Baryogenesis from baryon number violating scalar interactions.

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Abstract

In the following work we consider the possibility of explaining the observed baryon number asymmetry in the universe from simple baryon number violating modifications, involving massive scalar bosons, to the Standard Model. In these cases baryon number violation is mediated through a combination of Yukawa and scalar self-coupling interactions. Starting with a previously compiled catalogue of baryon-number violating extensions of the Standard Model, we identify the minimal subsets which can induce a $B - L$ asymmetry and thus be immune to sphaleron washout. For each of these models, we identify the region of parameter space that leads to the production of a baryon number asymmetry of the correct order of magnitude.
1. Introduction and Motivation

The excess of matter over antimatter observed in the present day universe can be explained by proposing that there existed suitable baryon number violating physical processes which operated in the early stages of the universe. These physical processes led to the situation whereby the production of baryons was slightly greater than the production of antibaryons thereby producing the required baryon-antibaryon asymmetry. In fact the quark-antiquark asymmetry required in the early universe ($t < 10^{-6}$ sec) to give rise to the present excess of matter over antimatter is very small:

$$\frac{n_q - n_{\bar{q}}}{n_q} \simeq 3 \times 10^{-8}. \quad (1)$$

Thus the processes leading to the quark-antiquark asymmetry need only be minute to account for baryogenesis.

The three basic requirements for baryogenesis, which were first compiled in [1], are: (a) Baryon number violation: if baryon number was not violated then the present day baryon asymmetry could only be explained by an initial baryon number asymmetry; (b) C and CP violation: even with the existence of baryon number violation, a baryon asymmetry would not have arisen unless there was a preference for the production of matter or antimatter in the early universe; and (c) Nonequilibrium conditions: if the universe were in a continual state of thermal equilibrium, the phase space densities of baryons and antibaryons would necessarily be equal, owing to the fact that by CPT invariance the baryon and antibaryon masses are always equal.

Ref[2] provides a catalogue of all of the simplest extensions of the Standard Model that explicitly violate baryon number. (With lepton number broken explicitly through neutrino Majorana masses, all of the global $U(1)$ symmetries of the Standard Model are broken and an understanding of charge quantisation results [3]. The charge quantisation issue provides a strong theoretical motivation for constructing all of the simplest Standard Model extensions which explicitly break $B$. See Ref[2] for further discussion.) In each of these models baryon number violation is mediated through scalar bosons, which are required to be massive so as to ensure that nucleon decay remain unobserved. In the following work we will be considering the possibility that the out of equilibrium decays of these massive scalar bosons in the early universe could account for the present day baryon number asymmetry. This work differs from similar calculations arising from GUT models [4] in that most of the models we will consider break baryon number through a combination of Yukawa interactions and scalar self-couplings rather than through the baryon number violating Yukawa interactions associated with a single massive particle.

Starting with the baryon number violating interactions proposed in [2] we are left with the task of both demonstrating the existence of and calculating the magnitude of the CP violation arising in each of the baryon number violating
models. In each case there will be two opposing decay paths for our massive scalar each giving rise to products with different baryon numbers. We can analyse this type of system using the following procedure:

If we assume for simplicity that our scalar boson $X$ has the two quark lepton final states $qq(B = -1/3)$ and $q\bar{q}(B = 2/3)$, then C and CP are violated if the branching ratio of the $X$ to the $q\bar{q}$ final state ($= r$) is unequal to the branching ratio of the $\bar{X}$ to the $ql$ final state ($= \bar{r}$), i.e. $r \neq \bar{r}$. We also know that CPT invariance requires that the total decay rates of $X$ and $\bar{X}$ be equal. Therefore we can write the branching ratios of the corresponding decays $X \to qq$ and $\bar{X} \to q\bar{q}$, as $1 - r$ and $1 - \bar{r}$ respectively. From a symmetric initial state consisting of $X$ and $\bar{X}$, provided there are no further baryon number violating reactions, a net baryon asymmetry will exist after all of the $X$ and $\bar{X}$ decay, with the mean net baryon number produced being given by,

$$\Delta B = -\frac{1}{3}r + \frac{2}{3}(1 - r) + \frac{1}{3}\bar{r} - \frac{2}{3}(1 - \bar{r}) = -(r - \bar{r}).$$  \hspace{1cm} (2)

We can therefore evaluate the mean net baryon number produced by calculating the difference in the branching ratio for boson and antiboson decay for just one of the decay channels.

We are hence essentially left with the task of calculating $r - \bar{r}$, which is a measure of the CP violation in the system. CP violating effects come from considering the interaction of the lowest order decay diagrams together with their loop corrections. It can be shown that the intermediate states in the loops must not only have CP violating complex couplings, but must also propagate on shell, thereby leading to complex Feynman amplitudes. Another important requirement is that the loop corrections involve baryon number violating interactions (see [5]). In general we expect $\Delta B$ to be of order $\alpha^N$, where $\alpha$ characterises the coupling constants of the loop particles, and $N$ is the number of loops in the lowest order diagram which interferes with the tree graph to give a non zero value for $\Delta B$.

2. The Model:

The following modifications to the standard model were devised with the primary aim of obtaining complete electric charge quantisation from the gauge invariance of the Lagrangian.

Complete charge quantisation through gauge invariance can be obtained provided there is only one unembedded $U(1)$ invariance, and we obtain the correct charge quantisation provided the generator of this $U(1)$ invariance is the standard weak hypercharge $Y$. If there is more than one unembedded $U(1)$ invariance then the actual weak hypercharge of the theory can be chosen to be some linear combination of the standard model hypercharge and these additional symmetries of the theory.
As it stands the three-generation minimal standard model has five $U(1)$ invariances, the standard weak hypercharge $Y$, baryon number $B$, and the three family lepton numbers $L_e$, $L_\mu$, and $L_\tau$. These five $U(1)$ invariances correspond to there being four classically undetermined electric charges. To remove these invariances we must construct extensions of the minimal standard model which break $B$ and $L_i$ but leave $Y$ exact.

The simplest and most interesting way to break $U(1)_L$ is to introduce non-zero neutrino masses. This is most easily done by introducing right handed neutrinos into the model, where we choose that our right and left handed neutrinos be related through Dirac and Majorana mass terms.

This leaves us with just one undetermined electric charge, which can be taken to be the electric charge of the down quark $Q(d)$, or equivalently the hypercharge of the down quark $y_d$. Our four parameter uncertainty has thus been reduced to a two parameter uncertainty by this simple extension of the lepton sector. For more information see [2].

We are therefore left with the task of breaking this unwanted baryon symmetry without affecting the $U(1)_Y$ hypercharge symmetry. This dual requirement is acheived by extending the standard model further to include either one or two exotic scalar multiplets together with a set of baryon number violating Yukawa and scalar-scalar self interactions. For further information on these models see [2].

The Yukawa interactions and the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ representations, where the hypercharge $Y$ is parameterised in terms of the unknown down quark hypercharge $y_d$, is listed below:

\[
\begin{align*}
\sigma_{1.1} & \sim \bar{Q}_L f_L \sim \bar{u}_R (e_R)^c \sim \bar{d}_R (\nu_R)^c \sim (3, 1, -y_d)(-1/3) \\
\sigma_{1.2} & \sim \bar{Q}_L (f_L)^c \sim (3, 3, -y_d)(-1/3) \\
\sigma_2 & \sim \bar{Q}_L f_R \sim (3, 2, -3 - y_d)(-1/3) \\
\sigma_{3.1} & \sim \bar{Q}_L (Q_L)^c \sim \bar{u}_R (d_R)^c \sim (3, 1, -2 - 2y_d)(-2/3) \\
\sigma_{3.2} & \sim \bar{Q}_L (Q_L)^c \sim (3, 3, -2 - 2y_d)(-2/3) \\
\sigma_{3.3} & \sim \bar{Q}_L (Q_L)^c \sim \bar{u}_R (d_R)^c \sim (6, 1, -2 - 2y_d)(-2/3) \\
\sigma_{3.4} & \sim \bar{Q}_L (Q_L)^c \sim (6, 3, -2 - 2y_d)(-2/3) \\
\sigma_4 & \sim \bar{u}_R (\nu_R)^c \sim (3, 1, -2 - 2y_d)(-1/3) \\
\sigma_5 & \sim \bar{d}_R f_L \sim (3, 2, -1 - y_d)(-1/3) \\
\sigma_{6.1} & \sim \bar{u}_R (u_R)^c \sim (3, 1, -4 - 2y_d)(-2/3) \\
\sigma_{6.2} & \sim \bar{u}_R (u_R)^c \sim (6, 1, -4 - 2y_d)(-2/3) \\
\sigma_7 & \sim \bar{d}_R (d_R)^c \sim (3, 1, -2y_d)(-2/3) \\
\sigma_8 & \sim \bar{d}_R (e_R)^c \sim (6, 1, -2y_d)(-2/3) \\
\end{align*}
\]

Note that we have used the following notation for the standard model fermions and right handed neutrinos:

\[
\begin{align*}
Q_L & \sim (3, 2, 1 + y_d), \quad u_R \sim (3, 1, 2 + y_d), \quad d_R \sim (3, 1, y_d). \\
\end{align*}
\]
The fermion interactions, $\bar{Q}_L(Q_L)^c$, $\bar{Q}_L(Q_L)^c$, $\bar{u}_R(u_R)^c$, and $\bar{d}_R(d_R)^c$ associated with the $\sigma_{3,2}, \sigma_{3,3}, \sigma_{6,1}$, and $\sigma_{7,1}$ scalars are flavour antisymmetric.

The simplest baryon number breaking scalar interactions were obtained by noting that certain pairs of scalars in the above list have group properties which can be related by conjugation subject to the required constraint $y_d = -2/3$. For these cases the two distinct scalars listed in Eq(3) can be considered as representing one particle with two sets of baryon number breaking Yukawa interactions. There are three such conjugate pairs which we list below:

$$\sigma_{1,1} = \sigma_{5,1}^c \sim (3,1,2/3)$$
$$\sigma_{1,2} = \sigma_{5,2}^c \sim (\bar{3},3,2/3)$$
$$\sigma_4 = \sigma_{7,1}^c \sim (\bar{3},1,-4/3).$$

Baryon number violation is also obtainable by considering the scalar-scalar interactions of the scalars listed above. By considering every possible scalar combination in Eq(3), two lists of possible charge quantising scalar potentials were compiled, corresponding to $\Delta B = 1$ baryon number violating processes and $\Delta B = 2$ baryon number violating processes respectively. The $\Delta B = 1$ list is shown below:

$$\begin{align*}
\sigma_1, \sigma_2 & \rightarrow \sigma_{1,2} \sigma_{1,2}^c \phi \\
\sigma_1, \sigma_3 & \rightarrow \sigma_{1,1} \sigma_{3,1} + \sigma_{1,1} \sigma_{1,1}^c \sigma_{1,1} \sigma_{3,1} + \sigma_{1,1} \sigma_{3,1} \sigma_{1,1}^c + \sigma_{1,1} \sigma_{3,1}^c \sigma_{1,1} \\
& \rightarrow \sigma_{1,2} \sigma_{3,2} + \sigma_{1,2} \sigma_{1,2}^c \sigma_{1,2} \sigma_{3,2} + \sigma_{1,2} \sigma_{3,2} \sigma_{1,2}^c + \sigma_{1,2} \sigma_{3,2}^c \sigma_{1,2} \\
\sigma_1, \sigma_5 & \rightarrow \sigma_{1,1} \sigma_{5,5} \\
& \rightarrow \sigma_{1,2} \sigma_{1,2} \sigma_{5,5}^c \\
\sigma_1, \sigma_6 & \rightarrow \sigma_{1,2} \sigma_{6,1} \phi \\
\sigma_1, \sigma_7 & \rightarrow \sigma_{1,2} \sigma_{7,1} \phi^c \phi^c \\
\sigma_2, \sigma_3 & \rightarrow \sigma_{2,2} \sigma_{3,2} \sigma_{3,2}^c \\
\sigma_2, \sigma_7 & \rightarrow \sigma_{2,2} \sigma_{7,1} \phi \\
\sigma_3, \sigma_4 & \rightarrow \sigma_{3,2} \sigma_{4,2} \phi \\
\sigma_3, \sigma_5 & \rightarrow \sigma_{3,1} \sigma_{5,5} \\
& \rightarrow \sigma_{3,2} \sigma_{3,2} \sigma_{5,5}^c \\
\sigma_3, \sigma_8 & \rightarrow \sigma_{3,2} \sigma_{8,1} \phi^c \phi^c \\
\sigma_4, \sigma_7 & \rightarrow \sigma_{4,2} \sigma_{7,1} + \sigma_{4,2} \sigma_{7,1} \sigma_{7,1} \sigma_{7,1} + \sigma_{4,2} \sigma_{7,1} \sigma_{7,1}^c + \sigma_{4,2} \sigma_{7,1}^c \sigma_{7,1} \\
\sigma_5, \rho & \rightarrow \sigma_{5,2} \sigma_{5,2} \sigma_{5,2} \\
\sigma_5, \sigma_5 & \rightarrow \sigma_{5,2} \sigma_{5,2} \sigma_{5,2} \\
\sigma_7, \sigma_5 & \rightarrow \sigma_{5,7,1} \phi \\
\sigma_6, \sigma_8 & \rightarrow \sigma_{6,1} \sigma_{8,1} \sigma_{6,1} \sigma_{8,1} \sigma_{6,1} \sigma_{8,1} + \sigma_{6,1} \sigma_{8,1} \phi^c + \sigma_{6,1} \sigma_{8,1}^c \phi^c \\
& \rightarrow \sigma_{6,1} \sigma_{8,1} + \sigma_{6,1} \sigma_{8,1} \sigma_{6,1} \sigma_{8,1} \sigma_{6,1} \sigma_{8,1} + \sigma_{6,1} \sigma_{8,1} \sigma_{6,1} \sigma_{8,1} \sigma_{6,1} \sigma_{8,1} + \sigma_{6,1} \sigma_{8,1} \phi^c + \sigma_{6,1} \sigma_{8,1}^c \phi^c
\end{align*}$$

where $\phi$ represents the SM Higgs scalar $\phi \sim (1,2,1)$, and $\rho$ represents a new Higgs-like scalar $\rho \sim (8,2,1)$. Similarly the $\Delta B = 2$ list of scalar potentials take
the form:

\[
\begin{align*}
\sigma_1, \sigma_3 & \rightarrow \sigma_1\sigma_3\sigma_1\sigma_3, \\
& \rightarrow \sigma_1\sigma_3\sigma_1\sigma_2, \\
& \rightarrow \sigma_1\sigma_3\sigma_1\sigma_3, \\
& \rightarrow \sigma_1\sigma_3\sigma_1\sigma_4, \\
& \rightarrow \sigma_1\sigma_3\sigma_1\sigma_1, \\
& \rightarrow \sigma_1\sigma_3\sigma_1\sigma_2, \\
& \rightarrow \sigma_1\sigma_3\sigma_1\sigma_3, \\
& \rightarrow \sigma_1\sigma_3\sigma_1\sigma_4. \\
\sigma_3, \sigma_7 & \rightarrow \sigma_3\sigma_3\sigma_7, \\
& \rightarrow \sigma_3\sigma_3\sigma_7, \\
& \rightarrow \sigma_3\sigma_3\sigma_7, \\
& \rightarrow \sigma_3\sigma_3\sigma_7, \\
& \rightarrow \sigma_3\sigma_3\sigma_7, \\
& \rightarrow \sigma_3\sigma_3\sigma_7, \\
\sigma_4, \sigma_7 & \rightarrow \sigma_4\sigma_7, \\
& \rightarrow \sigma_4\sigma_7, \\
\sigma_7, \sigma_7 & \rightarrow \sigma_7\sigma_7, \\
& \rightarrow \sigma_7\sigma_7.
\end{align*}
\]

(7)

The above scalar potential terms can be placed into groups consisting of quadratic, cubic and quartic terms. As shown in [2] each of these different groups of scalar interactions are of different phenomenological interest, with the higher order interactions (i.e. the quartics and cubics) having the least stringent experimental constraints (obtained from nucleon decay data) on \(m_\sigma\), the mass of the scalar. Each of these subgroups will also result in different outcomes as far as explaining baryogenesis is concerned. In the following work we will be determining which of the above interactions are able to account for baryogenesis. This will basically involve the calculation of \(r - \bar{r}\), the CP violation arising from each model and the subsequent use of standard cosmological arguments.

3. Calculation of Baryon Production:

As previously mentioned baryogenesis is only possible provided we have CP violation incorporated in such a way as to induce different partial decay rates for baryon number violating decays of particles and antiparticles. To obtain this CP violation we require the introduction of at least two baryon number violating scalar self interactions. For example for the \(\sigma_1, \sigma_5, \sigma_5\) class of scalar potentials which we will consider later, we are required to introduce two scalar interactions, \(\sigma_1\sigma_5\sigma_5\) and \(\sigma_1\sigma_5\sigma_5\), where the scalars \(\sigma_5\) and \(\sigma_5\) have identical group properties but different masses and couplings. There are two reasons
behind this requirement; the first being that if the exchanged scalar(s) \( Y \) in
the loop corrections have identical masses and properties to the decaying par-
ticle(s) \( X \), then the contribution to baryogenesis arising from the decay of \( X \)
with the exchange of \( Y \) will be cancelled by an equal and opposite contribution
made by the decay of \( Y \) with the exchange of \( X \); the second reason arises from
the requirement that there be an imaginary component to the product of cou-
pling constants associated with the CP violating tree and loop graphs, which is
impossible if each Yukawa coupling appears with its conjugate.

The above list of baryon number violating models can be narrowed down
if we consider the effects of sphaleron or other forms of damping processes on
any baryon number asymmetry produced by each model [6]. The rapid \( B + L \)
violating sphaleron transitions that are still occurring after the decays of our
\( \sigma \)-particles will quickly erase any \( B + L \) asymmetry. If we take this factor into
account we can rule out a number of the models listed in Eqs(5,6,7), by keeping
only those that produce a \( B-L \) asymmetry (these being immune from sphaleron
washout) we are left with the following much shortened list of possible baryon
asymmetry generating models:

\[
\begin{align*}
\sigma_1, \sigma_5 & \rightarrow \sigma_1 \sigma_5 \\
\sigma_2, \sigma_7 & \rightarrow \sigma_2 \sigma_7 \\
\sigma_3, \sigma_5 & \rightarrow \sigma_3 \sigma_5 \\
\sigma_6, \sigma_7 & \rightarrow \sigma_6 \sigma_7 \\
\end{align*}
\]

Thus if we allow for damping, we are left with just four interactions, one cubic,
and three Higgs containing cubic interactions.

It should also be noted that the version of the SM which we have used
includes Majorana masses for neutrinos, which can in themselves allow for the
production of a baryon asymmetry or the damping of a preexisting baryon
asymmetry. If \( M > m_\sigma \), where \( M \) is a Majorana neutrino mass, then out-
of-equilibrium decays of \( \nu_R \) may produce a \( \Delta L \) prior to the decays of the \( \sigma \)
bosons. However, this asymmetry will be erased, and in particular it will not
be reprocessed into a \( \Delta B \) through \( B + L \) violating sphaleron processes. This
is because the \( B - L \) violating \( \sigma \) interactions (which we assume still occur
rapidly) will combine with sphaleron processes to force both \( \Delta L \) and \( \Delta B \) to
vanish. If \( M < m_\sigma \), then any \( \Delta B \) produced by \( \sigma \) decays will be erased by the
combination of rapid \( L \) violating and \( B + L \) violating processes. Subsequent
out-of-equilibrium decays of \( \nu_R \) may produce a \( \Delta L \) which gets reprocessed into
a \( \Delta B \) by sphaleron effects, given that the \( \sigma \)-induced \( B - L \) violating processes
have already switched off. Although this is an interesting scenario (and has been
studied in other contexts [7]), it is not the one we wish to consider in this paper.
We will therefore require that \( M > m_\sigma \), so that only \( \sigma \)-decays contribute to the
\( \Delta B \) that survives to the present day. (We also need to assume that no baryon
asymmetry is produced during the electroweak phase transition. This, however, is assured provided that we assume the existence of only a single electroweak Higgs doublet $\phi$, see for example Ref[8] for a review.

Although any baryon number asymmetry arising from the conjugate pair and quadratic interactions will be damped away, we consider these systems first as an instructive exercise in how to do the calculations. The realistic cases will then follow by simple extension.

The conjugate pair interactions are the simplest class of interaction included in our catalogue. The calculation of the baryogenesis arising from these conjugate pair interactions is directly analogous to that obtained by [4] using a GUT model. CP violation arises from considering the tree and one loop corrections for the scalar decay [4]. For the $\sigma_{1,1} - \sigma_{3,1}$ system, our Yukawa terms have the following form,

$$\mathcal{L} = \lambda_1 (\bar{\epsilon}_R)^c \sigma u_R + \lambda_3 \bar{d}_R \sigma \sigma_R$$

where $\sigma \sim \sigma^i \sim (3,1,2/3)$ under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. The reasons for introducing two copies of $(3,1,2/3)$ are discussed in the first paragraph of Sec 2 above. This leads to the following expression for the baryon number production:

$$r - \bar{r} = \frac{4 Im Tr (\lambda_1^1 \lambda_1^0 \lambda_3)}{Tr (\lambda_1^1 \lambda_1^0) + Tr (\lambda_3^1 \lambda_3^0)} Im I(m_{\sigma^i}/m_{\sigma})$$

The function $I(m_{\sigma^i}/m_{\sigma})$ represents the Feynman integral for the exchange of the scalar $\sigma^i$ in the decay of the scalar $\sigma$, and it has the form

$$Im I(m_{\sigma^i}/m_{\sigma}) = - \frac{1}{16\pi} \left[1 - \frac{m_{\sigma^i}^2}{m_{\sigma}^2} \ln \left(1 + \frac{m_{\sigma^i}^2}{m_{\sigma}^2}\right) \right]$$

If we assume that the mass of $\sigma$ is much larger than the mass of $\sigma^i$ then we can ignore the contribution made by the decay of the $\sigma^i$ scalar and Eq(10) is essentially the complete contribution to baryogenesis.

In the quadratic models baryon number is violated through a scalar-scalar interaction. The analysis of the bilinear interaction coupling two different scalars is similar to that associated with the conjugate pair case, and we must again introduce two sets of interactions to obtain non zero CP violation. In our sample calculation we will use the $\sigma_{1,1}\sigma_{3,1}$ and $\sigma_{1,1}^0\sigma_{3,1}^0$ bilinears together with the following Yukawa interactions:

$$\mathcal{L} = \lambda_1 (\bar{\epsilon}_R)^c \sigma_1 u_R + \lambda_3 (\bar{d}_R)^c \sigma_3 u_R$$

The CP violating tree and one loop corrections for this model are shown in Fig(1). In this case we have a four vertex loop correction rather than the
three vertex loop correction we had for the conjugate pairs. The CP violating Feynman amplitude corresponding to $\sigma_1$ decay takes the form:

$$M(\sigma_1 \rightarrow \bar{u} + \bar{e}) = \lambda_1 + \frac{(\mu^2)(\mu'^2)^{\dagger}}{m_{\sigma_1}^2 - m_{\sigma_5}^2} \lambda_3^\dagger \lambda_1 \lambda_3^\dagger I(m_{\sigma'_1}/m_{\sigma_1}, m_{\sigma'_5}/m_{\sigma_1}),$$  \hspace{1cm} (13)$$

where $\mu^2$ is the coupling constant associated with the bilinear $\mu^2\sigma_1\sigma_3$, and $\mu'^2$ is the coupling constant associated with the bilinear $\mu'^2\sigma'_1\sigma'_3$, where $\mu$ and $\mu'$ have dimensions of mass. The factor $I(m_{\sigma'_1}/m_{\sigma_1}, m_{\sigma'_5}/m_{\sigma_1})$ again represents the contribution of the Feynman integral around the loop; in this case two scalars connected by a mass insertion are exchanged. By taking the difference between absolute value of Eq(13) squared and the corresponding squared amplitude of the antiparticle process, we arrive at the following expression for the baryon number production:

$$r - \bar{r} = \frac{4}{m_{\sigma_1}^2 (m_{\sigma_1}^2 - m_{\sigma_5}^2)} \frac{ImTr\left((\mu^2)(\mu'^2)^{\dagger}\lambda_3^\dagger \lambda_1 \lambda_3^\dagger I\right)}{Tr(\lambda_3^\dagger \lambda_1)} ImI(m_{\sigma'_1}/m_{\sigma_1}, m_{\sigma'_5}/m_{\sigma_1})$$ \hspace{1cm} (14)$$

The value of $I(m_{\sigma'_1}/m_{\sigma_1}, m_{\sigma'_5}/m_{\sigma_1})$ in this case is found to have the form:

$$ImI(\rho_1, \rho_3) = \frac{1}{32\pi} \frac{1}{\rho_1^2 - \rho_3^2} (-7\rho_1^2 + 7\rho_3^2 + \rho_1^2 \ln(1 + \rho_1^{-2})$$
$$+ 7\rho_3^2 \ln(1 + \rho_3^{-2}) - \rho_3^2 \ln(1 + \rho_3^{-2}) - 7\rho_3^4 \ln(1 + \rho_3^{-2}))$$ \hspace{1cm} (15)$$

where,

$$\rho_i = \frac{m_{\sigma'_i}}{m_{\sigma_i}}.$$ \hspace{1cm} (16)$$

Note the similarity between the two expressions for the imaginary components of the phases for the conjugate pair and quadratic cases, Eq(11) and Eq(15), particularly in the limit $m_{\sigma'_i}/m_{\sigma_i} \rightarrow \infty$. We assume that $m_{\sigma_5} > m_{\sigma_1} > m_{\sigma'_1,3}$, which as can be seen from Eqs(15) and (14) will allow us to ignore the contribution the decays of $\sigma'_1,3$ and $\sigma_3$ will make to baryogenesis. Thus Eq(14) can be regarded as constituting the entire contribution to baryogenesis.

It should be noted that we can also obtain baryogenesis in the special case where we have just three new exotic scalars. For example for $\sigma_1$ decay we can set $\sigma_3 = \sigma'_3$ and still obtain the required imaginary components to the Yukawa couplings and the Feynman integral.

We now turn to realistic models based on Eq(8). In addition to not having a sphaleron washout problem, these models are also of greater phenomenological interest because the nucleon-decay bounds on $m_{\sigma}$ are much weaker than those for the unrealistic conjugate pair and quadratic toy models just considered ($m_{\sigma} > 10^6$ GeV for the $\sigma_1,\sigma_3,\sigma_5$ cubic compared with $m_{\sigma} > 10^{15}$ GeV for the conjugate pair and quartic interactions [2]).
From a diagrammatic and calculational point of view these models differ from the above because baryon number violation arises from a cubic or trilinear term in the Higgs potential.

Consider first the $\sigma_{1,1} - \sigma_5$ system. We will initially introduce two copies of both the $\sigma_{1,1}$ and $\sigma_5$ scalars; we will require at least three of these four scalars to obtain the required CP violation. We will consider a simplified set of Yukawa interactions associated with the participating scalars as shown below:

$$\mathcal{L} = \lambda_1 (\bar{e}_R) \sigma_1 u_R + \lambda_5 \bar{\nu}_L \sigma_5 \sigma_1 d_R + \lambda_6 \bar{e}_L \sigma_5 d_R$$

where $\sigma_{5a}$ and $\sigma_{5b}$ designate the two $SU(2)$ components of $\sigma_5$. The above set of Yukawa interactions are simplified in that we have considered just a few of the Yukawa interactions, see Eq.(3), which may from group considerations be associated with the exotic scalars. The CP violating tree and loop corrected diagrams for the decay of the $\sigma_5$ multiplet are shown in Fig.(2). For these decays the lowest order loop corrections involve both the $\sigma_1 \sigma_5 \sigma_1$ and $\sigma_1 \sigma_5 \sigma_1'$ cubic interactions. (In the interests of simplicity we have ignored the contribution the $\sigma_1 \sigma_5 \sigma_1'$ interaction will make to baryogenesis.) Thus we require only one version of the $\sigma_1$ scalar to obtain baryogenesis from the decay of the $\sigma_5$ scalar, conversely we would require two versions of the $\sigma_1$ scalars and one $\sigma_5$ scalar to obtain baryogenesis from the decay of the $\sigma_1$ scalar. The loop corrected diagrams shown in Fig.(2) are complicated and as such won’t be evaluated. This omission is justified because we are interested in an order of magnitude calculation only. In any case, it is the coupling constants which will play the major part in determining the numerical value of the baryogenesis arising from these cubic interactions. Based on the results obtained for the conjugate pairs and the quadratic interactions it is reasonable to assume that the imaginary part of the Feynman integral will be of a similar form and hence give similar numerical values to those calculated in Eqs.(11) and (15); we must however allow for an extra $1/(2\pi)^3$ suppression factor for each additional loop order. For the diagrams given in Fig.(2) the imaginary component of our loop integral will thus be suppressed by an extra factor of $1/(2\pi)^3$ in comparison to the one loop integrals considered in Eqs.(11) and (15). The CP violating amplitudes for the decays shown in Fig.(2) are given by:

$$M(\sigma_5 \to \bar{d} + f) = \lambda_5 + \mu \mu^\dagger \lambda_1^\dagger \lambda_1^\dagger I(m_{\sigma_5}, m_{\sigma_5}, m_{\sigma_5}).$$

where $\mu$ represents the coupling constant associated with the cubic interaction (again with units of mass), and $I(m_{\sigma_1}, m_{\sigma_5}, m_{\sigma_5})$ represents the Feynman integral taken around the loops, which in this case will not be evaluated. From Eq.(18) we obtain the following expression for the total baryon number production arising from the decays of the $\sigma_{5a}$ and $\sigma_{5b}$ scalars, where
\[ m_{\sigma_\alpha} = m_{\sigma_{\alpha\alpha}} \simeq m_{\sigma_{\alpha}}; \]

\[ r - \bar{r} = \frac{8}{m_{\sigma_{\alpha}}^2} \frac{ImTr(\mu \mu^\dagger \lambda_3^\dagger \lambda_5^\dagger \lambda_5)}{Tr(\lambda_3^\dagger \lambda_5)} ImI(m_{\sigma}, m_{\sigma_{\alpha}}, m_{\sigma_{\alpha}'}) \]  \hspace{1cm} (19)

Note that we have included an extra factor of two in the above expression to allow for the fact that we have two equal (as \( m_{\sigma_{\alpha\alpha}} \simeq m_{\sigma_{\alpha}} \)) contributions to the baryon number production. If we assume that \( m_{\sigma_{\alpha}} > m_{\sigma_5} \) then we can ignore the contribution the decay of \( \sigma_5' \) will make to baryogenesis. Thus Eq(19) is in effect the complete contribution the system given in Eq(17) will make to baryogenesis.

The phenomenologically less interesting Higgs containing trilinears (\( m_{\sigma} > 10^{11}\text{GeV} \)) will lead to baryogenesis via CP violating loop diagrams such as that shown in Fig(3) for the \( \sigma_{3,1} \sigma_5 \phi \) cubic, where we have used the following Yukawa interactions,

\[ \mathcal{L} = \lambda_3 \langle \bar{u}_R \rangle \sigma_3 d_R + \lambda_5 \langle \bar{d}_L \rangle \sigma_{5a} d_R + \lambda_3' \langle \bar{u}_R \rangle \sigma_5' d_R + \lambda_5' \langle \bar{d}_L \rangle \sigma_{5a}' d_R + h.c. \]  \hspace{1cm} (20)

Note the similarity between the diagrams in Fig(2) and Fig(3). For \( \sigma_i \) decay resulting from the interactions \( \mu \sigma_3 \sigma_5 \phi \) and \( \mu' \sigma_3' \sigma_5' \phi \) we obtain an expression for \( r - \bar{r} \) of the form,

\[ r - \bar{r} = \frac{4}{m_{\sigma_{\alpha}}^2} \frac{ImTr(\mu \mu^\dagger \lambda_3^\dagger \lambda_5^\dagger \lambda_5')}{Tr(\lambda_3^\dagger \lambda_5')} ImI(m_{\sigma}, m_{\sigma_{\alpha}}, m_{\sigma_{\alpha}'}, m_{\sigma_{\alpha}'}, m_{\phi}). \]  \hspace{1cm} (21)

If \( \sigma_5 \) is significantly more massive than \( \sigma_3, \sigma_3' \) and \( \sigma_5' \) then Eq(21) is essentially the complete contribution to baryogenesis. In this case our expression for the Feynman loop integral will involve the Higgs scalar with \( m_{\phi} \ll m_{\sigma} \).

Note that in the special case of \( \sigma_3 = \sigma_3' \) we can still get baryon number violation from \( \sigma_5 \) decays, and vice-versa for the decay of the \( \sigma_3 \) scalar. Thus as in the trilinear case the simplest possible baryon number violating system requires the introduction of two Higgs containing cubic interactions and three exotic scalars.

4. Numerical estimates

From our expressions for the baryon number creating \( r - \bar{r} \) obtained in the previous section, we will attempt to numerically determine the likelihood of each model accounting for baryogenesis.

We know from [9] that the decay of super heavy bosons and antibosons at temperatures sufficiently well below their masses will produce a cosmic baryon entropy ratio of the form:

\[ \frac{k n_B}{s} = 0.28 \frac{N_X}{N} \Delta B, \]  \hspace{1cm} (22)
where $k$ is Boltzmann’s constant, $N_X/N$ is the ratio of the number of boson helicity states to the number of light particle helicity states ($N_X/N \approx 10^{-2}$) and $\Delta B$ represents the baryon number generation arising from the decay. Astronomical observations put $kn_B/s \approx 10^{-10}$ to $10^{-8}$, therefore $\Delta B$ must lie within the range $10^{-8}$ to $10^{-6}$, to account for the presently observed baryogenesis.

Our expression for the baryon number created by the cubic interactions Eq(19) can be expressed as

$$\Delta B = \frac{8\mu^\dagger}{m_{\sigma}^2}e\lambda^2 ImI(m_{\sigma},m_{\phi},m_{\sigma'}).$$  (23)

Similarly our expression for the baryogenesis arising from the Higgs containing cubics, see Eq(21), can be expressed as

$$\Delta B = \frac{4\mu^\dagger}{m_{\sigma}^2}e\lambda^2 ImI(m_{\sigma},m_{\sigma},m_{\sigma'},m_{\sigma'},m_{\phi}).$$  (24)

where $\lambda$ represents a typical value for the magnitude of the unknown Yukawa couplings in Eqs(19) and (21), and $\epsilon$ is a phase angle characterising the average strength of the CP violation associated with our scalar-scalar, and Yukawa interactions.

In our analysis we will assume that the scalar self interacting and Yukawa couplings are approximately equal in magnitude. Using this assumption we will set out to find the region of parameter space within which these coupling constants must fall in order to account for baryogenesis.

The value of the imaginary component of the Feynman integral for the above two cubic interactions is estimated from the corresponding expressions obtained from the conjugate pairs and quadratic interactions. After allowing for the extra $1/(2\pi)^2$ integration factor, it is thus assumed to fall within the range $(2\pi)^{-2}(10^{-3} - 10^{-2})$. The value for the lower bound arises from the equal mass $m_{\sigma} = m_{\sigma'}$ and $\rho_i = 1$ values of Eqs(11), and (15) respectively, and the upper bound arises from the consideration of the $m_{\sigma} \gg m_{\sigma'}$ limit of Eq(11).

By taking all of these assumptions on board (for both classes of cubic interaction), it is found that the coupling constants $\lambda$ and $\mu$ must lie within the following range to account for baryogenesis,

$$\lambda^e^{1/4} \approx \frac{|\mu|}{m_{\sigma}}e^{1/4} \approx 10^{-1} - 10^{-2}. $$  (25)

If we assume for example that $1 > \epsilon > 10^{-3}$ rads, then the couplings $\lambda$ and $|\mu|/m_{\sigma}$ must be of order $1 - 10^{-2}$ to account for baryogenesis.

4. Conclusion:
We have thus demonstrated that baryogenesis can be induced by each of the model types listed in Eq (8).

In the numerical analysis of the various interactions listed in Eq (8), it was found that both classes of cubic interactions can successfully account for baryogenesis provided the coupling constants fall within the range, \( \lambda e^{1/4} = \mu e^{1/4}/m_{\sigma} \approx 10^{-1} - 10^{-2} \). Given that the known Higgs Yukawa couplings range up to order unity for the top quark, it is not at all unreasonable that the unknown Yukawa’s may fall within this parameter space.

We have thus demonstrated that at least some of the baryon number violating models introduced in [2], with the explicit aim of obtaining complete charge quantisation from gauge invariance will also allow for baryogenesis. It should however be noted that whilst we only need one of the baryon number violating scalar interactions listed in Eqs (5, 6, 7) (and consequently two exotic scalar multiplets), to obtain charge quantisation, we require the introduction of at least two scalar-scalar interactions and at least three exotic scalars to account for baryogenesis. Thus the minimal requirement for charge quantisation is not sufficient to also give us baryogenesis.

References

[1] A. D. Sakharov, *JEPT Lett.* 5 24 (1967);


Figure Captions:

Figure 1:
CP violating tree and one loop corrected diagrams resulting from the decay of $\sigma_{1,1}$ in the $\sigma_{1,1}\sigma_{3,1}$ quadratic model.

Figure 2:
CP violating tree and loop corrected diagrams resulting from the decay of $\sigma_5$ in the $\sigma_{1,1}\sigma_5\sigma_5$ cubic model.

Figure 3:
CP violating tree and loop corrected diagrams resulting from the decay of $\sigma_5$ in the $\sigma_{3,1}\sigma_5\phi$ cubic model.
figure 1.
figure 2.
\[ \sigma_5 a \rightarrow \sigma_3 \rightarrow u_R \rightarrow d_R \rightarrow \sigma_3' \rightarrow \sigma_5 a \rightarrow \phi \rightarrow \nu_L \]

Figure 3.