Texture of fermion mass matrices in partially unified theories

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Abstract

We investigate the texture of fermion mass matrices in theories with partial unification (for example $SU(2)_L \times SU(2)_R \times SU(4)_c$) at a scale $\sim 10^{12}$ GeV. Starting with the low energy values of the masses and the mixing angles, we find only two viable textures with atmost four texture zeros. One of these corresponds to a somewhat modified Fritzsch textures. A theoretical derivation of these textures leads to new interesting relations among the masses and the mixing angles.

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Since the existence of a grand unifying scale is always well motivated, several attempts have been made to unify the $SU(3)_c \times SU(2)_L \times U(1)_Y$ in bigger groups e.g $SU(5)$, $SO(10)$, $E_6$ etc. But it is not decisive yet which one among them is "the grand unifying group". Also it is not clear whether there is no other scale, with some kind of symmetry being broken, in between the weak scale and the grand unifying scale. There can always be a partial unification happening in between the two scales, since the existence of a partially unifying scale around $10^{10} - 10^{12}$ GeV in the SUSY theories is also well motivated for different reasons. For example, one can naturally get a neutrino mass in the interesting region of 3-10 eV, which could serve as hot dark matter candidate. Such a neutrino mass may also be needed to explain the large scale structure of the universe [1]. The window, $10^{10} - 10^{12}$ GeV, is also the right size for a hypothetical PQ symmetry to be broken so as to solve the strong CP problem without creating any phenomenological or cosmological problem [2]. This scale also gives rise to lepton flavor violating processes like, $\mu \rightarrow e\gamma$ [3] and electric dipole moment of electron and neutron [4] which may make it possible to detect this scale in the near future. These lepton flavor violations and the edms get generated in these models without having to make any assumption regarding the location of the soft SUSY breaking terms. They can be anywhere between the GUT scale and the Planck or string scale. The GUT scale (with MSSM being the symmetry below the GUT scale) is lower than the string scale by a factor of 25. However using the intermediate scale around $10^{12}$ GeV, it is possible to push up the GUT scale to the string scale [5]. Also recently, an intermediate scale at $\sim 10^{12}$ GeV has been advocated [6] to produce monopoles that would explain the high energy cosmic ray spectrum. The partially unified models like $SU(2)_L \times SU(2)_R \times SU(4)_C$ gives rise to such monopoles naturally.

It is always a big challenge to find a justification for the the mass distribution of the fermions. So far attempts have been made from the GUT scale and the weak scale [7-9]. In this letter we try to generate the most predictive textures with minimum number of parameters at a scale $\sim 10^{12}$ GeV where the theory is partially unified. We do not use any effect of any particular grand unifying group, however for the partial unification group we
SU(2)_L \times SU(2)_R \times SU(4)_C symmetry. We write down the superpotential for these mass matrices and also we write some new relations among the masses and the mixing angles.

We use the bottom up approach as developed in the reference [9] in order to derive the mass matrices for the quarks and leptons at the scale \( \sim 10^{12} \) GeV. In this method one starts from the known and/or presumed known quark and lepton masses and the mixing angles at the low energy scale and then evolve these parameters, using the appropriate RGE’s for the MSSM [10], to the desired scale which in this case is situated at \( 10^{12} \) GeV. After evolving these parameters, the complex symmetric matrices \( M^U \) and \( M^D \) for the up and down are constructed with texture zeros [11]. For example

\[
M_u = U_u^\dagger D_u U_u = D_u + i\alpha x[D_u, H] - \frac{1}{2}\alpha^2 x^2[[D_u, H], H] + \ldots, \tag{1}
\]

and

\[
M_d = U_d^\dagger D_d U_d = D_d + i\alpha(x - 1)[D_d, H] - \frac{1}{2}\alpha^2(x - 1)^2[[D_d, H], H] + \ldots, \tag{2}
\]

where \( D_u \) and \( D_d \) are diagonal up and down mass matrices, \( \alpha \) is a real number and \( H \) is some hermitian matrix. Using Sylvester’s theorem the expression for \( \alpha H \) is given as:

\[
i\alpha H = \sum_{k=1}^{3} \frac{\ln(v_k)}{\prod_{i \neq k}(v_k - v_i)} \frac{\prod_{i \neq k}(V - v_i \times 1)}{\Pi_{i \neq k}(v_k - v_i)} \tag{3}
\]

where \( V \) is the CKM matrix, and \( v_i \)’s are its eigenvalues which are kept nondegenerate.

The parameter \( x \) can be varied from 0 to 1 to obtain different textures. The elements of those textures are then compared to remove the lowest ones in order to get the textures with maximum numbers of zeros. After constructing these textures, the masses and mixing angles are derived and then are evolved to the weak scale to be compared with the experimental values.

From our analysis described above, we have found only two types of textures (at the partial unification scale), for the quark mass matrices, that are consistent with the low energy data. Both type has at most four texture zeros. Type 1 in the symbolic form is:

\[
M^U = \begin{pmatrix}
a_u & 0 \\
0 & b_u \\
0 & 0 & c_u
\end{pmatrix}; \tag{4}
\]
and

\[ M^D = \begin{pmatrix} 0 & a_d & a'_d \\ a_d & b_d & b'_d e^{i\beta} \\ a'_d & b'_d e^{i\beta} & c_d \end{pmatrix} \] (5)

Here all the parameters are real. The Type 2 looks like:

\[ M^U = \begin{pmatrix} 0 & a_u & 0 \\ a_u & b_u & b'_u \\ 0 & b'_u & c_u \end{pmatrix} \] (6)

\[ M^D = \begin{pmatrix} 0 & a_d & 0 \\ a_d & b_d & b'_d e^{i\beta} \\ 0 & b'_d e^{i\beta} & c_d \end{pmatrix} \] (7)

For this type we also find the additional relation:

\[ \frac{b'_u}{b_u} = \frac{b'_d}{b_d} \] (8)

Consequently we have one less parameter in this type and obtain one extra prediction. So we will discuss this type 2 in the rest of the paper.

Now we will show analytically how the elements of this texture given by eqn. (6) and eqn. (7), can be related to the masses and the CKM angles and whether there is any further relation among the masses and the mixing angles. The elements of the texture has a hierarchy, \( c \gg b \sim b' \gg a \). Also the matrix \( M^D \) can be re-written by removing the phases: \( M^D = P_D M'^D P_D Q_D \), where P and Q are the phase matrices. To find the quark masses at the scale \( \sim 10^{12} \text{ GeV} \), the matrices \( M^U \) and \( M^D \) need to be diagonalized. We use the orthogonal transformation \( R M R^{-1} = M'^\text{diag} \) to get the eigenvalues \( m_u(d), -m_c(s), m_t(b) \). So the elements of the textures can be written in terms of the masses:

\[ c_u \approx m_d(r); c_d \approx m_b(r); b_u \approx -m_c(r); b_d \approx -m_s(r); a_u^2 \approx m_u(r) m_c(r); a_d^2 \approx m_d(r) m_s(r). \] (9)

where \( r \) is the partial unification scale, \( 10^{12} \text{ GeV} \). From Eqn. (8) and Eqn. (9), we obtain,
\[ \frac{b_u}{b_d} = \frac{b'_d}{b'_d} = \frac{m_c(r)}{m_s(r)} \]  

(10)

The matrix \( R \) looks like:

\[
R = \begin{pmatrix}
1 & s_1 & s_1 s_2 \\
-s_1 & 1 & s_2 \\
0 & -s_2 & 1
\end{pmatrix}
\]

(11)

For the up quark matrix, we obtain

\[ s_1^u \equiv \sin \phi_1^u \simeq \sqrt{\frac{m_u}{m_c}} \]

(12)

and

\[ s_2^u \equiv \sin \phi_2^u \simeq \frac{-b'_u}{c_u} \simeq \frac{-b'_u}{m_t}. \]

(13)

For the down sector we have

\[ s_1^d \equiv \sin \phi_1^d \simeq \sqrt{\frac{m_d}{m_s}} \]

(14)

and

\[ s_2^d \equiv \sin \phi_2^d \simeq \frac{-b'_d}{c_d} \simeq \frac{-b'_d}{m_b}. \]

(15)

Using Eqns (10), (13) and (15), we obtain

\[ \frac{s_2^d}{s_2^d} = \frac{m_t m_s}{m_c m_b} \]

(16)

All the above equations are valid at the partial unification scale, \( r \). So far, other than \( b'_u \) and \( b'_d \) and the phase \( \beta \), all the other elements have been determined in terms of the masses. Since \( b' \)'s are involved in the expressions for \( s_2 \)'s we use the CKM matrix elements. For the determination of phases we also use the CKM. At the scale \( r \) \( V_{\text{CKM}} \) is given by:

\[
V_{\text{CKM}}(r) = R_u \begin{pmatrix}
1 & e^{i\beta} \\
& & e^{i\theta}
\end{pmatrix} R_d^{-1},
\]

(17)
where $\sigma = -2\beta$ and $\tau = -\beta$. Then, in terms the transforming angles and the phases, the CKM element $V_{cb}(r)$ can be written as:

$$|V_{cb}(r)| = \left| s_2^d e^{i\sigma} - s_2^u e^{i\tau} \right|$$

(18)

The eqn. (19) can be approximated as:

$$|V_{cb}(r)| \approx s_2^d (1 - 2k)^{1/2}$$

(19)

where

$$k \equiv \frac{m_c m_b}{m_t m_s}$$

(20)

Using the magnitude of $V_{cb}(r)$, we can solve for $s_2^d$, since $k$ is already known in terms the mass ratios at the intermediate scale. For the determination of the phase we use the CKM element $V_{us}(r)$:

$$|V_{us}(r)| = \left| -s_1^u + s_2^d e^{i\sigma} \right|$$

(21)

Thus we have determined all the parameters of the model in terms of the masses and some of the CKM elements. The remaining CKM angles are the predictions of the model. For example, the model predicts:

$$|V_{ts}(r)| = |V_{cb}(r)|$$

(22)

and

$$V_{td}(r) = s_1^d V_{cb}(r); \quad V_{ub}(r) = s_1^u V_{cb}(r)$$

(23)

For the parametrization-invariant CP violation quantity $J$ [12], we obtain,

$$J = Im[V_{td}^* V_{tb} V_{ub}^* V_{ud}]$$

$$\approx s_1^u s_1^d s_2^d s_2^u \sin 2\beta,$$

(24)

where $J$ is determined at the weak scale. Now, we use the experimental values of the quark masses and the CKM elements $|V_{cb}|$ and $|V_{us}|$ at the weak scale to determine the parameters
of the model. We use \( m_t = 180 GeV \), and the light quark mass ratios \([13] \frac{m_u}{m_d} = 0.55, \frac{m_c}{m_d} = 18.8 \) along with \( m_u(1 GeV) = 5.1 MeV, m_d(1 GeV) = 9.3 MeV, m_e = 1.27 GeV, m_s(1 GeV) = 0.175 GeV, m_b(m_b) = 4.32 GeV, m_s(m_s) = 1.78 GeV. \) The ratio of the values of the Yukawa coupling at the \( m_t \) scale to the 1 GeV or to the corresponding pole mass scale is given by \( \eta \). The values of the \( \eta \) we use are \( \eta_u = 2.4, \eta_d = 2.4, \eta_s = 2.4, \eta_c = 2.1, \eta_t = 1.0158 \). The strong coupling \( \alpha_s \) is taken to be 0.118 along with \( sin^2 \theta_w \) to be 0.2321 and \( \alpha = 1/127.9 \) at the \( M_Z \) scale. Also we use \( |V_{cb}| = 0.040 \) and \( |V_{us}| = 0.22 \) at the \( m_t \) scale. Now we need to evolve these masses and mixing elements to determine their values at the scale \( r \). If we use a large \( tan \beta (tan \beta = 48) \) scenario where all the third generation Yukawa coupling i.e top, bottom and \( \tau \) are same, we get the masses as follows:

\[
\begin{align*}
  m_t(r) &= 158.4 GeV; m_c(r) = 0.33 GeV; m_u(r) = 0.001 GeV; \\
  m_d(r) &= 0.0019 GeV; m_s(r) = 0.043 GeV; m_b(r) = 3.30 GeV.
\end{align*}
\]  

(25)  

(26)  

At the scale \( r \), \( |V_{cb}| \) becomes 0.03 and \( |V_{us}| \) is 0.22. We use these to solve for \( b' \) and the phase \( \beta \). We obtain

\[
\begin{align*}
  s_2^b &= 0.0382; s_2^u = 0.006; s_1^s = 0.059; s_1^d = 0.231 \text{ and } \sin 2\beta = 0.96;
\end{align*}
\]  

(27)  

Since the texture is already determined, we can determine the other CKM parameters and \( J \). Using the above values, the predictions from the model for the other CKM elements and \( J \) at the weak scale are:

\[
\begin{align*}
  V_{td}(m_t) &= 0.009; V_{ts}(m_t) = 0.039; V_{ub}(m_t) = 0.003; J = 3 \times 10^{-5};
\end{align*}
\]  

(28)  

One also can use a low \( tan \beta \) scenario just like the large one.

The superpotential for this texture can easily be written with some discrete symmetry:

\[
W = \lambda_{33} F_3 \bar{F}_3 H_3 + \left( \frac{s_1}{M} \right) \lambda_{32} F_3 F_2 H_2 + \left( \frac{s}{M} \right)^2 \lambda_{22} \bar{F}_2 F_2 H_2 + \left( \frac{s_1}{M} \right)^3 \lambda_{21} \bar{F}_2 F_1 H_1 + h.c.
\]  

(29)  

The fields \( F_i \) and \( \bar{F}_i \) correspond to the matter superfield (2,1,4) and (1,2,\bar{4}). One can choose \( H_1 \) and \( H_3 \) to be the bidoublet Higgs superfields and \( H_2 \) to have the representation (2,2,15)
under the $2_L 2_R 4_e$ symmetry in order to produce a reasonable lepton mass matrix and s’s are the singlet Higgs superfields. The quantum numbers for the field under the discrete symmetry we invoke can be as follows:

$$F_1 : 9; F_2 : -11; F_3 : 2; s : 5; s_1 : -3; H_1 : 11; H_2 : 12; H_3 : -4.$$  \hspace{1cm} (30)

$F_i$ and $\bar{F}_i$ transform in the same way under the discrete symmetry. There can be other set of choices of the quantum numbers. However for this particular choice of quantum numbers the $11$ element of the up and down quark matrices get generated for the 10th power of $s_1$, which can easily be neglected.

In conclusion, we have derived two types of textures at the scale $\sim 10^{12}$ GeV which can give rise to the correct values for the masses of the quarks and the leptons and the CKM angles and the CP violation parameter $J$. These textures are different from those obtained at the GUT scale. Out of these two types we found one has less number of parameters and some new relations among the mixing angles and masses. This particular texture also looks like the Fritzsch texture with the “22” element being non-zero. We also write a superpotential for this texture supported by a model with string unification and an intermediate scale around $10^{12}$ GeV.

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