The current picture of Glueballs

Frank E. Close*
Rutherford Appleton Laboratory
Chilton, Didcot, OX11 0QX, England

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Abstract

Some recent developments in the phenomenology of the lightest scalar glueball are summarised. Tools for determining the gluonic content of a resonance of known mass, width and $J^{PC}$ from its branching fraction in radiative quarkonium decays and production cross section in $\gamma\gamma$ collisions are presented. Two $q\bar{q}-G$ mixing schemes for $J^{PC} = 0^{++}$, inspired by the lattice, are shown to lead to similar phenomenology that may be tested at BEPC and in $\gamma\gamma$ production at LEP2.

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*e-mail: fec@v2.rl.ac.uk
1 Introduction

A quarter of a century after glueballs were first proposed, it is now looking likely that the scalar glueball has revealed itself, though with a novel twist. After searching for a single example, we now have the luxury of having to choose between two candidates! Furthermore it is of particular interest to this conference that $p\bar{p}$ annihilation has been the major player in the experimental developments.

In the emerging picture four states are of particular interest:

- $f_0(1500)[1, 2, 3]$
- $f_J(1710)[4]$ where $J = 0$ or $2[5]$
- $\xi(2230)[6]$
- $\eta(1440)[7]$, now resolved into two pseudoscalars.

The interest in these states as glueball candidates is motivated on both phenomenological and theoretical grounds. First, phenomenologically, these states satisfy **qualitative** criteria expected for the production of glueballs[8]:

1. Glueballs should be produced in proton-antiproton annihilation, where the destruction of quarks creates opportunity for gluons to be manifested. This is the Crystal Barrel [9], and E760 [10] production mechanism, in which detailed decay systematics of $f_0(1500)$ have been studied. The empirical situation with regard to $f_J(1710)$ and $\xi(2230)$ is currently under investigation. The $\eta(1440)$ is clearly seen in $p\bar{p}$ annihilation[11, 12]

2. Glueballs should be favoured over ordinary mesons in the central region of high energy scattering processes, away from beam and target quarks. The $f_J(1710)$ and possibly the $f_0(1500)$ have been seen in the central region in $pp$ collisions[13, 14].

3. Glueballs should be enhanced compared to ordinary mesons in radiative quarkonium decay. In fact, all four of these resonances are produced in radiative $J/\psi$ decay at a level typically of $\sim 1$ part per thousand.

The latter mechanism has a special role as recent work[15, 16] has **quantified** the production rate of conventional mesons ($q\bar{q}$) and glueballs ("$G$") in
the radiative decay of vector quarkonium, as a function of their mass, angular momentum, and width. If the data on the radiative production of these states are correct, then [16] finds that

(i) The $f_0(1500)$ is probably produced at a rate too high to be a $q\bar{q}$ state. The average of world data suggests it is a glueball-$q\bar{q}$ mixture.

(ii) The $f_J(1710)$ is produced at a rate which is consistent with it being $q\bar{q}$, only if $J = 2$. If $J = 0$, its production rate is too high for it to be a pure $q\bar{q}$ state but is consistent with it being a glueball or mixed $q\bar{q}$-glueball having a large glueball component.

(iii) The $\xi(2230)$, whose width is $\sim$ 20 MeV, is produced at a rate too high to be a $q\bar{q}$ state for either $J = 0$ or 2. If $J = 2$, it is consistent with being a glueball. The assignment $J = 0$ would require $\text{Br}(J/\psi \to \gamma\xi) \lesssim 3 \times 10^{-4}$, which already may be excluded.

(iv) The enhancement once called $\eta(1440)$ has been resolved into two states[16, 17]. The higher mass $\eta(1480)$ is dominantly $s\bar{s}$ with some glue admixture, while the lower state $\eta(1410)$ has strong affinity for glue.

I shall begin with a brief summary of this quantification. Then I shall review ideas inspired by the lattice QCD with reference to $f_0(1500)$ and $f_{J=0}(1710)$. The latest developments involve mixing schemes motivated by the lattice; we shall see that two different schemes[2, 18] have similar implications which may be tested in experiment.

2 Production in $\psi \to \gamma R$

Ref.[16] has used the measured radiative quarkonium production rates and gamma-gamma decay widths to make quantitative estimates of the gluonic content of isosinglet mesons. In particular it applies the relationship of ref.[15] between the branching fraction for a resonance $R$ in radiative quarkonium decay, $b_{\text{rad}}(Q\bar{Q}_V \to \gamma + R) \equiv \Gamma(Q\bar{Q}_V \to \gamma + R)/\Gamma(Q\bar{Q}_V \to \gamma + X)$ and its branching fraction to gluons, $\text{br}(R \to gg) \equiv \Gamma(R \to gg)/\Gamma(R \to \text{all})$:

$$b_{\text{rad}}(Q\bar{Q}_V \to \gamma + R_J) = \frac{c_R x [H_J(x)]^2 m_R}{8\pi(\pi^2 - 9)} \frac{m_R}{M_V^2} \text{br}(R_J \to gg),$$

where $M_V$ and $m_R$ are masses of the initial and final resonances, and $x \equiv 1 - \frac{m_R^2}{M_V^2}$; $c_R$ is a numerical factor and $H_J(x)$ a loop integral whose magnitude is
shown in ref.\[16\]. For a resonance of known mass, total width (\(\Gamma_{tot}\)), and \(J^{PC}\), a relationship such as eq. (1) would determine \(\text{br}(R \to gg)\) if \(\text{br}_{rad}(Q\bar{Q}V \to \gamma + R)\) were known. One may expect

\[
\text{br}(R[q\bar{q}] \to gg) = 0(\alpha_s^2) \simeq 0.1 - 0.2
\]

Thus knowledge of \(\text{br}(R \to gg)\) would give quantitative information on the glueball content of a particular resonance. Known \(q\bar{q}\) resonances (such as \(F_2(1270)\)) satisfy the former\[16\].

In the \(x\) regime of immediate interest, \(x \sim 0.5 - 0.75\), one finds \[16\] that \(\frac{\pi^2|x|^2}{3N_c} \sim O(1)\). This enables us to manipulate the above into a scaled form that exhibits the phenomenological implications immediately. Specifically, for scalar mesons

\[
10^3\text{br}(J/\psi \to \gamma 0^{++}) = \left(\frac{m}{1.5 \text{ GeV}}\right)\left(\frac{\Gamma_{R \to gg}}{96 \text{ MeV}}\right) x |H_S(x)|^2 \frac{35}{35}.
\]

This is to be compared with the analogous formula for a tensor meson:

\[
10^3\text{br}(J/\psi \to \gamma 2^{++}) = \left(\frac{m}{1.5 \text{ GeV}}\right)\left(\frac{\Gamma_{R \to gg}}{26 \text{ MeV}}\right) x |H_T(x)|^2 \frac{34}{34}.
\]

For pseudoscalars we find:

\[
10^3\text{br}(J/\psi \to \gamma 0^{-+}) = \left(\frac{m}{1.5 \text{ GeV}}\right)\left(\frac{\Gamma_{R \to gg}}{50 \text{ MeV}}\right) x |H_{PS}(x)|^2 \frac{45}{45}.
\]

Having scaled the expressions this way, because \(\frac{\pi^2|x|^2}{3N_c} \sim O(1)\) in the \(x\) range relevant for production of 1.3 - 2.2 GeV states, we see immediately that the magnitudes of the branching ratios are driven by the denominators 96 and 26 MeV for \(0^{++}\) and \(2^{++}\), and 50 MeV for \(0^{-+}\). Thus if a state \(R_J\) is produced in \(J/\psi \to \gamma X\) at \(O(10^{-3})\) then \(\Gamma(R_J \to gg)\) will typically be of the order 100 MeV for \(0^{++}\), \(O(25 \text{ MeV})\) for \(2^{++}\), and \(O(50 \text{ MeV})\) for \(0^{-+}\).

This immediately shows why the \(2^{++} q\bar{q}\) states are prominent: A \(2^{++}\) state with a total width of \(O(100 \text{ MeV})\) (typical for \(2^{++} q\bar{q}\)'s in this mass range\[2, 19\]) will be easily visible in \(J/\psi \to \gamma 2^{++}\) with branching fraction \(O(10^{-3})\), while remaining consistent with

\[
\text{br}(R[q\bar{Q}] \to gg) = 0(\alpha_s^2) \simeq 0.1 - 0.2.
\]

Eqs. 3 - 5 not only indicate which \(q\bar{q}\) states will be prominent in \(J/\psi \to \gamma R\), but they also help to resolve an old paradox concerning \(0^{++}\) production.
It was recognised early on that when the gluons in the absorbive part of $J/\psi \to \gamma gg$ are classified according to their $J^{PC}$, the partial wave with $2^{++}$ was predicted to dominate. The waves with $0^{-+}$ and $0^{++}$ were also predicted to be significant and of comparable strength to one another [20]. When extended to include the dispersive part[15, 21] the $0^{++}$ was predicted to be prominent over a considerable part of the kinematic region of interest. States with $J \geq 3$ were predicted to have very small rate in this process. Experimentally, all but one of these appeared to be satisfied. There are clear resonant signals in $2^{++}$ and $0^{++}$, and no unambiguous signals have been seen with $J \geq 3$. However no $0^{++}$ signal had been isolated.

From our relations above, we see that for a $0^{++}$ to be produced at the $10^{-3}$ level in $J/\psi$ radiative decay it must either have a large gluonic content and width $O(100)$ MeV or, if it is a $q\bar{q}$ meson, it must have a very large width, $\gtrsim 500$ MeV. Taking this into account, along with the following points, the puzzle of the absence of $0^{++}$ signal has been resolved:

(i): The width of $^3P_0$ $q\bar{q}$ is predicted to be $\sim 500$ MeV[2, 19]. Thus production at the level $\text{br}(J/\psi \to \gamma(gg))_{0^+} \sim 10^{-3}$ is consistent with $\text{br}(R \to gg) = 0(\alpha_s^2) \simeq 0.1 - 0.2$, but the $\sim 500$ MeV wide signal is smeared over a large kinematic ($x$) range.

(ii): The $\sim 100$ MeV wide $f_0(1500)$ signal seen in $J/\psi \to \gamma 4\pi$ was originally misidentified as $0^{-+}$, but is now understood to be $0^{++}$[3].

(iii): The $f_2(1710)$ which was originally believed to be $J = 2$ may contain a contribution with $J = 0[3, 5]$.

Detailed analysis of the data on $\psi$ radiative decay, when combined with the above formulae, lead to the conclusions on the gluonic status of the cited states as listed in the introduction. I refer you to ref.[16] for more details. Here I would like to report on recent developments arising from the predictions of lattice QCD concerning the mass and other properties of the lightest scalar glueball. In particular I shall be interested in the impact of the $q\bar{q}$, $J^{PC} = 0^{++}$ nonet being in the vicinity of the scalar glueball. I shall also end with some speculations on the role that $S$-wave decays into pseudoscalar meson pairs may have on $0^{++}$ mesons in the presence of a glueball.

3 Lattice QCD and Mixing

Lattice QCD predicts that the lightest “ideal” (i.e., quenched approximation) glueball be $0^{++}$, with state-of-the-art mass predictions of $1.55 \pm 0.05$
GeV[22] and $1.74 \pm 0.07$ GeV[4]. A consistent fit to both analyses gives[23] $1.61 \pm 0.07 \pm 0.13$ GeV where the first error is statistical and the second is systematic. That lattice QCD is now concerned with such fine details represents considerable advance in the field and raises both opportunity and enigmas. First, it encourages serious consideration of the further lattice predictions that the $2^{++}$ glueball lie in the 2.2 GeV region, and hence raises interest in the $\xi(2230)$. Secondly, it suggests that scalar mesons in the $1.5 - 1.7$ GeV region merit special attention independent of, and supplementary to the previous discussion. The $f_0(1500)$[2] and $f_J(1710)$ (if $J = 0$)[26] are the two candidates that have created most interest recently.

A significant new result from the lattice[26] is that the two body width of a scalar glueball is $\sim O(100)$ MeV and not $\sim O(1000)$ MeV. In principle the glueball could have been extremely wide and for practical purposes unobservable. The lattice shows that the scalar glueball should be a reasonably sharp signal which is an important guide in helping to eliminate candidates. The width for decay of the scalar glueball into pseudoscalar pairs was predicted[26] to be $108 \pm 28$ MeV. The $f_0(1500)$ has $\Gamma_{tot} = 120 \pm 20$ MeV[24] with the decays into pseudoscalar pairs comprising $\sim 60$ MeV of this. The $f_J(1710)$ has $\Gamma_{tot} = 140 \pm 12$ MeV. The lattice prediction of the width guides us towards these states (if $f_J(1710)$ has $J = 0$) but does not of itself discriminate between them.

Amsler and Close[2] have pointed out that the $f_0(1500)$ shares features expected for a glueball that is mixed with the nearby isoscalar members of the $^3P_0$ $q\bar{q}$ nonet. In particular we noted that this gives a destructive interference between $s\bar{s}$ and $n\bar{n}$ mixing whereby the $K\bar{K}$ decays are suppressed. This appears to be the case empirically[25]. The suppression of $K\bar{K}$ together with the lack of suppression for $\eta\eta$ is a significant indicator for a glueball - $q\bar{q}$ mixture[2].

The properties of the $f_J(1710)$ become central to completing the glueball picture. If the $f_J(1710)$ proves to have $J = 2$, then it is not a candidate for the ground state glueball and the $f_0(1500)$ will be essentially unchallenged. On the other hand, if the $f_J(1710)$ has $J = 0$ it becomes a potentially interesting glueball candidate. Indeed, Sexton, Vaccarino and Weingarten[26] argue that $f_{J=0}(1710)$ should be identified with the ground state glueball, based on its similarity in mass and decay properties to the state seen in their lattice simulation. The prominent scalar $f_0(1500)$ was interpreted by ref.[26] as the $s\bar{s}$ member of the scalar nonet, however this identification does not fit easily with the small $K\bar{K}$ branching ratio and the dominant decays to pions.
3.1 Mixing of Scalar Glueball and Quarkonia

Whereas the spin of the $f_J(1710)$ remains undetermined, it is now clearly established that there are scalar mesons $f_0(1370)$ and $f_0(1500)$ [1] which couple to $\pi\pi$ and $KK$ and so must be allowed for in any analysis of this mass region.

The presence of $f_0(1370)$, $a_0(1450)$, $K_0(1430)$ reinforce the expectation that a $q\bar{q}$ $^3P_0$ nonet is in the $O(1.3-1.7)$ GeV mass region. It is therefore extremely likely that an ‘ideal’ glueball at $\sim 1.6$ GeV [23] will be degenerate with one or other of the $^3P_0$ states given that the widths of the latter are $O$(hundreds MeV). This has not been allowed for in any lattice simulation so far.

Ref[2] has considered the phenomenology of an ideal glueball lying in the midst of the scalar nonet and finds that the ensuing mixings between the glueball and the nonet lead to a state with enfeebled coupling to $KK$ (in line with the $f_0(1500)$). This scenario also leads to significant gluonic components in the nearby $n\bar{n}$ and $s\bar{s}$ states. Ref.[2] proposed that “if the $f_J(1710)$ is confirmed to have a $J = 0$ component in $KK$ but not in $\pi\pi$, this could be a viable candidate for a $G_0 - s\bar{s}$ mixture, completing the scalar meson system built on the glueball and the quarkonium nonet”.

Recently Weingarten[18] has proposed what at first sight appears to be a different mixing scheme based on estimates for the mass of the $s\bar{s}$ scalar state in the quenched approximation. Whereas ref[2] supposed that the ideal glueball lies within the nonet, ref[18] supposed it to lie above the nonet. I shall now start with the general expressions of ref[2] and compare the two schemes. This will reveal some rather general common features.

3.2 Three-State Mixings

An interesting possibility is that three $f_0$’s in the $1.4-1.7$ GeV region are admixtures of the three isosinglet states $gg$, $s\bar{s}$, and $n\bar{n}$[2]. At leading order in the glueball-$q\bar{q}$ mixing, ref[2] obtained

\begin{align}
N_G|G\rangle &= |G_0\rangle + \xi(\sqrt{2}|n\bar{n}\rangle + \omega|s\bar{s}\rangle) \\
N_s|\Psi_s\rangle &= |s\bar{s}\rangle - \xi\omega|G_0\rangle \\
N_n|\Psi_n\rangle &= |n\bar{n}\rangle - \xi\sqrt{2}|G_0\rangle
\end{align}

(7)

where the $N_i$ are appropriate normalisation factors, $\omega \equiv \frac{E(G_0) - E(dd)}{E(G_0) - E(s\bar{s})}$ and the mixing parameter $\xi \equiv \frac{E(G_0) - E(dd)}{E(G_0) - E(s\bar{s})}$. The analysis of ref[16] suggests that the
$gg \to q\bar{q}$ mixing amplitude manifested in $\psi \to \gamma R(q\bar{q})$ is $O(\alpha_s)$, so that qualitatively $\xi \sim O(\alpha_s) \sim 0.5$. Such a magnitude implies significant mixing in eq. (7) and is better generalised to a $3 \times 3$ mixing matrix. Ref.[18] defines this to be
\[
\begin{pmatrix}
m^0_G & z & \sqrt{2}z \\
z & m^0_s & 0 \\
\sqrt{2}z & 0 & m^0_n
\end{pmatrix}
\]
where $z \equiv \xi \times (E(G_0) - E(\bar{d}\bar{d}))$ in the notation of ref.[2].

Mixing based on lattice glueball masses lead to two classes of solution of immediate interest:

(i) $\omega \leq 0$, corresponding to $G_0$ in the midst of the nonet[2]

(ii) $\omega > 1$, corresponding to $G_0$ above the $q\bar{q}$ members of the nonet[18].

The model of Genovese[27] is a particular case where $\xi \to 0; \omega \to \infty$ with $\xi \omega \to 1$.

We shall denote the three mass eigenstates by $R_i$ with $R_1 = f_0(1370)$, $R_2 = f_0(1500)$ and $R_3 = f_0(1710)$, and the three isosinglet states $\phi_i$ with $\phi_1 = m\bar{n}$, $\phi_2 = s\bar{s}$ and $\phi_3 = gg$ so that $R_i = f_{ij}\phi_i$.

There are indications from lattice QCD that the scalar $s\bar{s}$ state, in the quenched approximation, may lie lower than the scalar glueball [28, 18]. Weingarten[18] has constructed a mixing model based on this scenario. The input “bare” masses are $m^0_n = 1450; m^0_s = 1516; m^0_G = 1642$ and the mixing strength $z \equiv \xi \times (E(G_0) - E(\bar{d}\bar{d})) = 72$ MeV. The resulting mixtures are
\[
\begin{pmatrix}
f^{(n)}_{i1} & f^{(s)}_{i2} & f^{(G)}_{i3} \\
f_0(1370) & 0.87 & 0.25 & -0.43 \\
f_0(1500) & -0.36 & 0.91 & -0.22 \\
f_0(1710) & 0.34 & 0.33 & 0.88
\end{pmatrix}
\]

It is suggested, but not demonstrated, that the decays of the $f_0(1500)$ involve significant destructive interference between its gluonic and $s\bar{s}$ components whereby the $K\bar{K}$ suppression and $2\pi, 4\pi$ enhancements are explained. The disparity between $K\bar{K}$ suppression and the strong coupling to $\eta\eta$ remains an open question here.

Recent data on the decay $f_0(1500) \to K\bar{K}[25]$ may be interpreted within the scheme of ref[2] as being consistent with the $G_0$ lying between $n\bar{n}$ and $s\bar{s}$ such that the parameter $\omega \sim -2$. (In this case the $\eta\eta$ production is driven by
the gluonic component of the wavefunction almost entirely, see ref[2]). If for illustration we adopt \( \xi = 0.5 \sim \alpha_s \), the resulting mixing amplitudes are

\[
\begin{array}{ccc}
 f_0^{(n)} & f_0^{(s)} & f_0^{(G)} \\
 f_0(1370) & 0.86 & 0.13 & -0.50 \\
 f_0(1500) & 0.43 & -0.61 & 0.61 \\
 f_0(1710) & 0.22 & 0.76 & 0.60 \\
\end{array}
\]

The solutions for the lowest mass state in the two schemes are similar, as are the relative phases and qualitative importance of the \( G \) component in the high mass state. Both solutions exhibit destructive interference between the \( n\bar{n} \) and \( s\bar{s} \) flavours for the middle state.

This parallelism is not a coincidence. A general feature of this three way mixing is that in the limit of strong mixing the central state tends towards flavour octet with the outer (heaviest and lightest) states being orthogonal mixtures of glueball and flavour singlet, namely

\[
\begin{array}{c}
 f_0(1370) |q\bar{q}(1)\rangle - |G\rangle \\
 f_0(1500) |q\bar{q}(8)\rangle + \epsilon |G\rangle \\
 f_0(1710) |q\bar{q}(1)\rangle + |G\rangle \\
\end{array}
\]

where \( \epsilon \sim \xi^{-1} \rightarrow 0 \).

In short, the glueball has leaked away maximally to the outer states even in the case (ref[2]) where the bare glueball (zero mixing) was in the middle of the nonet to start with. The leakage into the outer states becomes significant once the mixing strength (off diagonal term in the mass matrix) becomes comparable to the mass gap between glueball and \( q\bar{q} \) states (i.e. either \( \xi \geq 1 \) or \( \xi \omega \geq 1 \)). Even in the zero width approximation of ref[2] this tends to be the case and when one allows for the widths being of \( O(100) \) MeV while the nonet masses and glueball mass are spread over only a few hundred MeV, it is apparent that there will be considerable leakage from the glueball into the \( q\bar{q} \) nonet. It is for this reason, \textit{inter alia}, that the output of refs[2] and [18] are rather similar. While this similarity may make it hard to distinguish between them, it does enable data to eliminate the general idea should their common implications fail empirically.

If we make the simplifying assumption that the photons couple to the \( n\bar{n} \) and \( s\bar{s} \) in direct proportion to the respective \( \alpha_s^2 \) (i.e. we ignore mass effects and any differences between the \( n\bar{n} \) and \( s\bar{s} \) wavefunctions), then the corresponding
two photon widths can be written in terms of these mixing coefficients:

\[ \Gamma(R_i) = |f_{i1} \frac{5}{9\sqrt{2}} + f_{i2} \frac{1}{9} R_i|, \]

where \( \Gamma \) is the \( \gamma \gamma \) width for a \( q\bar{q} \) system with \( \epsilon_q = 1 \). One can use eq. (8) to evaluate the relative strength of the two photon widths for the three \( f_0 \) states with the input of the mixing coefficients[16]. These are (ignoring mass dependent effects)

\[ f_0(1370) : f_0(1500) : f_0(1710) \sim 12 : 1 : 3 \]  

(9)

in the Amsler - Close scheme [2] to be compared with

\[ f_0(1370) : f_0(1500) : f_0(1710) \sim 13 : 0.2 : 3 \]  

(10)

in Weingarten[18]. At present the only measured \( \gamma \gamma \) width in this list is that of the \( f_0(1370) = 5.4 \pm 2.3 \) keV[5]. Using this to normalise the above, we anticipate \( f_0(1500) \rightarrow \gamma \gamma \sim 0.5 \) keV [2] or \( \sim 0.1 \) keV [18]. Both schemes imply \( \Gamma(f_0(1710) \rightarrow \gamma \gamma) = 1 - 2 \) keV.

This relative ordering of \( \gamma \gamma \) widths is a common feature of mixings for all initial configurations for which the bare glueball does not lie nearly degenerate to the \( n\bar{n} \) state. As such, it is a robust test of the general idea of \( n\bar{n} \) and \( s\bar{s} \) mixing with a lattice motivated glueball. If, say, the \( \gamma \gamma \) width of the \( f_0(1710) \) were to be smaller than the \( f_0(1500) \), or comparable to or greater than the \( f_0(1370) \), then the general hypothesis of significant three state mixing with a lattice glueball would be disproven. The corollary is that qualitative agreement may be used to begin isolating in detail the mixing pattern.

The production of these states in \( \psi \rightarrow \gamma f_0 \) also shares some common features in that \( f_0(1710) \) production is predicted to dominate. The analysis of ref.[16] predicts that

\[ \text{br}(J/\psi \rightarrow \gamma \Sigma f_0) \geq (1.5 \pm 0.6) \times 10^{-3}. \]  

(11)

In [2] the \( q\bar{q} \) admixture in the \( f_0(1500) \) is nearly pure flavour octet and hence decouples from \( gg \). This leaves the strength of \( \text{br}(J/\psi \rightarrow \gamma f_0(1500)) \) driven entirely by its \( gg \) component at about 40% of the pure glueball strength. This appears to be consistent with the mean of the world data (for details see ref.[16]).

Thus, in conclusion, both these mixing schemes imply a similar hierarchy of strengths in \( \gamma \gamma \) production which may be used as a test of the general idea of
three state mixing between glueball and a nearby nonet. Prominent production of $J/\psi \rightarrow \gamma f_0(1710)$ is also a common feature. When the experimental situation clarifies on the $J/\psi \rightarrow \gamma f_0$ branching fractions, we can use the relative strengths to distinguish between the case where the glueball lies within a nonet, ref[2], or above the $s\bar{s}$ member, ref[18].

In the former case this $G_0 - q\bar{q}$ mixing gives a destructive interference between $s\bar{s}$ and $n\bar{n}$ whereby decays into $K\bar{K}$ are suppressed. However, even in the case where the $s\bar{s}$ lies below the $G_0$ we expect that there will be $K\bar{K}$ destructive effects due to mixing with not only with the $s\bar{s}$ that lies below $G_0$ (as in ref.[18]) but also with a radially excited $n\bar{n}$ lying above it (not considered in ref.[18]). Unless $G_0$ mixing with the radial state is much suppressed, this will give a similar pattern to that of ref.[2] though with more model dependence due to the differing spatial wavefunctions for the two nonets. In the final section I shall assume that the $G_0 - q\bar{q}$ mixing leads to $n\bar{n} - s\bar{s}$ in the wavefunction.

4 Glue, $q\bar{q}$ and Mesons: A Hierarchy

Scalar quarkonium mesons are $P$-wave $q\bar{q}$ but decay in $S$-wave to pseudoscalar meson pairs. Tornqvist[29] has argued that this can distort the meson nonet considerably, in particular dragging the $s\bar{s}$ and $I = 1$ bare states down to the vicinity of $K\bar{K}$ threshold (to be identified with the $f_0(980)$ and $a_0(980)$ respectively). His analysis has not included the possible role of a primitive glueball.

On the other hand the analyses in refs.[2, 18] have considered mixing of $gg$ and $q\bar{q}$ without inclusion of meson pairs. It is amusing to consider the qualitative picture that might emerge when the parton [2] and meson[29] are combined.

The main question is how the glueball couples to the hadronic sector. In ref.[2] we argued that mixing into (nearly) $q\bar{q}$ may play a leading role, at least for scalar glueball. This will cause a suppression of $K\bar{K}$ if the mixing is dominantly with nearest neighbours: this is clear in the scheme of ref[2] where the glueball lies in the middle of a nonet, but will also happen if the glueball lies above the $s\bar{s}$ as proposed by Weingarten[18]. The point is that in the $q\bar{q}$ spectroscopy (where $N$ denotes the radial excitation quantum number) the $N(s\bar{s})$ will lie below, and near to, $(N + 1)(n\bar{n})$. Consequently we may expect that in general a glueball will lie either between $n\bar{n}$ and $s\bar{s}$ of a single multiplet, or between $s\bar{s}$ and $n\bar{n}$ of adjacent multiplets. While the overlapping
of multiplets for the higher mass $2^{++}$ (which can also involve $^3P_2$ as well as radial excitation of $^3P_2$) may break this flavour ordering, it is a rather general feature in potential models for the 1.5 GeV region for scalars.

If we adopt this $K\bar{K}$ suppression, we find that (at least for the $q\bar{q}$ decays) the $\eta\eta$ and $\eta'\eta'$ are also suppressed. The $\pi\pi$ threshold is far away from the 1.6 GeV region and the intrinsic Clebsch Gordan coefficient coupling $\frac{1}{\sqrt{2}}(n\bar{n} - s\bar{s})\cos\theta + \sin\theta G_0$ to $\pi\pi$ is also somewhat reduced ($\cos\theta/2$) due to this mixing. A major effect may be expected from the $\eta\eta'$ threshold. This channel couples strongly to the $n\bar{n} - s\bar{s}$ mixture[2] and possibly also to $G_0$ through glue content in the $\eta$ system. Furthermore the $\eta\eta'$ threshold of 1510 MeV is near to the glueball mass, according to lattice QCD[22, 23, 26]. Thus it may be natural to find a strong attraction of the glueball to the 1500 MeV mass region. It may be interesting to extend the $G_0 \to q\bar{q}$ mixing analysis of ref[2] to include the $q\bar{q} \to 0^-0^-$ unitarisation of ref.[29]. This leads to a more complete description of the scalar mesons and S-wave thresholds in the 1 to 2 GeV region[30]. The implications for $\gamma\gamma$ couplings, in particular, could then enable the role and parameters of the scalar glueball to be quantified.

In parallel a study of flavour decays based on the lattice results of ref[26] and the mixing schemes of section 3.2 is warranted. The focus now is on ways to disentangle the glueball dynamics from the scalar mesons in the 1.4 to 1.8 GeV region through a dedicated programme concentrating on the $f_0(1500)$ and $f_{J/\psi}(1710)$ in particular. The emergent data are remarkably consistent with models based on lattice QCD. This is real progress compared even to two years ago and is due in no small part to the remarkable data that have emerged from $p\bar{p}$ annihilation at LEAR.

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