Vacuum Nucleon Loops and Naturalness

R. J. Furnstahl

Department of Physics
The Ohio State University, Columbus, OH 43210

Brian D. Serot

Department of Physics and Nuclear Theory Center
Indiana University, Bloomington, IN 47405

Hua-Bin Tang

Department of Physics
University of Minnesota, Minneapolis, MN 55455

(October, 1996)

Abstract

Phenomenological studies support the applicability of naturalness and naive
dimensional analysis to hadronic effective lagrangians for nuclei. However,
one-baryon-loop vacuum contributions in renormalizable models give rise to
unnatural coefficients, which indicates that the quantum vacuum is not de-
scribed adequately. The effective lagrangian framework accommodates a more
general characterization of vacuum contributions without reference to a Dirac
sea of nucleons.

PACS number(s): 24.85+p,21.65.+f,12.38.Lg
Variations of the Walecka model are widely applied in descriptions of nuclear structure and reactions [1–4]. These models are appealing both for their phenomenological success and for the simplicity and economy of the physics at the mean-field level. Most practical applications used for quantitative nuclear structure phenomenology include contributions from valence nucleons only; the framework is often labeled a “no sea” approximation to indicate that the Dirac sea of nucleons is neglected [2].

The approach to the relativistic nuclear many-body problem known as quantum hadrodynamics (QHD) was originally based on renormalizable field theory [1,4], so that systematic calculations with a finite number of parameters are possible, at least in principle. In particular, one-baryon-loop vacuum effects can be included as a way to add vacuum dynamics to the “no sea” physics. These effects have a natural interpretation as the response of a filled Dirac sea of nucleons to the presence of valence nucleons [1]. The resulting “relativistic Hartree approximation” (RHA), however, does not provide an acceptable description of the properties of finite nuclei, at least by the standards of modern successful mean-field models [5–9].

The deficiencies of the RHA could imply that one simply needs to work harder to describe vacuum dynamics within the QHD framework. It is known, however, that including higher-order corrections in a simple loop expansion makes the phenomenology worse [6], so that any improvement (if it exists) would require significantly more complicated diagrams, such as those involving vertex corrections or short-range correlations [10,11]. Moreover, one would like to structure the hadronic many-body framework so that the appealing, intuitive, and successful mean-field theory is indeed a good starting point. This suggests an effective field theory (EFT) approach based on the “modern” viewpoint of nonrenormalizable lagrangians [12–17]. This viewpoint is well known in the particle physics community (for systems at zero density) and is adopted in applications of chiral perturbation theory to meson–meson, meson–nucleon, and nucleon–nucleon scattering. However, it is not well known or applied in the community of nuclear physicists who use relativistic models for nuclear structure.

From the EFT perspective, a description including only valence nucleons and classical
("mean") meson fields can still incorporate vacuum dynamics, hadron compositeness, and many-body correlations, albeit approximately [9,18,19]. The price to be paid is that all possible terms consistent with the underlying symmetries (excluding redundancies) must appear in the effective lagrangian. Nevertheless, by relying on the concept of "naturalness" (as defined below), it is possible to systematically truncate the effective lagrangian, leaving only a finite number of unknown parameters; moreover, recent fits to empirical nuclear properties using this framework give strong evidence that the model parameters are indeed natural [9,18,19].

From this point of view, the RHA in renormalizable QHD models is simply one specific prescription for determining an infinite number of parameters in the effective theory, namely, the coefficients in the scalar effective potential and of terms involving derivatives of the boson fields. Here we assess the relevance of the RHA prescription by examining the size of these coefficients. We find that the RHA leads to unnaturally large coefficients, in disagreement with results obtained from fits to empirical nuclear properties [19]. This implies that although it may be possible to explicitly include vacuum dynamics by calculating baryon vacuum loops, it is much more efficient to include them implicitly in the small number of natural parameters contained in the truncated effective lagrangian.

There have been more formal criticisms of the RHA: (1) the RHA vacuum contributions violate $N_c$ counting rules motivated from quantum chromodynamics (QCD) [20,21], (2) the RHA neglects the compositeness of the nucleons [22–24], and (3) the treatment of the vacuum in terms of $NN$ pairs alone is simply wrong, or at best incomplete [22,24–27]. It seems much more compelling to us that one should avoid explicit baryon-loop calculations simply because the empirical properties of nuclei show that dynamical vacuum effects are quite modest and can be described with a few adjustable parameters.

Georgi and Manohar [28,15] have proposed a naive dimensional analysis (NDA) for assigning a coefficient of the appropriate size to any term in an effective lagrangian for the strong interaction. This NDA has been extended to effective hadronic lagrangians for nuclei [19]. The basic assumption of "naturalness" is that once the appropriate dimensional scales
have been extracted using NDA, the remaining overall dimensionless coefficients should all be of order unity. For the strong interaction, there are two relevant scales: the pion-decay constant \( f_\pi \approx 93 \text{ MeV} \) and a larger scale \( 0.5 \lesssim \Lambda \lesssim 1 \text{ GeV} \), which characterizes the mass scale of physics beyond Goldstone bosons. The NDA rules prescribe how these scales should appear in a given term in the lagrangian density:

1. Include a factor of \( 1/f_\pi \) for each strongly interacting field.
2. Assign an overall normalization factor of \( f_\pi^2 \Lambda^2 \).
3. Multiply by factors of \( 1/\Lambda \) to achieve dimension (mass)\(^4\).
4. Include appropriate counting factors (such as \( 1/n! \) for \( \phi^n \)).

The appropriate mass for \( \Lambda \) might be the nucleon mass \( M \) or a non-Goldstone boson mass; the difference is not important for numerical assessments of naturalness, but will be relevant for the \( N_c \) counting arguments considered later.

As an example of the NDA prescription, a term in the scalar effective potential takes the form

\[
\kappa_n \frac{1}{n!} f_\pi^2 \Lambda^2 \left( \frac{\phi}{f_\pi} \right)^n.
\]

(1)

The coupling constant \( \kappa_n \) is dimensionless and of \( O(1) \) if naturalness holds. Until one can derive the effective lagrangian from QCD, the naturalness assumption must be checked by fitting to empirical nuclear data. Such fits give strong support for naturalness \cite{19}.

We can assess the naturalness of the RHA vacuum contributions by matching the results in a renormalizable model to an effective mean-field theory in which the Dirac sea degrees of freedom are excluded by construction. The matching to the RHA is conveniently made with an effective action, in which the Dirac sea contribution is easily isolated. (See, for example, Ref. \cite{9}.) The one-loop Dirac sea effective action \( \Gamma' \) in models with Yukawa couplings to the nucleon takes the (unrenormalized) form

\[
\Gamma' \equiv -i\hbar \text{ Tr } \ln(i\gamma^\mu \partial_\mu - M^* - g_\nu \gamma^\mu V_\mu),
\]

(2)
where $M^* \equiv M - g_\phi \phi$, $\phi$ and $V^\mu$ are neutral scalar and vector fields, $g_\phi$ and $g_v$ are their couplings to the nucleon, and the trace is over spatial and internal variables. For time-independent background meson fields, the effective action is proportional to the energy.\(^1\)

The expression in Eq. (2) can be renormalized and evaluated as a derivative expansion \(^{29}\) in the $\phi$ and $V^\mu$ fields:

$$
\Gamma'_{\text{ren}} = \int d^4x \left[ -U_{\text{eff}}(\phi) + \frac{1}{2} Z_{1s}(\phi) \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} Z_{2s}(\phi)(\Box \phi)^2 
+ \frac{1}{4} Z_{1v}(\phi) F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} Z_{2v}(\phi)(\partial_\alpha F^{\alpha\mu})(\partial_\beta F_{\beta\mu}) + O(1/M^3) \right].
$$

This expansion in inverse powers of $M^*$ converges rapidly for finite nuclei \(^{30-33}\). The effective potential $U_{\text{eff}}$ and the coefficient functions $Z_i$ can be further expanded in (infinite) polynomials in $\phi$ with well-behaved coefficients; that is, there is a local expansion of $\Gamma'$, which can be absorbed into an effective lagrangian for nuclei. Thus the finite Dirac sea contribution can be reproduced by an effective lagrangian treated at the mean-field level with only valence nucleons, as long as all possible terms (generally nonrenormalizable) are included.

The effective potential $U_{\text{eff}}(\phi)$ is found by evaluating the trace in Eq. (2) with constant fields. This expression is divergent and must be regularized and renormalized to obtain a finite result, which takes the general form \(^1\)

$$
U_{\text{eff}}(\phi) = -\frac{\gamma}{(4\pi)^2} \left( M^* \text{ln} \frac{M^*}{\mu} + \sum_{n=0}^4 \alpha_n M^{1-n}(g_\phi)^n \right),
$$

where $\gamma$ is the spin-isospin degeneracy and the $\alpha_n$ are dimensionless constants. The scale parameter $\mu$ is typically chosen to be $M$. The counterterms $\alpha_0$ and $\alpha_1$ are fixed by requiring $U_{\text{eff}}$ to be zero and a minimum in the vacuum ($\phi = 0$). The others are fixed by prescription.\(^3\)

\(^1\)The contribution to the energy from Eq. (2) can be written more transparently as a sum over single-particle energies of occupied states in the filled Dirac sea \(^1\). In nuclear matter, this (unrenormalized) energy density is $\Delta E = V^{-1} \sum_{k,\lambda} \left[ (k^2 + M^*)^{1/2} - (k^2 + M^2)^{1/2} \right]$. 

5
tion. Note that in a renormalizable model, only the first four powers of $\phi$ are available as counterterms. In an effective lagrangian, however, all powers are present.

The most common prescription has been to choose the $a_n$ to cancel the first four powers of $\phi$ appearing in the expansion of the logarithm [1]. One finds in this case (for $\gamma = 4$)

$$U_{\text{eff}}(\phi) = -\frac{1}{4\pi^2} [(M - \Phi)^4 \ln(1 - \frac{\Phi}{M}) + M^3\Phi - \frac{7}{2} M^2\Phi^2 + \frac{13}{3} M\Phi^3 - \frac{25}{12} \Phi^4]$$

$$= \frac{M^4}{4\pi^2} \left\{ \frac{\Phi^5}{5M^5} + \frac{\Phi^6}{30M^6} + \frac{\Phi^7}{105M^7} + \cdots + \frac{4!(n-5)!}{n!} \frac{\Phi^n}{M^n} + \cdots \right\},$$

(5)

where $\Phi \equiv g_n\phi$. When $\phi$ (or $M^*)$ is determined at each density by minimization, $U_{\text{eff}}(\phi)$ is the finite shift in the baryon zero-point energy that occurs at finite density and is analogous to the “Casimir energy” that arises in quantum electrodynamics.

To evaluate the size of the one-loop vacuum correction, we apply the NDA. Based on the scaling rules discussed above, a term of $O(\phi^5)$ should be scaled as

$$\frac{M^2}{5f_\pi^2} \phi^5,$$

(6)

where we have associated $\Lambda$ with the nucleon mass $M$. (See, however, the comments on $N_c$ scaling below.) If this contribution is natural, any residual overall constant should be of order unity. However, if we perform a similar scaling on the leading term in Eq. (5), we find

$$\frac{M^4}{4\pi^2} \frac{g_n^5\phi^5}{5M^5} \rightarrow \frac{4}{5} \frac{M^2}{f_\pi^2} \phi^5 = 96 \left(\frac{M^2}{5f_\pi^2} \phi^5 \right),$$

(7)

where we used $4\pi f_\pi \approx M$ and $g_n \approx M/f_\pi$. Thus the one-baryon-loop contribution to the vacuum energy is roughly two orders of magnitude larger than naturalness requires. It is not hard to show from Eq. (5) that all higher powers of $\Phi$ contain essentially the same large overall factor.

We can make an instructive comparison of natural and unnatural coefficients using the linear sigma model, generalized to include a neutral vector meson. Variations of this model have been used to investigate the role of chiral symmetry in nuclear structure [34,1,4]. After

2The limitations of this approach are discussed in Refs. [8] and [9].
following conventional procedures to introduce a nonzero expectation value for the scalar field and then shifting the field, we obtain the model lagrangian (with $m_\pi = 0$ for simplicity):

$$
\mathcal{L}_{\sigma\omega} = \bar{\psi}[i\gamma_\mu \partial^\mu - g_\nu \gamma_\mu V^\mu - (M - g_\pi \sigma) - ig_\pi \gamma_5 \mathbf{T} \cdot \mathbf{\pi}]\psi \\
+ \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4}(\partial_\mu V_\nu - \partial_\nu V_\mu)^2 + \frac{1}{2}m_\pi^2 V_\mu V^\mu \\
+ \frac{1}{2}\partial_\mu \mathbf{\pi} \partial^\mu \mathbf{\pi} + g_\pi \frac{m_\sigma^2}{2M}\sigma(\sigma^2 + \mathbf{\pi}^2) - g_\pi^2 \frac{m_\sigma^2}{8M^2}(\sigma^2 + \mathbf{\pi}^2)^2 ,
$$

(8)

where the associations $\sigma \rightarrow \phi$, $g_\pi \rightarrow g_8$, and $m_\sigma \rightarrow m_8$ should be made for our discussion. Comparing to Eq. (1), with $\Lambda$ identified as $m_8$, we find $\kappa_3 = -\kappa_4 = -3$, so that the nonlinear parameters are natural at the mean-field level. However, if one includes the one-baryon-loop vacuum corrections, renormalized in a fashion that preserves the chiral symmetry [34,8], one finds unnatural corrections to the cubic and quartic couplings: $\Delta\kappa_3 = 2M^2/\pi^2 f_\pi^2 \approx 20$, $\Delta\kappa_4 = -8M^2/\pi^2 f_\pi^2 \approx -80$. The quintic and higher corrections are exactly the same as in Eq. (5).

These unnatural coefficients generate correspondingly large corrections to the conventional linear Walecka model mean-field theory (MFT). If we adjust the model parameters to reproduce the standard nuclear matter properties in the RHA (equilibrium at Fermi momentum $k_F^0 = 1.30 \text{fm}^{-1}$ with a binding energy of 15.75 MeV), the baryon effective mass at equilibrium becomes $M^*/M \approx 0.73$. This translates into a change in the scalar potential $\Phi$ from 430 MeV in the MFT to 250 MeV in the RHA. This is a large effect, particularly since it is the $\phi^0$ term that makes the difference. When a general effective lagrangian is fit to nuclear properties, terms of this order play essentially no role [19].

Contributions to the scalar potential at equilibrium density from $U_{\text{eff}}(\phi)$ and from the original tree-level lagrangians are plotted in Fig. 1 for the Walecka model and in Fig. 2 for the linear sigma model. In each figure, the crosses indicate contributions to the energy density estimated using naturalness, with error bars allowing for a range of $\Lambda$ and $M^*/M$. A steady

\[3\text{This is to be expected; the model builds in at tree level a realization of QCD chiral symmetry and Goldstone-boson physics, which is the same physics behind the NDA.}\]
FIG. 1. Contributions to the scalar potential per particle in nuclear matter from the $n^{\text{th}}$-order terms of the form $\Phi^n$ for the RHA model. The crosses are estimates based on Eq. (1). The arrow indicates the total binding energy $\epsilon_0 = 15.75$ MeV.

decrease with increasing $n$ is evident, which motivates the truncation of effective lagrangians with natural coefficients. $M^*/M$ in natural models (and phenomenologically successful ones) is typically between 0.60 and 0.66. For those values, the RHA $O(\Phi^5)$ contribution would be as large as a typical $O(\Phi^3)$ contribution in a natural model (see Fig. 1) and would prevent a successful description of nuclear structure. In fact, the unnatural and unbalanced $O(\Phi^5)$ term drives $M^*/M$ to its self-consistent value of 0.73. Higher-order contributions are essentially negligible, because of the natural factor of $\phi/f_\pi$ (and a combinatoric factor) that accompanies each higher power. In the linear sigma model, which has unnatural $O(\Phi^3)$ and $O(\Phi^4)$ contributions from the RHA, $M^*/M$ is driven to 0.9.

It is possible to devise a prescription [35] that leads to an effective mass $M^*/M$ and a nuclear compressibility consistent with a reasonable (although not optimal) fit to properties of finite nuclei (see Ref. [18] for the criteria). However, this requires choosing $\alpha_n$ coefficients in Eq. (4) to achieve sensitive cancellations between terms of different order in the effective potential, so as to neutralize the effect of the unnatural $\phi^5$ contribution. The unnaturalness
FIG. 2. Contributions to the scalar potential per particle in nuclear matter from the $n^{\text{th}}$-order terms of the form $\Phi^n$ for the linear sigma model plus RHA. The crosses are estimates based on Eq. (1). The arrow indicates the total binding energy $\epsilon_0 = 15.75$ MeV.

of the vacuum nucleon loop contributions is generally characterized by the changes in the coefficients that accompany $O(1)$ changes in the scale $\mu$ [say, from $\mu$ to $\epsilon \mu$ in Eq. (4)], as in Ref. [28]. The subsequent changes in the $\alpha_n$ coefficients are obtained by expanding $\gamma M^*/(4\pi^2)$. These changes are large compared to the natural size implied by Eq. (1).

Based on the strong empirical evidence for naturalness, we conclude that the treatment of the quantum vacuum at the one-baryon-loop level is, at best, inadequate. Although the concept of a Dirac sea is compelling for nuclear physicists because of the analogy to the Fermi sea, the explicit calculation of these effects prejudices the description of the vacuum dynamics and (to date) has not yielded results consistent with nuclear structure phenomenology. Moreover, the self-consistent, valence-nucleon-only theory is covariant, causal, and internally consistent by itself, and the empirical evidence shows that the vacuum degrees of freedom can be included implicitly by a small number of local interactions among the mesons and valence nucleons. Note that the omission of explicit dynamical contributions from the Dirac sea does not mean that one can discard negative-energy solutions entirely; they must be
retained to ensure the completeness of the Dirac wave functions in certain calculations of density-dependent effects (for example, linear response) [36,37].

It should also be emphasized that an effective lagrangian allows for a more general characterization of the vacuum dynamics than that arising from baryon loops. The explicit calculation of counterterms in the effective lagrangian is unnecessary, since the end result is simply an infinite polynomial in the scalar field, with finite, unknown, and apparently natural coefficients arising from the underlying dynamics of QCD. To have predictive power, one must rely either on the truncation scheme provided by naturalness (see Ref. [19]), so that only a small, finite number of unknown coefficients are relevant, or on some other dynamics to constrain the form of the renormalized scalar potential. For example, a simple model is used in [9] to show how the broken scale invariance of QCD leads to dynamical constraints on the scalar potential, and fits to the properties of finite nuclei also generate coefficients that are natural.

Another potential difficulty in the explicit calculation of baryon vacuum loops is that RHA vacuum contributions violate $N_c$ counting rules motivated by consistency with QCD [20,21]. The expected $N_c$ scaling of the coupling for an $n$-meson vertex is $O(N_c^{1-n/2})$ [38]. The $N_c$ scaling property of a vertex in an effective lagrangian or the RHA is established with the associations $M \propto O(N_c)$, $m_s \propto O(1)$, and $g_s^2 \propto O(N_c)$ [38]. [Note also that $f_\pi \propto O(N_c^{1/2})$.] Thus one may simply inspect the coefficient of $\phi^n$. For the RHA, the result is $O(N_c^{1-n/2})$, which exceeds the expected scaling by a factor of $N_c^3$ [20,21].

In contrast, the effective lagrangian approach, with our definition of naive dimensional analysis and naturalness, is consistent with the scaling expected from QCD if one associates $\Lambda$ in the $\phi^n$ terms with $m_s$ (and not $M$). For example, applying the $N_c$ scaling rules to the coefficient of $\phi^5$ produces

$$\kappa_5 \frac{1}{5!} f_\pi^2 \Lambda^2 \frac{\phi^5}{f_\pi^5} \longrightarrow \kappa_5 \frac{1}{5!} (m_s^2 \phi^2) \frac{\phi^5}{f_\pi^3} \longrightarrow O(1) \times O(N_c^{-3/2}) \propto O(N_c^{-3/2}) ,$$

which is consistent with the usual $N_c$ counting. It is important here that the normalization factor $\Lambda^2 f_\pi^2$ comes from the meson mass term, with the meson mass $m_s$ being $O(1)$ rather
than $O(N_c)$. Thus $N_c$ counting implies that the vacuum response is $\bar{q}q$ in nature; the response of a $NN\bar{N}$ vacuum is unlikely to agree with the large $N_c$ limit.

Finally, we comment on the statement often made or implied in the literature that hadronic field theories treat nucleons as point particles and so do not account for the com-

positeness of nucleons. The use of effective lagrangians with fields for composite particles is by now well established (e.g., chiral perturbation theory with nucleons and pions). In Ref. [19], the electromagnetic structure of the nucleon manifestly appears at the mean-field level using a hadronic effective lagrangian for nuclei. The key features are the inclusion of nonrenormalizable interaction terms and the use of a derivative expansion to incorporate the nucleon compositeness; indeed, the allowed freedom in the definition of the boson fields shows that even renormalizable terms like $\phi^3$ and $\phi^4$ implicitly include the effects of nucleon substructure. Thus, the deficiencies of the RHA are not intrinsic to the use of nucleons (and the Dirac equation) to describe nuclei, but arise because the implied vacuum dynamics is incorrect or incomplete.

In summary, we have examined the relativistic Hartree approximation (RHA) to renor-

malizable hadronic field theories in the context of modern effective field theory. The param-

eters obtained in phenomenologically successful mean-field models of finite nuclei exhibit naturalness, as anticipated from naive dimensional analysis of the strong interaction. How-

ever, the parameters implied from vacuum loops in the RHA are unnatural. Since the phenomenological parameters in successful models implicitly include the effects of vacuum dynamics, we conclude that the explicit treatment of the vacuum in the RHA, which involves the Dirac sea of nucleons, is inadequate. In contrast, a natural effective model with only valence nucleons, including all possible (nonredundant) terms, is consistent with nucleon compositeness, $N_c$ counting, and nuclear structure phenomenology. Extensions to include higher-order vacuum loops will be discussed in subsequent work.
ACKNOWLEDGMENTS

We thank P. Ellis, S. Rudaz, and J. Rusnak for useful discussions. This work was supported in part by the Department of Energy under Contracts No. DE-FG02-87ER40365 and DE-FG02-87ER40328, the National Science Foundation under Grants No. PHY-9511923 and PHY-9258270, and the A. P. Sloan Foundation.
REFERENCES


    C. M. Fraser, Z. Phys. C 28 (1985) 101;


