ 800 GeV and a luminosity around $10^{33}$ cm$^{-2}$ s$^{-1}$ are particularly well suited for the study of electromagnetic and weak interactions, by observing the production of leptons and lepton pairs. Cross-sections for the reactions pp + $\mu^+\mu^-$ + anything, and pp + $\nu\bar{\nu}$ + anything can be conservatively estimated using current theoretical models (see report 5), and a relatively simple experimental apparatus is discussed in report 6. Rates as high as $\sim$ 2 muons/hour are expected from the reaction pp + $\mu\bar{\nu}$ + anything, with the invariant mass of the $\nu\bar{\nu}$ system in excess of 300 GeV, in the presence of very low background. At this value, the weak interaction cross-section reaches the unitarity limit, whose effects could therefore be studied. On the other hand, if a $W^\pm$ boson exists, much higher single muon rates would result, at muon momenta around 0.5 $M_W$. As an example, for $M_W = 100$ GeV/c$^2$, approximately 200 muons/hour would be detected in the momentum interval between 40 and 60 GeV/c. These events would create a bump on the continuum, with a signal to noise ratio of $\sim$ 100 to 1. A fundamental discovery in this field is, therefore, almost unavoidable.

2) The yield of high transverse momentum hadrons was estimated by a reasonable extrapolation of ISR results (see report 7). At luminosities of $10^{33}$ cm$^{-2}$ s$^{-1}$, secondary pions with $p_T$ above 40 GeV/c can be observed at a rate of $\sim$ 1/hour. The study of these events, under conditions almost free from kinematics constraints (since $p_T \ll \sqrt{s}/2$), should help to understand the mechanism responsible for this type of collisions. In particular, it may become possible to answer the question whether these events result from scattering of hard, point-like constituents of the protons.
3) Measurements of the pp total cross-section appear feasible, with precisions comparable to those achieved at the ISR (see report 8). In view of the huge range of $\sqrt{s}$ values available, it becomes possible to obtain a precise determination of the energy dependence of $\sigma_{\text{TOT}}$. Fits to the ISR data predict values of $\sigma_{\text{TOT}}$ between 65 and 75 mb at $\sqrt{s} = 800$ GeV.

4) Particle production will benefit greatly from the range of $\sqrt{s}$ values offered by the machine, and from the large interval of rapidity ($y \approx \pm 3$ at $\sqrt{s} = 800$ GeV). The study of diffractive dissociation at $\sqrt{s} = 800$ GeV is particularly interesting, since states of very high mass (up to $\sim 200$ GeV/$c^2$) can be coherently excited in the collisions. Experiments to study these phenomena appear quite feasible (see report 9).

In addition to the topics listed above, some members of the study group, in collaboration with U. Amaldi and A. Minten, studied the experimental possibilities of pp colliding beams at present ISR energies. The methods to fill one of the two ISR rings with $\bar{p}$ from the SPS, as well as estimates of the luminosity are contained in report 10. A discussion of the physics programme which could be carried out with this facility, is contained in report 11. The main conclusion here is that, because of the low luminosity foreseen ($L \leq 10^{26}$ cm$^{-2}$ s$^{-1}$), only studies of pp interactions with cross-sections larger than a few percent of the total cross-section can be performed.
List of Documents for the Study


2) D.A. Swenson; Geometries for a Superconducting Storage Ring in the ISR Tunnel, CERN/ISR-GS/74-35.

3) H.G. Hereward and E. Keil; Measurement of the Luminosity for Beams Crossing at Small Angles in the Low-β Section, CERN/ISR-DI/TH/74-41.

4) G. Matthiae; Measurement of the Luminosity at the Super-ISyR.

5) N. Cabibbo; Production of Leptons at Very High Energies.

6) L. Camilleri; Lepton Production at the SISR.

7) L. Di Lella; Particle Production at High Transverse Momenta.

8) A.N. Diddens; Elastic Scattering and Total Cross Sections in a 400 + 400 GeV Proton Storage Ring.

9) P. Darriulat and J.C. Sens; Multiple Production Studies at PRINCE*

10) K. Hübner; The Storage of Antiprotons in the ISR, CERN/ISR-TH/73-19.

11) U. Amaldi, et al.; Study of Experimental Possibilities of \( \bar{p}p \) Colliding Beams.

*) PRINCE stands for Proton Rings Intersecting at New CERN Energies