Cosmology in a String-Dominated Universe

David Spergel
Princeton University Observatory, Princeton, NJ 08544

and Ue-Li Pen
Harvard-Smithsonian Center for Astrophysics, 60 Garden St., Cambridge, MA 02138

ABSTRACT

The string-dominated universe locally resembles an open universe, and fits dynamical measures of power spectra, cluster abundances, redshift distortions, lensing constraints, luminosity and angular diameter distance relations and microwave background observations. We show examples of networks which might give rise to recent string-domination without requiring any fine-tuned parameters. We discuss how future observations can distinguish this model from other cosmologies.

1. Introduction

Most theoretical cosmologists prefer flat universe models. While this preference was initially based on extensions of the Copernican principle (Dicke 1970), it has been strengthened by the theoretical successes of the inflationary universe paradigm (see Linde 1990 for discussion). While it is possible to construct inflationary models with $\Omega < 1$ (e.g., Linde and Mezhlumian 1994), these models are less aesthetically appealing than the flat universe models.

Observations, however, suggest that the matter density of the universe is not sufficient to make $\Omega = 1$: measurements of the Hubble constant (Freedman, Madore & Kennicutt 1997) and estimates of the age of the universe (Bolte and Hogan 1995) suggest that $H_0 t_0 > 2/3$; measurements of the baryon to dark matter ratio in clusters, together with estimates of the baryon density from big bang nucleosynthesis imply $\Omega_0$, the energy density in matter, is much less than 1 (White et al. 1993); and the power spectrum of large scale structure is best fit by models with $\Omega_0 h_0 = 0.25$ (Peacock & Dodds 1994). Here, $h_0 = H_0/(100 \text{ km/s/Mpc})$. For several decades, it has been observed that the Mass-to-Light ratio in clusters of galaxies suggests $\Omega_0 \sim 0.2$ (Bahcall, Lubin & Dorman 1995). The simplest COBE normalized parameter-free Harrison-Zeldovich-Peebles power spectrum slope $n = 1$ predicts local peculiar velocities and cluster abundances in $\Omega_0 = 1$ and $\Lambda$
universes which are significantly higher than observed (Strauss and Willick 1995, Eke, Cole and Frenk 1996, Pen 1996a, Viana and Liddle 1995), which can be resolved by lowering the matter density $\Omega_0$.

This contradiction has led cosmologists to consider exotic equations of state for the universe. The most studied modification of the standard matter dominated cosmology is the vacuum dominated model. While there is no particle physics motivation for positing a vacuum energy of $10^{-121}M_{\text{Pl}}^4$ (Weinberg 1996), the model does appear to be consistent with a number of observations (Ostriker and Steinhardt 1995). However, recent measurements of $q_0$ using distant supernova (Perlmutter et al. 1996) and limits based on the statistics of gravitational lensing (Kochanek 1996) are encouraging cosmologists to consider alternative models. A novel technique of distance determination using cluster hydrostatic equilibrium measurements also indicates positive values of $q_0$ (Pen 1996b).

A string dominated cosmology is an intriguing alternative to the standard model. In this model, the energy density in strings scales with the expansion factor, $a$, as $a^{-2}$, decaying faster than a vacuum energy term, but slower than the energy density in matter (which decays as $a^{-3}$). In this model, strings form at near the electroweak symmetry breaking scale. Unlike the much heavier GUT scale strings (see e.g., Vilenkin and Shellard 1993), these light strings do not seed structure formation. Individual strings in this model have too low a density to be observable separably. A typical string density would be $10^{-5}$ kg/m. However, their culmative effect is to alter the expansion of the universe. Locally, they make a flat universe appear to have many of the properties of an open universe model. The energy density of such a string network arises naturally to be near the critical energy density today.

If the universe today is string-dominated, then the strings must be produced near the electroweak scale, a scale at which there must be new physics. These electroweak strings are very light and would be undetected through their gravitational lensing as their bending angle is only $(M/M_{\text{Pl}})^2 \sim 10^{-32}$ radians. Here, $M$ is the symmetry breaking scale associated with string formation and $M_{\text{Pl}}$ is the Planck scale. While these strings are light, they are expected to be numerous. The characteristic separation between strings is expected to be the bubble size during the phase transition, which in the case of the electroweak phase transition is typically $10^{-3}$ of the horizon size (Moore and Prokopec 1996), 0.1 A.U.(comoving). Thus, there would be many light strings in our own Solar System. If these light strings are associated with baryogenesis (Starkman & Vachaspati 1996) or are superconducting (Vilenkin 1989), then they may be directly detectable.

Only some cosmic string models lead to a string dominated universe. In theories where cosmic strings can intercommute, their evolution obeys a “scaling solution”: their energy
density scales as $a^{-3}$ during matter domination and as $a^{-4}$ during radiation domination. In these theories, strings never dominate the energy density of the universe. On the other hand, if strings do not intercommute nor pass through each other, then the network can “freeze-out” and the energy density in strings can scale as $a^{-2}$ (Vilenkin 1984). Initial interest in string dominated universes was spurred by the possibility that Abelian strings might not intercommute effectively and might dominate the energy density of the universe (Kibble 1976; Vilenkin 1984; Kardashev 1986). However, numerical simulations showed that even complicated Abelian string networks (Vachaspati and Vilenkin 1987) intercommuted effectively and rapidly approached the scaling solution. Despite the lack of a model that had non-intercommuting strings, the interesting astrophysical implications of a string dominated universe led to a number of papers investigating their cosmological properties (Turner 1985; Charlton and Turner 1987; Gott and Rees 1987; Dabrowski and Stelmach 1989; Tomita and Watanabe 1990; Stelmach, Dabrowski and Byrka 1994; Stelmach 1995) and the properties of cosmologies with similar equations of state (Steinhardt 1996; Coble, Dodelson & Frieman 1996). We review some of these results in section 3 and compare string-dominated flat cosmologies to observations of large-scale structure, microwave background fluctuations, observations of rich clusters, and other cosmological probes. In this section, we show that a string-dominated universe with $H_0 \sim 60 - 65$ km/s and $\Omega_0 \sim 0.4 - 0.6$ agrees remarkably well with a broad class of observations.

Our interest in string-dominated universes was stimulated by our numerical simulations of the evolution of Non-Abelian cosmic strings (Pen and Spergel 1996). In theories in which a non-Abelian symmetry is broken to a discrete sub-group, multiple types of cosmic strings can be produced (Mermin 1979). Topological constraints prevents these different types of strings from intercommuting (Toulouse 1976; Poenaru and Toulouse 1977), which led to the speculation that they could potentially dominate the energy density of the universe (Kibble 1980). There are a number of phenomenologically interesting particle physics models that utilize these non-Abelian symmetries (Chkareuli 1991; Dvali & Senjanovic 1994). These complex string networks are not merely flights of theoretical fantasy: they can be seen in biaxial nematic liquid crystals (De’Neve, Kleman and Navard 1992). In section 2, we discuss the physics of non-Abelian strings and summarize the results of our numerical simulations.
2. Physics of Non-Abelian Strings

2.1. What are Non-Abelian Strings?

Strings are created when the lowest energy state of an order parameter is degenerate and its vacuum manifold not simply connected. A simple such example is given by nematic liquid crystals. Each crystal is a needle with perfect reflection symmetry. The unbroken symmetry state above the liquid crystal phase transition is one where each molecule can point in any random direction in space, which is described by the rotation group $G = SO(3)$. In the liquid crystal phase, neighboring elements prefer to point in the same direction, but rotation around the needle axis is not distinguishable, nor are reflections around the the plane perpendicular to its axies. The broken symmetry group is $H = O(2)$. The vacuum manifold is given by $\mathcal{M} = G/H$, and its first homotopy group satisfies the exact sequence

$$\pi_1(H) \to \pi_1(G) \to \pi_1(G/H) \to \pi_0(H) \to \pi_0(G).$$

Since $\pi_0(SO(3)) = 0$ and $\pi_0(H) = Z_2$, we know that $\pi_1(G/H)$ must be non-trivial. In fact, $\mathcal{M}$ is just the projective 2-sphere, the unit sphere with antipodes identified. We know that $\pi_1(\mathcal{M}) = Z_2$, which is an Abelian group with only two elements, one of which is the identity element. All strings correspond to the other element, allowing any two strings to intercommute.

The situation gets more interesting when different types of strings can be formed. Two strings corresponding to distinct group elements of $\pi_1$ can intercommute, i.e. exchange partners, only if they correspond to the same element, or to the inverses of each other. When that is not the case, they can still pass through each other if their corresponding elements commute. In a non-Abelian system there exist elements which do not commute, and two strings which attempt to cross each other result in a configuration where an umbilical cord is formed between the points where they crossed.

In general, each string type may have a different tension $\mu$, and strings can decay into factors if that is energetically favorable. If the tension in the umbilical cord is larger than twice the tension in either of the intersection strings, it is energetically favorable to have an umbilical cord of zero length, which macroscopically appear like the junction of four string segments at one vertex. Similarly, a vertex joining any number of strings may be formed, depending on the exact structure of $\pi_1$ and the distribution of string tensions.
2.2. Dynamics of a Biaxial Nematic Crystal

A particularly illustrative example of a system exhibiting non-Abelian string defects are the biaxial nematic crystals. Each crystal element is in this case triaxial, with symmetry for 180 degree rotation about any of the three axes. This broken symmetry state is described by the four element dihedral group $H = D_4$, one element corresponding to no rotation, and the other three to a 180 degree rotation about each coordinate axis. Such a system is available commercially, and has been studied experimentally by Zapotocky, Goldbart and Goldenfeld (1995).

The exact homotopy sequence (1) describes the system. We have $0 \to 0 \to \mathbb{Z}_2 \to \pi_1(\mathcal{M}) \to D_4 \to 0$. We thus know that $\pi_1(\mathcal{M})$ must be an 8 element group, which in this case is the quaternion group $Q_8$ with 8 elements $(1, i, j, k, -1, -i, -j, -k)$ and the multiplication properties $i^2 = j^2 = k^2 = -1$, $ij = k$, $jk = i$, $ki = j$. The non-Abelian property is exhibited by $ij = -ji$ etc., which derives from the commutation property of the rotation group. We have seven different strings in this system, which can have up to four different tensions. In a liquid crystal system, the string corresponding to $-1$ can always decay into two strings from the generator with the smallest tension. The only non-commuting strings are those corresponding to $i, j, k$, and when they cross, they in principle create an umbilical cord with charge $-1$, but energetically they prefer to stick, forming a four leg vertex. Three leg vertices form at the junction of $i, j, k$ strings (or their inverses).

If we neglect the presence of sticking strings (four leg vertices), the system is quite similar to the $Z_3$ monopole-string network studied by Vilenkin and Vachaspati (1987). Whenever two three-leg vertices come together, they annihilate and release two disconnected strings. To simulate these and other networks more realistically, we have developed a global string code which simulates non-Abelian strings using a nonlinear $\sigma$ model on a lattice.

2.3. Dynamics of More Complex String Networks

To simplify the simulation while capturing the essentials of a wide range of non-Abelian string dynamics, we chose a modification of the non-linear $\sigma$ model from Pen, Spergel and Turok (1995) (hereafter PST). In this model, we have a classical field $\phi$ defined at every lattice point $\vec{x}$, which takes on values in the range $[0,\pi) \times \mathbb{Z}_N$. The field has both a continuous component, and a discrete index in the range $0..N-1$. The multiple leaves of semicircles are to be thought of as a Rolodex file: Whenever we examine the dynamics of two leaves, we open the system such that the two leaves form a full circle, and treat the
dynamics as lying in a unit circle. Since the evolution equation only require the pairwise force, any two points always lie on some such circle.

For $N = 2$, we recover the standard global strings from PST. For $N = 3$, a system very similar to the biaxial nematics and the $Z_3$ strings is obtained, with three different types of strings, and three point vertices which annihilate pairwise. Two strings get stuck when they try to pass through each other, just like the biaxial nematic liquid crystals. We show such a network in figure 1, where we have represented the string corresponding to each of the three generators by a different color. Each of the three semicircle leaves are either red, green or blue. Since each string contains a complete rotation which covers two leaves, the strings appear as composite colors, green+red=yellow, etc. There are three such possible pairs.

In general, we have a system of $N(N - 1)/2$ strings. Strings join at three-point vertices, of which there are $N(N - 1)(N - 2)/6$ different types. When two vertices join, there is a one in $3/N$ chance that they can annihilate and result in two disconnected strings. Otherwise, the two vertices can pass through one another and result in a new configuration which still contains the same number of vertices.

We can now vary the number of string generators $N$ to correspond to a one parameter class of non-Abelian strings. We expect strings to become more strongly tangled as we increase $N$. This is indeed observed, as shown in figure 2. The global field dynamics differs systematically from gauged strings in the fact that global strings exert long-range forces on each other, which can cause the network to move even when the configuration is neutrally stable. A neutrally stable solution, such as a sheet of hexagonal tilings, is sufficient to cause full entanglement for cosmological purposes, since the damping due to the universal expansion would prevent the structure from collapsing.

We have found that for $N = 3$ strings, the solution scales much like the $Z_3$ monopole-string network, which would suggest that the biaxial nematic liquid crystal system would also exhibit scaling behaviour (Pen and Spergel 1996). We also see that the network does seem to stop disentangling for large $N$. While no current model of the electro-weak phase transition predicts cosmic strings, electro-weak baryogenesis calculations have argued for the presence of more complicated symmetry structures. It would be conceivable that both the baryon asymmetry and the present day vacuum energy be caused by the electro-weak symmetry breaking, which may have testable laboratory consequences in the near future.
Fig. 1.— The network of a $N = 3$ string system, which exhibits dynamics very similar to biaxial nematic liquid crystals. The strings are color coded according to the generator they belong to.
Fig. 2.— This figure shows the evolution of the string density (normalised to the scaling density) as a function of time in different models. In scaling solutions, the string density should asymptote to a constant value in this plot. Note that the large $N$ models do not scale and become tangled.
3. Astrophysics of String Dominated Universe

3.1. Expansion of the universe: $H_0$, $q_0$, $\Omega_0$ and $t_0$

Non-commuting strings formed at low energies have only one basic effect on the universe: they add an additional term to the Friedman equation that governs the expansion of the universe:

$$H^2 = \left( \frac{8\pi G}{3} \right) \left[ \frac{\rho_{m0}}{a} + \frac{\rho_{\gamma 0}}{a} + \frac{\rho_{s0}}{a} \right]$$

where $H$ is the Hubble rate, $a$ the expansion factor and $\rho_{m0}$, $\rho_{\gamma 0}$, and $\rho_{s0}$ is the current energy density in matter, radiation and strings. Since this additional term has the same $a$ dependence as the presence of space curvature, a string-dominated flat universe is observationally similar to matter-dominated open universe. Since we are focusing on a flat universe, we define $\Omega_0 \equiv 8\pi G \rho_{m0}/3H_0^2$, $\Omega_s \equiv 8\pi G \rho_{s0}/3H_0^2$, $\Omega_r \equiv 8\pi G \rho_{\gamma 0}/3H_0^2$, and assume $\Omega_0 + \Omega_r + \Omega_s = 1$

As in a curvature dominated universe, we can divide the history of the universe into three epochs: a radiation dominated epoch, a matter dominated epoch and a string-dominated epoch. In the matter and string dominated epochs, we can express the evolution of the universe in terms of the conformal time, $\eta \equiv \eta_s \cosh^{-1}(2\Omega^{-1} - 1)$ (Peebles 1993):

$$a = \frac{\Omega_0}{2(1 - \Omega_0)} \left[ \cosh \frac{\eta}{\eta_s} - 1 \right]$$

$$H_0 t = \frac{\Omega_0}{2(1 - \Omega_0)^{3/2}} \left[ \sinh \frac{\eta}{\eta_s} - \frac{\eta}{\eta_s} \right]$$

where we have defined $a_0 = 1$ and $\eta_s^{-1} = H_0 \sqrt{1 - \Omega_0}$.

Thus, the relationship between the age of the universe, $t_0$, the energy density in matter, and the Hubble constant is the same as in a curvature dominated universe:

$$H_0 t_0 = \frac{\Omega_0}{2(1 - \Omega_0)^{3/2}} \left[ \frac{2}{\Omega_0} (1 - \Omega_0)^{1/2} - \cosh^{-1}(2\Omega_0^{-1} - 1) \right]$$

(Kolb and Turner 1990). We also recover the familiar relationship for the deceleration parameter:

$$q_0 = \frac{\Omega_0}{2}$$

Thus, the string dominated cosmology makes the same predictions for most of the classical cosmological tests as the open universe model.
Because the curvature of the string-dominated universe is flat, its angular diameter-redshift relationship differs from an open universe. In an open universe, the angular size distance out to a redshift $z_e$ is

$$H_0 r(z_e) = \frac{1}{\sqrt{1 - \Omega_0}} \sinh \chi$$

(Peebles 1993), while in a string dominated flat universe,

$$H_0 r(z_e) = \frac{\chi}{\sqrt{1 - \Omega_0}}$$

where

$$\chi = \int_{a_e}^{1} \frac{da}{\sqrt{\frac{\Omega_0}{1 - \Omega_0} a + a^2}}$$

This altered angular diameter distance affects number count predictions, the probability of gravitational lensing and the predictions for microwave background fluctuations. For both models, the number count statistics can be computed from equation (13.61) in Peebles (1993):

$$\frac{dN}{dz} = \frac{n_0}{H_0} \frac{r(z_e)^2}{\sqrt{\Omega_0 (1 + z_e)^{-3} + (1 - \Omega_0) (1 + z_e)^{-2}}}$$

where $n_0$ is the comoving density of galaxies. The string-dominated flat model predicts fewer galaxy counts per unit redshift than both the open universe model and the vacuum energy dominated model.

The statistics of gravitational lensing in this model differs significantly from the predicted statistics in a vacuum dominated model. Current observations already place strong constraints on the vacuum dominated model, which predicts too many small lens events, particularly with small angular separation (Turner 1990; Kochanek 1996). The absence of large number of lenses in the HST snapshot survey (Maoz et al. 1993) and in radio surveys implies that $\Omega_\Lambda < 0.6$ and rules out most of the interesting parameter space for vacuum dominated models. Because of the very different relation between redshift and distance in string dominated models, it predicts many fewer gravitational lenses than the vacuum dominated models. A recent analysis by Bloomfield-Torres & Waga (1995) finds that string-dominated flat universes are excellent fits to the observed lens statistics in the HST snapshot survey.

Observations of supernova at high redshift are another powerful probe of cosmology. Perlmutter et al. (1996) have already been able to rule out cosmological constant models
with $\Omega_0 < 0.6$ at the 95% confidence level with their supernova data. Thus, there are no cosmological constant models compatible with this observation, measurements of large scale structure, measurements of the Hubble constant and the constraint that the age of the universe exceed 11 Gyr. At the redshifts probed by the supernova study, the distance redshift relation in a string-dominated universe is close to, but not identical to the distance-redshift relation in an open universe. Using the relations given in equation (7), the Perlmutter et al. (1996) observations imply that $\Omega_0 > 0.15$ in a flat string-dominated cosmology.

3.2. Microwave Background Fluctuations

Because the strings only make significant contributions to the energy density of the universe at very late times, they have no effect on the physics at the surface of last scatter. However, since the strings alter the expansion rate of the universe, they have two effects on the detailed shape of the microwave background spectrum: (1) the decay of potential fluctuations at late times produces additional fluctuations on large angular scales; and (2) since the conformal distance to the surface of last scatter is smaller, the Doppler peaks are shifted to larger angular scales (Stelmac, Dabrowski, and Byrka 1994). Since identical effects occur in a vacuum dominated universe, it will be difficult to distinguish a string dominated universe from a vacuum dominated universe based on CMB observations. On the other hand, it will be very easy to distinguish a flat cosmology from an open universe due to the large differences in the angular diameter distance relation (Kamionkowski & Spergel 1994).

We have calculated the predicted CMB spectrum in a string-dominated universe using a modified version of a Boltzmann code developed by Seljak and Zaldarriaga (1996). Figure 3 shows the predicted multipole spectrum for various string dominated cosmologies. While the three spectra can not yet be distinguished by current observations, future CMB maps should be able easily distinguish between the curves in figure 3.

Most observations of large-scale structure are effectively measurements of the galaxy power spectrum. Peacock & Dodds (1994) have shown that most galaxy surveys are consistent with a standard CDM power spectrum with $\Gamma \equiv \Omega_0 h \exp(-\Omega_b - \Omega_k/\Omega_0) = 0.25 \pm 0.05$. The Las Campanas redshift survey is also compatible with a power spectrum with $\Gamma = 0.2 - 0.3$ (Lin et al. 1996). Figure 4 shows that this constraint alone is sufficient to rule out much of parameter space. Note that standard CDM in a matter dominated flat universe is ruled out unless $H_0 \sim 30$ km/s/Mpc.
Fig. 3.— This figure compares the predicted multipole spectrum for three different models: a flat standard CDM model with $\Omega_0 = 1.0$ and $H_0 = 50$ km/s/Mpc (solid line); a string-dominated flat cosmology with $\Omega_0 = 0.4$ and $\Omega_0 = 0.6$. Because COBE did not detect a large quadrupole, the relative likelihood of the $\Omega_0 = 0.4$ to the $\Omega_0 = 1.0$ model is 0.05.
This figure combines constraints from various astrophysical measurements. The vertically shaded region lie outside the best determinations of the Hubble Constant: $H_0 = 73 \pm 6 \pm 8 \text{ km/s/Mpc}$ (Freedman, Madore & Kennicutt 1997); the horizontally shaded regions do not agree with measurements of the shape of the galaxy power spectrum, $\Gamma = 0.25 \pm 0.05$ (Peacock & Dodds 1994), and with measurements of the fluctuation amplitude from clusters, $\sigma_8 \Omega_0^{0.6} = 0.6 \pm 0.1$ (Eke et al. 1996; Viana and Liddle 1996; Pen 1996a); and the region shaded with lines at 45° angle corresponds to cosmic ages less than 11 Gyr.
We use the CMB spectrum to normalise the standard inflationary model (scale-invariant, $\Omega_0 h^2 = 0.0125$) in this cosmology to the COBE observations. Once this normalization is fixed, there is no free parameters left in the model, so that it can be compared directly to observations of matter power spectrum.

Observations of clusters are powerful probes of the matter power spectrum. Gravitational lensing observations, X-ray observations of hot gas, and studies of galaxy kinematics in clusters, all probe the velocity distribution in clusters. Thus, they can constrain the distribution of mass, rather than the distribution of light. A number of studies (Eke et al. 1996, Viana & Liddle 1996; Pen 1996a) have concluded that these cluster observations place very strong constraints on the amplitude of mass fluctuations on the $8h^{-1}$ Mpc scale: $\sigma_8 = 0.6 \pm 0.1 \Omega_0^{-0.6}$. Figure 4 shows that most string-dominated models fit all the constraints.

4. Conclusions

String-dominated cosmologies have a number of very attractive features.

For $\Omega_0 \sim 0.4 - 0.6$ and $H_0 \sim 60 - 70$ km/s/Mpc, the model is consistent with current observations. It fits observations of the CMB, measurements of the shape of galaxy power spectrum and measurements of the amplitude of the mass power spectrum, and is compatible with age limits. Unlike cosmological constant models, string dominated cosmology is also consistent with observations of high redshift supernova and gravitational lensing statistics.

Unlike cosmological constant models, which require new physics at a very low energy scale, $\sim 10^{-4}$ eV, the string dominated model requires the introduction of new physics at the TeV scale, where unitarity arguments in the standard model require new physics (Wilczek 1996).

The observational predictions of the string dominated model are intermediate between the open universe model and the vacuum energy (cosmological constant) model. CMB observations can easily distinguish between an open universe model and the flat universe models (string-dominated, matter-dominated, or vacuum energy-dominated): the open universe model predicts that the Doppler peak should occur at $l \sim 220 \Omega^{-1/2}$. Low redshift measurements can distinguish between the matter, vacuum energy and string dominated models: the string dominated model predicts $q_0 = \Omega_0 / 2$, while the vacuum dominated model predicts $q_0 = 3\Omega_0 / 2 - 1$. Thus, future observations should be able to determine the equation of state of the universe.
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