Higher-order QCD corrections in hadron collisions: soft-gluon resummation and exponentiation

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Abstract

I briefly review some recent results on soft-gluon resummation and present applications to heavy-quark and jet production in hadron collisions.

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1 INTRODUCTION

Detailed studies on strong-interaction physics and the evaluation of the background for new-physics signals at high-energy colliders require accurate calculations in perturbative QCD. In this talk I shall describe some recent progress [1, 2] in the study of a particular class of higher-order QCD contributions to hadronic collisions. This class concerns multiple emission of soft gluons.

Some basic features of soft-gluon resummation are recalled in Sects. 2.3. These Sections provide a brief introduction to the theoretical analysis of Ref. [1], which is summarised in Sect. 4. Phenomenological implications are reviewed in Sect. 5 and some comments are left to Sect. 6.

2 SOFT-GLUON EFFECTS

The finite energy resolution of any particle detector implies that physical cross sections are always inclusive over arbitrarily-soft particles produced in the final state. This inclusiveness is essential in QCD calculations. Higher-order perturbative contributions due to virtual gluons are infrared divergent: the divergences are exactly cancelled by radiation of undetected real gluons.

More precisely, if \( 1 - z \) denotes the fraction of the centre-of-mass energy \( \sqrt{S} \) carried by unobserved final-state particles, virtual \((v)\) and real \((r)\) soft gluons affect the cross section by the following emission probabilities

\[
\frac{dw_\nu(z)}{dz} = -2a \delta(1-z) \int_0^{1-z} \frac{dz'}{1-z'} \ln \frac{1}{1-z'} ,
\]

\[
\frac{dw_r(z)}{dz} = 2a \frac{1}{1-z} \ln \frac{1}{1-z} \Theta(1-z-\epsilon) ,
\]

where \( a = \alpha_s / \pi \) and the coefficient \( C \) depends on the process. These double-logarithmic (DL) expressions arise from the combination of the customary bremsstrahlung spectrum \( d\omega / \omega \) with the spectrum \( d\theta^2 / \theta^2 \) for collinear radiation. Here I have introduced an unphysical cutoff \( \epsilon \) on the minimum energy fraction of both gluons. Adding the real and virtual terms, the physical limit \( \epsilon \to 0 \) can be safely performed, leading to a finite differential probability:

\[
\frac{dw(z)}{dz} = 2a \left[ \frac{1}{1-z} \ln \frac{1}{1-z} \right]_+ ,
\]

where, as usual, the notation \( [g(z)]_+ \) stands for \( \int_\epsilon^{1} dz f(z) [g(z)]_+ \equiv \int_\epsilon^{1} dz [f(z) - f(1)]g(z) \), which defines a well-behaved distribution acting on any smooth function \( f(z) \) (such as cross sections or parton densities) at \( z = 1 \).

Note that the virtual term contributes only at \( z = 1 \) while the real one, besides regularizing the virtual probability at \( z = 1 \), is spread out up to the kinematical boundary as given by the energy fraction \( x \) of the tagged final state. Thus the total soft-gluon contribution to the cross section is proportional to the following quantity

\[
\int_x^{1} dz \frac{dw(z)}{dz} = -a \ln^2(1-x) .
\]

This perturbative correction is the finite heritage of the cancellation of the infrared singularity.

If we force the tagged final state to carry most of the total energy (i.e. we consider the quasi-elastic limit \( x \to 1 \)), we strongly suppress the radiative tail of the real emission. Thus the virtual term is unbalanced and the double-logarithmic factor in Eq. (3) can become large, \( \alpha_s \ln^2(1-x) \gtrsim 1 \), even if the coupling constant is in the perturbative region \( \alpha_s \ll 1 \). In this kinematic regime, multiple soft-gluon radiation produces higher-order contributions of the type

\[
C_m \alpha_s^n \ln^m(1-x) , \ m \leq 2n ,
\]

that have to be resummed in order to obtain reliable theoretical predictions.

In the case of hadron collisions, this large-\( z \) region is encountered in the production of systems of high mass \( M^2 \) near threshold. Outstanding examples of these systems are the hadronic final state in deep-inelastic lepton-hadron scattering (DIS) (here \( x = x_B \) is the Bjorken variable), lepton pairs with large
invariant mass $Q^2$ produced via the Drell-Yan (DY) mechanism ($x \equiv \tau = Q^2/S$) [3], heavy quark-antiquark pairs ($x \equiv \rho = 4m_Q^2/S$) [4] and pairs of jets at large transverse momentum ($x \equiv \tau = M_{jj}^2/S$) [1].

Similar soft-gluon effects occur both in hadron and lepton collisions when one measures detailed properties ($Q_\perp$-distributions [5], event shapes [6], multijet rates [7]) of the hadronic final state.

Although the logarithmically-enhanced contributions in Eqs. (3,4) are expected to be very relevant when $x \to 1$, the actual size of the soft-gluon corrections in cross section calculations depends on the coefficients $C_{nm}$ and on the $x$-shape of the parton densities. Thus soft-gluon effects can be substantial also before reaching this kinematic region. This is the main motivation in Refs. [8, 9] for evaluating soft-gluon effects for top quark production at Tevatron [10, 11, 12] where the inelasticity variable $\rho = 4m_T^2/S$ is as small as $\rho = 0.04$.

3 RESUMMATION AND EXPONENTIATION

In order to discuss soft-gluon resummation in hadron collisions, let me consider the DY process, which is the most studied process (see [3] and references therein) so far. The DY cross section $\sigma(Q^2, \tau)$ is obtained by convoluting the parton densities $f(x_i, Q^2)$ of the colliding hadrons with a partonic cross section $\hat{\sigma}(Q^2, z)$ as follows

$$\sigma(Q^2, \tau) = \int_0^1 dz \int_0^1 dx_1 dx_2 \ f(x_1, Q^2) f(x_2, Q^2) \ \delta(x_1 x_2 z - \tau) \ \Delta(z, \alpha_s(Q^2)) \sigma_0(Q^2).$$

(5)

Here, I have omitted all parton indices and factorized the partonic cross section in the Born-level contribution $\sigma_0$ times the term $\Delta$ that takes into account all the radiative corrections. The latter is computable in QCD perturbation theory as a series in $\alpha_s$. In Eq. (5) I have also explicitly introduced energy conservation: the lepton pair carries the fraction $z = Q^2/(x_1 x_2 S)$ of the centre-of-mass energy squared $x_1 x_2 S$ available in the partonic subprocess.

If the lepton pair is produced close to the hadronic threshold ($\tau \to 1$), energy conservation forces the partonic subprocess towards $z = 1$ (as well as the parton densities towards $x_i = 1$) and the radiative corrections in $\Delta(z, \alpha_s)$ are dominated at lowest order by the soft-gluon probability in Eq. (2). The resummation of higher-order soft-gluon effects is conveniently carried out in $N$-moment space. One can introduce the the $N$-moments $\sigma_N(Q^2)$ of the cross section in Eq. (5) (and likewise for any other function of longitudinal-momentum fractions) by performing a Mellin transformation at fixed $Q^2$:

$$\sigma_N(Q^2) \equiv \int_0^1 d\tau \, \tau^{N-1} \, \sigma(Q^2, \tau).$$

(6)

Working in $N$-moment space, any $z$-distribution is replaced by a function of $N$ and, due to the weight factor $\tau^{N-1}$, the threshold region is sampled by the limit $N \to \infty$. However, the main reason for using $N$-moments is not the replacement of distributions by functions, but rather the fact that one can easily implement momentum conservation. For instance, the energy-conservation constraint in Eq. (5) is exactly diagonalized and the $N$-moments of the cross section have the following factorized expression

$$\sigma_N(Q^2) = f_N(Q^2) f_N(Q^2) \Delta_N(\alpha_s(Q^2)) \sigma_0(Q^2).$$

(7)

This kinematical factorization plays an essential role in the resummation programme. The radiative factor $\Delta(z, \alpha_s)$ is obtained as follows

$$\Delta(z, \alpha_s) = \delta(1-z) + \sum_{n=1}^{+\infty} \int_0^1 dz_1 \ldots dz_n \ \frac{dw_n(z_1, \ldots, z_n)}{dz_1 \ldots dz_n} \ \Theta_{PS}(z; z_1, \ldots, z_n),$$

(8)

where the probability $dw_n$ for producing $n$ soft gluons is integrated over the phase-space region (symbolically denoted by $\Theta_{PS}$) that is available according to the actual definition of the cross section. The first relevant ingredient to perform the summation in Eq. (8) has a dynamical origin. Owing to the factorization properties of multi-gluon QCD amplitudes in the soft limit, the $n$-gluon probability factorizes in the product of the single-gluon contributions $dw(z_i)$ in Eq. (2):

$$\frac{dw_n(z_1, \ldots, z_n)}{dz_1 \ldots dz_n} \sim \frac{1}{n!} \prod_{i=1}^n \frac{dw(z_i)}{dz_i}.$$
The second relevant information for the summation regards the kinematics. In general, the phase-space function $\Theta_{\text{PS}}$ depends in a non-trivial way on the multi-gluon configuration so that it cannot be handled in a simple manner. However, in the case of total cross sections, like in the DY process, the only relevant constraint is longitudinal-momentum conservation. This constraint is exactly factorizable going to $N$-space:

$$\Theta_{\text{PS}}(z; z_1, \ldots, z_n) = \delta(z - z_1 \ldots z_n) \rightarrow z_1^{N-1} \ldots z_n^{N-1}$$ (10)

Using Eqs. (9) and (10) one can straightforwardly recast Eq. (8) in exponential form:

$$\Delta_N(\alpha_S) = \exp \left[ \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \ln \frac{1}{1-z} \right] .$$ (11)

In summary, the physical basis for the exponentiation of soft-gluon contributions are the factorization of both the multi-gluon amplitudes and the phase-space. The first property follows from QCD dynamics and is completely general. The second property depends on the actual definition of the cross section and can be easily violated even in QED [13]. In the case of total cross sections in hadron collisions, because of energy conservation, phase-space factorization can only be achieved by working in $N$-space. There are no physical basis for exponentiation directly in $x$-space.

Inserting Eq. (2) into Eq. (11) and performing the large-$N$ limit, one finds the exponential of a DL expression:

$$\Delta_N(\alpha_S) = \exp \left[ 2a \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \ln \frac{1}{1-z} \right]$$ (12)

$$= \exp \left[ -a \left( \ln^2 N + O(\ln N) \right) \right] .$$ (13)

The derivation of Eq. (13) that I have sketched would be complete in massless QED (neglecting the effect of soft-fermion pairs). The only simplification I have introduced in the QCD case regards the factorization formula (9). Unlike photons, which have no electric charge, gluons carry colour charge and multiple gluon emission is affected by dynamical correlations. Nonetheless, using general properties related to gauge invariance and unitarity, one can prove that, in the DY process and similar total cross sections, a generalized exponentiation theorem is valid in the following form [3]

$$\ln \Delta_N(\alpha_S) = \ln N \ g_1(\alpha_S \ln N) + \ g_2(\alpha_S \ln N) + \alpha_S g_3(\alpha_S \ln N) + \ldots .$$ (14)

Since the functions $g_i$ depend only on $\alpha_S \ln N$, in this context exponentiation means that all the logarithmic corrections of the type $\alpha_S^n \ln^m N$ with $n + 1 < m \leq 2n$, allowed by Eq. (4), are simply taken into account by the exponentiation of lowest-order terms and thus cancel in the exponent of $\Delta_N$. Owing to the general structure of Eq. (14), one can control the resummation of the logarithmically-enhanced terms by introducing an improved perturbative expansion. The function $g_1$ gives the leading logarithmic (LL) contributions $\alpha_S^n \ln^{n+1} N$, $g_2$ contains the next-to-leading logarithmic (NLL) terms $\alpha_S^n \ln^n N$, etc.

The functions $g_i$ essentially represent the QCD correction to the factorization in Eq. (9). Most of the correction is simply taken into account by introducing running coupling effects. Roughly speaking, this amounts to the replacement $a = C\alpha_S(Q^2)/\pi \rightarrow C\alpha_S[(1 - z)Q^2]/\pi$ at the integrand level in Eq. (12). Thus, because of the logarithmic behaviour $\alpha_S(\mu^2) \sim 1/\ln(\mu^2/L^2)$, integrating exactly in $z$ up to $z = 1$ one eventually hits the Landau pole at $z = 1 - \Lambda^2/Q^2$. This singularity, called infrared renormalon [14, 15], signals the onset of non-perturbative phenomena and has to be properly regularized.

4 IMPLEMENTING RESUMMED FORMULAE IN HADRON COLLISIONS: THE MINIMAL PRESCRIPTION

The theoretical analysis in Ref. [1] deals with difficulties that arise when one tries to apply resummed formulae of the type discussed in Sect. 3 to the actual evaluation of physical cross sections.

The first difficulty is due to the fact that these formulae are provided in $N$-space. Cross sections like that in Eq. (5) are not experimentally measurable in the entire kinematic range $0 \leq \tau \leq 1$ and thus theoretical predictions for the $N$-moments cannot directly be compared with data. In any phenomenological application one has to go back to $x$-space and compute the radiative factor $\Delta(z, \alpha_S)$.
The regularization of the running coupling singularity noticed at the end of Sec. 3 leads to the second difficulty. One has to introduce a regularization prescription that does not spoil the main perturbative features.

As discussed in Ref. [1], several methods used in the literature to solve these difficulties are theoretically unjustified and, typically, enhance soft-gluon effects long before the hadronic-threshold region is actually approached.

In particular, we pointed out the differences between the above difficulties and we warned of the danger of exploiting the formal equivalence $N \sim 1/(1 - x)$ to straightforwardly transform $N$-space resummed formulae into $x$-space resummed formulae. For instance, following Refs. [4, 8, 9], one would replace the DL expression (13) with the $x$-space version:

$$\Delta(z, \alpha_s) \simeq - \frac{d}{dz} \left[ \Theta(1 - z - \epsilon) e^{-a \ln^2(1 - z)} (1 + \ldots) \right],$$

(15)

where the dots stand for ‘subleading’ logarithmic terms, i.e. terms at most of the order of $\alpha_s^2 \ln^n(1 - z)$. Expanding Eq. (15) as a power series in $\alpha_s$ and identifying $\epsilon$ with the unphysical cutoff in Eq. (1), one obtains the lowest-order probability in Eq. (2). Equation (15) is formally valid [6] in terms of the logarithmic expansion in $\ln(1 - z)$, but the identification $\ln N \to \ln 1/(1 - z)$ and the ensuing approximation of neglecting subleading logs is correct only if $\alpha_s \ln 1/(1 - z) \lesssim 1$. Care has to be taken before extending this approximation to higher values of $z$.

In particular, it is quite dangerous to insert Eq. (15) into the factorization formula (5), where the $z$-integration range extends up to $z = 1$. In the case of hadron collisions, after having introduced the parton densities $f(x_i, Q^2)$ in customary factorization schemes (like the DIS or $\overline{\text{MS}}$ schemes), the coefficient $C$ in $a = C \alpha_s/\pi$ turns out to be negative. Thus the expression $\exp[-a \ln^2(1 - z)] = \exp[a \ln^2(1 - z)]$ diverges faster than any power of $(1 - z)$ and is not integrable at $z = 1$: in the resummed expression (15) the unphysical cutoff $\epsilon$ can no longer be removed without producing a singularity. Note that such a singularity is not related to the hadronic threshold. The inelasticity variable $\tau$ controls the lower limit of the $z$-integral in Eq. (5): the point $z = 1$ is inside the integration region even asymptotically far ($\tau \to 0$) from threshold.

The origin of this singularity was studied in detail in Ref. [1] (see also [16]) and traced back to the same-sign factorial divergence of the ‘subleading’ terms systematically neglected in the derivation of the $x$-space formula (15) from Eq. (13). No analogous divergence is present in the original $N$-space formula. Soft-gluon exponentiation in $N$-space is related to momentum conservation. Using exponentiation directly in $x$-space, one violates infinitely many times momentum conservation and can build up a (kinematical) divergence.

In Ref. [1] we specified a prescription for the implementation of soft-gluon resummation formulae in the computation of cross sections in hadronic collisions. We used the generalized exponentiation formula in Eq. (14) and performed numerically the inversion from $N$-space to $x$-space without further approximations. As for the regularization of the Landau pole in the running coupling, we pointed out that the functions $g_1(\alpha_s \ln N)$ are well-defined analytic functions of $N$ in the complex plane. The Landau pole only produces a branch point at $N = Q/\Lambda$. In the numerical inversion we used a prescription that leaves this non-perturbative branch cut to the right of the integration contour.

As proved in Ref. [1], with our prescription no factorial divergence is introduced in the resummed expansion. Factorially growing terms related to infrared renormalons are likely to be present in the full perturbative QCD series, but as shown in Ref. [14], their identification and evaluation require an analysis whose accuracy goes beyond that used to prove the exponentiation of logarithmically-enhanced contributions. Since our prescription fulfills the relevant kinematical constraints and does not introduce large corrections that are not justified by the logarithmic expansion, we call it the ‘Minimal Prescription’ (MP). This prescription can be used to extend perturbative QCD calculations towards the threshold region by consistently taking into account logarithmically-enhanced soft-gluon effects.

5 PHENOMENOLOGICAL STUDIES

In Ref. [1], the MP was applied to the DY process and to the production of heavy quarks and high-$p_T$ jets in hadron collisions. Since the last two processes are becoming increasingly topical, in this Section I briefly review our results. In both cases, in the resummation of soft-gluon corrections we included only
the LL contributions (i.e. the analogue of the function \( g_1 \) in Eq. (14)). In the case of heavy-quarks, this corresponds (roughly speaking) to the \( N \)-space version of the resummation considered in Ref. [4].

### 5.1 Heavy-quark production

The importance of the resummation effects for top quark production in \( p\bar{p} \)-collisions was studied by computing the following quantities

\[
\frac{\delta_{gg}}{\sigma_{NLO}}^{(gg)}, \quad \frac{\delta_{qq}}{\sigma_{NLO}}^{(qq)}, \quad \frac{\delta_{gg} + \delta_{qq}}{\sigma_{NLO}^{(gg)} + \sigma_{NLO}^{(qq)}},
\]

that are plotted in Fig. 1. Here \( \delta_{ab} \) is equal to our MP-resummed hadronic cross section in which the terms of order \( \alpha_S^2 \) and \( \alpha_S^3 \) have been subtracted, and \( \sigma_{(ab)}^{(NLO)} \) is the hadronic cross section in full next-to-leading order [17] (i.e. including both soft and hard radiation up to order \( \alpha_S^3 \)). The labels \( ab \) refer to the various partonic channel that contribute to the cross sections.

At Tevatron energies one can see that the contribution of resummation is very small, being of the order of 1% for \( m_t \approx 175 \text{GeV} \) (in the calculation of Ref. [9] the resummation effect is about 10%, i.e. one order of magnitude higher; similar quantitative effects are found with the method of Refs. [4, 8]). Thus, as expected by the small value of \( \rho = 4m_t^2/S \approx 0.04 \), we concluded that top quark production at Tevatron is reliably estimated by the next-to-leading order (NLO) QCD calculation. Based upon these findings, we updated the computation of Ref. [18]. Full details of our calculation are given in Ref. [2]. The QCD prediction is conveniently parametrized as follows

\[
\sigma_{tt}(1.8 \text{ TeV}) = e^{\frac{\ln m_t}{\ln 1.7}} \times (4.75^{+0.73}_{-0.62}) \text{ pb}.
\]

We evaluated others heavy-quark production cross sections. In general we found [1] that, in most experimental configurations of practical interest, soft-gluon resummation effects are not dominant either because they are very small or because they are well below the (estimated) uncertainty due to higher-order (non-soft) corrections.

### 5.2 Jet production

The interest in the effects of soft-gluon resummation on the behaviour of jet cross sections is prompted by the discrepancy between the single-inclusive jet distribution at large \( p_T \), as measured by CDF [19], and the result of the NLO QCD predictions [20]. This discrepancy is the topic of ongoing investigations and discussions [21] on both experimental and theoretical aspects.

Owing to its kinematic similarity with DY and heavy-flavour production, we studied [1] the soft-gluon corrections to the invariant-mass distribution of a jet pair. For this distribution an analogous discrepancy...
Figure 2: Contribution of gluon resummation at order $\alpha_S^3$ and higher, relative to the truncated $O(\alpha_S^2)$ result, for the invariant-mass distribution of jet pairs at the Tevatron.

between data and theory has been reported [22]. Figure 2 shows our results for the following quantities

\[
\frac{\delta_{gg}^{(4)}}{\sigma^{(3)}}, \quad \frac{\delta_{qg}^{(4)}}{\sigma^{(3)}}, \quad \frac{\delta_{gq}^{(4)}}{\sigma^{(3)}}, \quad \frac{\delta_{gg}^{(4)} + \delta_{qg}^{(4)} + \delta_{gq}^{(4)}}{\sigma^{(3)}},
\]

where, analogously to Eq. (16), $\delta^{(4)}$ is equal to the MP resummed hadronic cross section with terms of order $\alpha_S^3$ subtracted. Unlike in Eq. (16), $\sigma^{(3)}$ is not the full NLO cross section but rather its approximation as obtained by truncating the resummation formula at order $\alpha_S^3$. One can see that the resummation effect leads to $10 \div 15\%$ increase of the cross section when the inelasticity variable $\tau = M_{jj}/S$ is as large as $\tau \sim 0.5$.

These results should only be taken as an indication of the order of magnitude of the correction. This is because we did not include a study of the resummation effects on the determination of the parton densities, we only considered LL resummation and we did not implement detailed experimental cuts.

Note also that other distributions, such as the $p_T$ of the jet, have a rather different structure from the viewpoint of soft-gluon resummation because of additional jet-broadening effects due to final-state radiation. Calculations of these effects are in progress.

6 SUMMARY AND OUTLOOK

The MP can be used to evaluate the effect of soft-gluon resummation on cross sections in hadron collisions. Using the MP we find that the logarithmically-enhanced soft-gluon terms become important only fairly close to the hadronic threshold ($x \sim 0.5$). Further investigations of resummation effects in this kinematic region are warranted. These include the consistent determination and $Q^2$-evolution of the parton densities by using resummed anomalous dimensions [3] and the evaluation of NLL corrections [23].

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