Strangeness in the Nucleon on the Light-Cone

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Abstract

Strange matrix elements of the nucleon are calculated within the light-cone formulation of the meson cloud model. The $Q^2$ dependence of the strange vector and axial vector form factors is computed, and the strangeness radius and magnetic moment extracted, both of which are found to be very small and slightly negative. Within the same framework one finds a small but non-zero excess of the antistrange distribution over the strange at large $x$. Kaon loops are unlikely, however, to be the source of a large polarized strange quark distribution.

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I. INTRODUCTION

There has been considerable discussion recently about strange matrix elements of the nucleon [1]. Much impetus for this was generated by the deep-inelastic scattering experiments with polarized targets at CERN and more recently at SLAC [2], which seemed to imply a large polarized strange quark distribution in the proton. At about the same time a measurement of the elastic neutrino–proton scattering cross section [3] at lower values of $Q^2$ had also suggested a non-zero value for the strange axial vector form factor of the proton [4].

These observations spurred many investigations of other processes in which traces of strangeness in the nucleon could be detected. It was argued [5] for example that semi-leptonic neutral current scattering experiments could be used to extract the strange vector as well as axial vector form factors. Parity-violating electron scattering experiments were proposed as a means of probing neutral current form factors [6]. Approved parity-violating experiments at MIT-Bates [7] and Jefferson Lab [8] will provide information on the strangeness form factors at low and intermediate $Q^2$ values, and more precisely determine the strange radius and magnetic moment of the nucleon. At higher $Q^2$, the CCFR Collaboration [9] has recently investigated a possible asymmetry between the strange and antistrange quark distributions in neutrino deep-inelastic scattering. Perturbative QCD alone would be expected to produce identical $s$ and $\bar{s}$ distributions, while any asymmetry would imply the presence of non-perturbative effects in the nucleon at deep-inelastic scales.

Of course the strangeness content of the nucleon is not a scale-invariant concept — strange matrix elements are not renormalization group invariant. The evolution with $Q^2$ of the moments of the polarized deep-inelastic strange quark distribution can be studied using the Altarelli-Parisi equations, so that a zero value for $\Delta s$ at a low scale is not necessarily incompatible with a non-zero value at a higher scale [10]. Naturally, extending such analyses to very low values of $Q^2$ is problematic — at some stage non-perturbative techniques or models of QCD need to be invoked when describing data. Recently Tang and Ji [11] attempted to link the strangeness radius of the nucleon with the densities of strange and antistrange deep-inelastic quark distributions, arguing on the basis of quark “locality” that the distributions in coordinate space and momentum space should be correlated. In order to investigate this proposal more quantitatively, we will explore this relationship within a simple model, namely the meson cloud model.

In the meson cloud model, the strangeness of the nucleon is assumed to be carried by the kaon–hyperon components of the physical nucleon. This model has in the past been utilized to calculate corrections to low energy nucleon properties [12–18], and has also been invoked to describe flavor symmetry breaking in nucleon sea quark distributions [19–22]. However, direct comparison of results for the strange matrix elements in meson cloud models has thus far not been possible due to the fact that form factors and structure functions are usually calculated within different theoretical frameworks. While the natural framework for analyzing strange quark distributions is the light-cone [21,22], strange form factor calculations have usually been performed within covariant perturbation theory in instant-form quantization [12,13,16–18]. (A light-cone constituent quark model was used in Ref. [18] to obtain the coupling of the photon to the kaon–hyperon cloud of the nucleon, however the kaon dressing itself was not described in terms of light-cone dynamics.) Meaningful comparison
of meson cloud corrections to light-cone quark distributions and low-$Q^2$ form factors obviously requires a single framework to be used for both. In the present work we shall present for the first time a light-cone analysis of the strange nucleon form factors, using the same framework in which structure functions are calculated [21,22]. We believe this is the only systematic way to test the hypothesis of Ref. [11] that the properties of the nucleon sea at low and high $Q^2$ might somehow be correlated.

In Sec.II we briefly outline the pertinent features of the meson cloud model on the light-cone. More detailed accounts can be found in Ref. [21]. We explain that the advantage of this framework lies in that it allows for a more consistent treatment of the hadronic meson-baryon vertex functions. Unlike previous calculations within covariant perturbation theory, which have usually had to make ad hoc assumptions about the dependence of the vertex functions on various loop momenta, one can formally avoid this on the light-cone, while simultaneously satisfying charge and momentum conservation. The kaon cloud model is applied in Sec.III to the calculation of the $Q^2$ dependence of the strange vector and axial vector form factors of the nucleon, from which the strangeness radius, strange magnetic moment and strange axial charge are extracted. Sec.IV discusses the contribution from the kaon cloud to the deep-inelastic strange quark distribution, and to a possible strange-antistrange asymmetry in the nucleon. We find that the asymmetry is consistent with the latest neutrino deep-inelastic scattering data, within experimental errors. We also estimate the contribution to the polarized strange quark distribution from kaon loops. Finally our findings are summarized in Sec.V.

II. LIGHT-CONE MESON CLOUD MODEL OF THE NUCLEON

The basic hypothesis of the meson cloud model of the nucleon is that the nucleon on the light-cone has internal meson and baryon degrees of freedom. The physical nucleon state (momentum $P$) can then be expanded (in the one-meson approximation) as a series involving bare nucleon and two-particle meson-baryon states:

$$|N(P)\rangle_{\text{phys}} = \sqrt{Z} \{ |N(P)\rangle_{\text{bare}} + \sum_{B,M} \int dy \, d^2k_\perp \, g_{MB}(y,k_\perp) \, |B(y,k_\perp);M(1-y,-k_\perp)\rangle \}.$$  (1)

The function $\phi_{BM}(y,k_\perp)$ is the probability amplitude for the physical nucleon $N$ to be in a state consisting of a baryon $B$ and meson $M$, having transverse momenta $k_\perp$ and $-k_\perp$, and carrying longitudinal momentum fractions $y = k_+/P_+$ and $1-y = (P_+ - k_+)/P_+$, respectively. The bare nucleon probability is denoted by $Z$, and $g_{MB}$ is the $MBN$ coupling constant. The one-meson approximation in Eq.(1) is valid as long as the meson cloud is relatively soft ($Z \lesssim 1$). It will progressively break down for harder $MBN$ vertex functions, at which point one will need to include two-meson and higher order Fock state components in Eq.(1).

The strangeness of the nucleon in this model is carried by the $|\Sigma;K\rangle$, $|\Lambda;K\rangle$, etc. Fock state components, and we shall consider only these henceforth. Therefore the properties of the bare nucleon state (such as its intrinsic form factor, or its structure function) will not be relevant for considerations of the nucleon strangeness content. For the sake of presentation we will consider the $\Lambda$ hyperon as representative in our discussions, although contributions
from the Σ will be included in the numerical results in Secs. III & IV. All expressions for
the $K\Sigma$ distributions can be obtained from the $KA$ by appropriate replacement of masses, coupling
constants and isospin factors. Contributions from heavier mesons and hyperons, such as $K^*$, $Y^*$, etc.
can easily be added, although their role is insignificant as long as the $MNB$ vertex functions are not very hard [21].

According to Eq.(1), the probability to find a $\Lambda$ inside a nucleon with light-cone momentum fraction $y$ is given by the $N \rightarrow KA$ splitting function (to leading order in the coupling constant):

$$f_{\Lambda K}(y) = g_{K\Lambda}^2 \int d^2k_\perp |\phi_{\Lambda K}(y,k_\perp)|^2. \quad (2)$$

Charge and momentum conservation require that this must also be the probability to find a kaon inside a nucleon with light-cone momentum fraction $1 - y$. The kaon distribution function, $f_{KA}(y)$, should therefore be related to the baryon distribution function by

$$f_{KA}(y) = f_{\Lambda K}(1 - y), \quad (3)$$

for all $y$, if the above interpretation is valid. This guarantees equal numbers of mesons emitted by the nucleon, $\langle n \rangle_{KA} = \int_0^1 dy f_{KA}(y)$, and virtual baryons accompanying them, $\langle n \rangle_{\Lambda K} = \int_0^1 dy f_{\Lambda K}(y)$:

$$\langle n \rangle_{KA} = \langle n \rangle_{\Lambda K}. \quad (4a)$$

This is just the statement that the nucleon has zero net strangeness. Momentum conservation imposes the further requirement that

$$\langle y \rangle_{KA} + \langle y \rangle_{\Lambda K} = \langle n \rangle_{KA}, \quad (4b)$$

where $\langle y \rangle_{KA} = \int_0^1 dy y f_{KA}(y)$ and $\langle y \rangle_{\Lambda K} = \int_0^1 dy y f_{\Lambda K}(y)$ are the average momentum fractions carried by kaon and $\Lambda$, respectively. Equations (4a) and (4b), and in fact similar relations for all higher moments of $f(y)$, follow automatically from Eq.(3). Any consistent treatment of the meson cloud model must therefore reproduce this symmetry relation.

The advantage of a light-cone formulation [14,15,21–23] is that one can construct a covariant framework in which Eq.(3) is manifestly preserved, regardless of the dynamics of the meson–baryon system. If the dynamics are parametrized in terms of an effective hadronic meson–baryon vertex function, the symmetry relations (3)–(4) can be satisfied if the vertex functions are functions of the invariant mass squared, $M^2$, of the $KA$ system:

$$M^2 = (p_K + p_\Lambda)^2 = \frac{k_\perp^2 + M_\Lambda^2}{y} + \frac{k_\perp^2 + m_K^2}{1 - y}, \quad (5)$$

where $M_\Lambda$ and $m_K$ are the masses of $\Lambda$ and kaon, respectively, and $p_K$ and $p_\Lambda$ their four-momenta. Note that the variable $M^2$ is related to the virtualities of the kaon ($t = (p_\Lambda - p_N)^2$) and $\Lambda$ ($u = (p_K - p_N)^2$) by:

$$M^2 + t + u = M^2 + m_K^2 + M_\Lambda^2, \quad (6)$$

where $t = -(k_\perp^2 + (1 - y)(M_\Lambda^2 - yM^2))/y$ and $u = -(k_\perp^2 + y(m_K^2 - (1 - y)M^2))/(1 - y)$. A hadronic vertex function which is a function of $M^2$ will automatically have the correct crossing symmetry structure in the $t$ and $u$ channels [24,25] (see Eqs.(19), (20) below).
In instant-form calculations, on the other hand, the functions $f(y)$ are usually evaluated in terms of $K\Lambda$ vertex functions taken from fits to $N\Lambda \rightarrow N\Lambda$ scattering data [26], where the vertex functions are assumed to depend only on the virtuality $t$ of the off-shell kaon. In this case the $t \leftrightarrow u$ symmetry is lost, and the relation (3) cannot be maintained, making verification of the charge and momentum conservation relations, Eqs.(4), very difficult [21,27]. Actually, the non-preservation of the relation (3) is not surprising since the splitting functions $f(y)$ refer to probability distributions of specific particles, and a probabilistic interpretation does not hold in all reference frames (since particle number is not preserved by Lorentz boosts). Indeed, for structured particles the factorization of the $\gamma^* N$ cross section itself into $\gamma^* K$ and $K N$ (or $\gamma^* \Lambda$ and $\Lambda N$) cross sections is not valid in all frames of reference [27]. Such factorization, or convolution, can in fact only be achieved by eliminating antiparticle degrees of freedom, which can formally be done only in the infinite momentum frame [28], or on the light-cone.

III. STRANGE FORM FACTORS

We will be interested in the strange matrix elements $\langle N|\bar{s}\Gamma s|N\rangle$ of operators $\Gamma$, where $\Gamma = \gamma_\mu$ or $\gamma_\mu\gamma_5$. While these have been investigated in earlier studies within the meson cloud model [12,13,16-18], we will present here the first analysis of the strange form factors on the light-cone, in which the distribution functions of kaons and hyperons in the nucleon have the correct symmetry properties as specified in Eq.(3). This will be our main contribution to the form factor discussion. Other aspects of the problem, such as the modeling of the intrinsic $K$ and $\Lambda$ form factors, or the truncation of the Fock state expansion in Eq.(1), are not developed here beyond what exists already in the literature.

The momentum dependence of the matrix elements of the strange vector current $J_\mu^s = \bar{s}\gamma_\mu s$ is conventionally written:

\[
\langle N(P')|J_\mu^s(0)|N(P)\rangle = \bar{u}(P') \left( \gamma_\mu F_1^s(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M} F_2^s(Q^2) \right) u(P),
\]

where $P$ and $P'$ are the initial and final nucleon momenta, and $q = P - P'$ is the momentum transferred to the nucleon. The form factors $F_1^s$ and $F_2^s$ are the Dirac and Pauli strange form factors, and $u(P)$ is the free Dirac spinor of the nucleon. For convenience we work in a Lorentz frame (viz. the Breit frame) where the photon momentum $q$ is purely transverse, so that $q^2 = -q_\perp^2 \equiv -Q^2$. The various particle momenta are parameterized as in Ref. [29]. Zero net strangeness in the nucleon requires the vanishing of the Dirac form factor at the on-shell photon point, $F_1^s(0) = 0$. On the light-cone the form factors can be evaluated by choosing the $\mu = +$ (or “good”) component of the current $J_\mu^s$ [23].

Usually one introduces more convenient combinations of $F_{1,2}^s$, in the form of the Sachs electric ($G_E^s$) and magnetic ($G_M^s$) form factors:

\[
G_E^s(Q^2) = F_1^s(Q^2) - \frac{Q^2}{4M^2} F_2^s(Q^2),
\]

\[
G_M^s(Q^2) = F_1^s(Q^2) + F_2^s(Q^2).
\]

The conventional definition of the strangeness radius of the nucleon is in terms of $G_E^s$. 
The strange magnetic moment of the nucleon is defined by:

\[ r_s^2 \equiv \int d^3r \langle N|s\gamma_0 s|N \rangle \]
\[ = -6 \frac{dG_s^s(Q^2)}{dQ^2} \bigg|_{Q^2=0} \]  

(9a)

(9b)

An alternative definition of the strangeness radius is in terms of the Dirac form factor:

\[ r_{s,\text{Dirac}}^2 = -6 \frac{dF_1^s(Q^2)}{dQ^2} \bigg|_{Q^2=0} \]

(10)

The strange magnetic moment of the nucleon is defined by:

\[ \mu_s = \frac{1}{2} \int d^3r \langle N|(r \times \bar{s}\gamma_5 s)|N \rangle \]
\[ = F_2^s(0) = G_M^s(0) \]  

(11a)

(11b)

The contributions of the kaon cloud to the strange form factors are represented in Fig.1. To maintain the analogy with the structure function calculations in Sec.IV, we will present the results for \( F_{1,2}^s \), and evaluate \( G_{E,M}^s \) from these. Therefore we define:

\[ F_i^s(Q^2) = F_i^{(s)}(Q^2) + F_i^{(K)}(Q^2), \quad i = 1, 2, \]

(12)

where \( F_i^{(s)} \) and \( F_i^{(K)}(Q^2) \) refer to the \( \Lambda \) and \( K \) interaction diagrams in Figs.1(a) and (b), respectively. The \( \Lambda \) contribution to \( F_i^s \) can be written:

\[ F_i^{(s)}(Q^2) = Q_{\Lambda} \int_0^1 dy \, f_i^{(s)}(y, Q^2) \, H_{\Lambda}(Q^2), \quad i = 1, 2, \]

(13)

where the \( \Lambda \) light-cone distribution functions \( f_i^{(s)} \) are:

\[ f_1^{(s)}(y, Q^2) = \frac{g_{NK}^2}{16\pi^3} \int \frac{d^2k}{y^2(1-y)} \frac{\mathcal{F}(M_{\Lambda K,i}^2)}{M_{\Lambda K,i}^2-M^2}(\mathcal{F}(M_{\Lambda K,f}^2)) \times \left( k^2 + (M_\Lambda -yM)^2 - (1-y)^2 \frac{Q_1^2}{4} \right), \]

(14a)

\[ f_2^{(s)}(y, Q^2) = \frac{g_{NK}^2}{16\pi^3} \int \frac{d^2k}{y^2(1-y)} \frac{\mathcal{F}(M_{\Lambda K,i}^2)}{M_{\Lambda K,i}^2-M^2}(\mathcal{F}(M_{\Lambda K,f}^2)) \times (-2M)(1-y)(M_\Lambda -yM). \]

(14b)

For the \( KN\Lambda \) vertex we assume a pseudoscalar \( i\gamma_5 \) interaction (the same results are obtained with a pseudovector coupling), with \( g_{NK}^2 \) the coupling constant and \( \mathcal{F}(M_{\Lambda K}^2) \) the hadronic vertex function. Note that identical expressions to these can be obtained using time-ordered perturbation theory in the infinite momentum frame [21,28]. For the sign of the strangeness we adopt the convention of Jaffe [30], so that \( Q_\Lambda = +1 \) is the strangeness charge of the \( \Lambda \). The intrinsic \( \Lambda \) form factor \( H_{\Lambda}(Q^2) \) contains possible \( Q^2 \) dependence in the \( \gamma^*\Lambda \) interaction vertex. The squared center of mass energies in Eqs.(14) are:
with $M^2$ defined in Eq.(5).

The contributions to $F_{1,2}^s$ from the coupling to the kaon in Fig.1(b) are written:

$$F_i^{(K)}(Q^2) = Q_K \int_0^1 dy \, f_i^{(K)}(y, Q^2) \, H_K(Q^2), \quad i = 1, 2,$$

where $Q_K = -1$ is defined to be the strangeness charge of the kaon, and where the kaon light-cone distribution functions $f_{1,2}^{(K)}$ are:

$$f_1^{(K)}(y, Q^2) = \frac{g_{K\Lambda}^2}{16\pi^3} \int \frac{d^2k_\perp}{y^2(1-y)} \frac{\mathcal{F}(M_{K\Lambda,i}^2)}{\mathcal{F}(M_{K\Lambda,i}^2)} \frac{\mathcal{F}(M_{K\Lambda,f}^2)}{\mathcal{F}(M_{K\Lambda,f}^2)} \left( k_\perp^2 + (M_\Lambda - yM)^2 - y^2 g_{K\Lambda}^2 \right),$$

$$f_2^{(K)}(y, Q^2) = \frac{g_{K\Lambda}^2}{16\pi^3} \int \frac{d^2k_\perp}{y^2(1-y)} \frac{\mathcal{F}(M_{K\Lambda,i}^2)}{\mathcal{F}(M_{K\Lambda,i}^2)} \frac{\mathcal{F}(M_{K\Lambda,f}^2)}{\mathcal{F}(M_{K\Lambda,f}^2)} \left( 2My(M_\Lambda - yM) \right).$$

The $K\Lambda$ squared center of mass energies are:

$$M_{K\Lambda,i}^2 = M^2 + \frac{q_\perp}{1-y} \cdot \left( y \frac{q_\perp}{4} + k_\perp \right),$$

$$M_{K\Lambda,f}^2 = M^2 + \frac{q_\perp}{1-y} \cdot \left( y \frac{q_\perp}{4} - k_\perp \right).$$

As discussed in Sec.II, on the light-cone it is natural to take the $K\Lambda$ vertex function to be a function of $M_{K\Lambda}^2$, which guarantees local gauge invariance [15,24] as well as energy-momentum conservation, as embodied in Eq.(3). In instant-form approaches gauge invariance has sometimes been enforced through introduction of seagull diagrams [16-18], which can be generated according to various prescriptions [31] in order to satisfy the Ward-Takahashi identity. However, as observed by Musolf and Burkardt [16], such prescriptions are not unique, since additional seagull terms which individually satisfy the Ward-Takahashi identity are also allowed. Furthermore, inconsistencies with some of the prescriptions, when applied to loop calculations, have also recently been pointed out by Wang and Banerjee [32].

With $M^2$ dependent vertex functions the functions $f_1^{(K)}$ and $f_1^{(\Lambda)}$ in Eqs.(17a) and (14a) at $Q^2 = 0$ satisfy the relation (3) explicitly, which they must because of strangeness conservation. The functions $f_2^{(K)}$ and $f_2^{(\Lambda)}$ at $Q^2 = 0$, on the other hand, do not need to satisfy such a relation, because they have their origin in the different spin couplings of the $\gamma^*$ to the spin-0 $K$ and spin-1/2 $\Lambda$. Indeed, the contribution to the strange magnetic moment of the nucleon would be zero if $f_2^{(K,\Lambda)}$ exhibited such a symmetry. From Eqs.(14b) and (17b) one can see in fact that $\mu_s$ in the kaon cloud model is always $< 0$.

Although it is clear that in order to preserve the relation (3) the vertex function should be a function of the $K\Lambda$ invariant mass, its specific functional dependence is not prescribed,
and several forms have been suggested in the literature [21,22,29]. In our numerical studies we will use a simple monopole-type function:

$$\mathcal{F}(M_{KA}^2) = \left( \frac{\Lambda_{KA}^2 + M^2}{\Lambda_{KA}^2 + M_{KA}^2} \right),$$

where $\Lambda_{KA}$ is the cut-off mass parameter. In the light-cone formulation the cut-off directly determines the probability of finding the physical nucleon in a $KA$ configuration, which provides a constraint on the range of $\Lambda_{KA}$ for which the average kaon number density can be viewed as reliable. (In covariant instant-form calculations the probability for each individual Fock state component is frame-dependent, since particle number is not invariant under Lorentz boosts.) For values larger than $\sim 1.5$ GeV the average probability to find kaons in the nucleon would be $\langle n \rangle_{KA} \gtrsim 10\%$, so that without including higher Fock state components, the one-meson approximation in Eq.(1) could not be considered trustworthy. Therefore cut-off masses between $\Lambda_{KA} = 0.7$ and 1.3 GeV, for which $\langle n \rangle_{KA} \approx 3-7\%$, can be considered as representative of a reasonable range for which the model (1) is a valid approximation (for these values the corresponding probability to find pions in the nucleon is $\sim 30\%-50\%$).

On the other hand, one can also try to constrain $\Lambda_{KA}$ phenomenologically by fitting, within the kaon cloud model, the available strange quark distribution data [20,21,33], as well as the inclusive $pp \to AX$ production data [34]. For the monopole parametrization (19) one typically finds values of $\Lambda_{KA} \lesssim 1.3$ GeV, with larger cut-offs difficult to accommodate [21] (see also Sec.IV). In instant-form calculations [16-18] one has often tried to connect the vertex function cut-offs with those obtained from NA potential model fits [26], where the vertex function is assumed to depend on the virtuality $t$ of the kaon,

$$\mathcal{F} = \left( \frac{\Lambda_{KA}^2 - m_K^2}{\Lambda_{KA}^2 - t} \right).$$

(20)

Here the vertex function cut-offs are found to be typically $\Lambda_{KA} \sim 1.2-1.5$ GeV. Note that the form (20) is an approximation to the $M^2$ dependent light-cone vertex function in Eq.(19), obtained by taking $u \to M_{\Lambda}^2$ and $\Lambda_{KA}^2 \to \Lambda_{KA}^2 + M^2 + m_K^2$, see Eq.(6). A typical value of $\Lambda_{KA} \sim 1.4$ GeV would therefore correspond to $\Lambda_{KA} \sim 1$ GeV.

The function $H_K(Q^2)$ in Eq.(16), like $H_{\Lambda}(Q^2)$ in Eq.(13), reflects the possible $Q^2$ dependence in the interaction vertex in Fig.1(b). The simplest approximation is to assume point-like $\gamma^*K$ and $\gamma^*\Lambda$ couplings, $H_K(Q^2) = H_{\Lambda}(Q^2) = 1$ [16]. Of course the virtual $K$ and $\Lambda$ do have finite size, and their coupling to $\gamma^*$ should in reality exhibit some $Q^2$ dependence. The most natural way to model this $Q^2$ dependence is through the vector meson dominance model, in which the virtual photon at low $Q^2$ couples to the kaon or $\Lambda$ through its fluctuations into correlated $q\bar{q}$ pairs [17,30,35-37]. Since the photon has $J^{PC} = 1^{-+}$, the states with the correct quantum numbers to which the $\gamma^*$ can couple are the vector mesons, $\rho^0$, $\omega$ and $\phi$. Indeed, the $\omega$ and $\phi$ mesons can act as an extra source of strangeness in the nucleon through this mechanism. A detailed treatment of the vector meson dominance model in strange form factors was presented in Refs. [17,30,36,37], where it was found that $r_s^2$ increased by a factor $\sim 2$ with respect to the kaon loop only result ($\mu_s$ is of course not affected by any changes in the $Q^2$ dependence). Other, quark-type models have also been used to estimate the intrinsic form factors of the bare particles [16,18,38]. However, as outlined in Sec.I, our aim...
here is more to explore the relationship between strange quark distributions in coordinate and momentum space within a single specific model, rather than present a comprehensive analysis of the form factors and structure functions themselves. Therefore in order to avoid diluting the result from the kaon cloud model by introducing extra degrees of freedom into the calculation, for our purposes we will begin by investigating the contributions to the $Q^2$ dependence of the strange form factors arising from kaon loops alone.

The $Q^2$ dependence of the strange Sachs electric and magnetic $G^*_E$ and $G^*_M$ form factors is shown in Figs.2 and 3 for cut-off masses $\Lambda_{KA} = 0.7, 1$ and 1.3 GeV. The values of the masses and couplings used in the numerical calculations are $M = 939$ MeV, $M_A = 1116$ MeV, $M_{Z'} = 1190$ MeV, $m_K = 494$ MeV and $g_{KNA} = -13.98$, $g_{KN\Sigma} = 2.69$ [26]. The results in Figs.2 & 3 contain contributions from both $K\Lambda$ and $K\Sigma$ components, with the latter contributing $\sim 4\%$ of the total, which reflects the ratio $g_{K\Sigma}/g_{K\Lambda} \sim -1/5$ expected from SU(3) symmetry. Contributions from $KY^*$ loops to the strange form factors, where $Y^*$ represents the $J = 3/2$ decouplet hyperons, are suppressed by more than two orders of magnitude relative to the $K\Lambda$ loops. The sign of $G^*_E$ is positive, while that of $G^*_M$ negative, in agreement with the instant-form kaon cloud model results of Refs. [16,17]. The magnitude depends strongly on $\Lambda_{KA}$, although even for the largest cut-off of 1.3 GeV it is still somewhat smaller than in Refs. [16,17]. The strangeness (Sachs) radius is found to be small and negative, ranging from $r_s^2 \approx -0.004$ fm$^2$ for $\Lambda_{KA} = 0.7$ GeV to $r_s^2 \approx -0.008$ fm$^2$ for $\Lambda_{KA} = 1.3$ GeV. This is slightly smaller than that obtained in previous kaon cloud model calculations [16-18], although still within the same order of magnitude. The strange Dirac radius is somewhat smaller still, $r_{s,\text{Dirac}} \approx -0.0007 \rightarrow -0.0012$ fm$^2$ for the two cut-off masses. The strange magnetic moment is also found to be negative as in earlier studies [16,17] although somewhat smaller, $\mu_s \approx -0.4 \rightarrow -0.1$. To obtain the same numerical values for the form factors and radii as in Refs. [16-18] would require values for the vertex function cut-off of $\Lambda_{KA} \approx 3$ GeV, as seen in Fig.4, where the $\Lambda_{KA}$ dependence of $r_s^2$, $r_{s,\text{Dirac}}^2$ and $\mu_s$ is plotted (note that $\mu_s$ is scaled down by a factor 10). For such hard vertex functions, however, one would find it difficult to be consistent with the data on the $s-\overline{s}$ asymmetry, as we shall discuss in the next Section. Furthermore, the average number of kaons in the nucleon for such a cut-off mass would be $\langle n \rangle_{KA} \approx 0.3$, which is clearly beyond the limits of a perturbative treatment of the meson cloud.

The matrix elements of the strange axial vector current $J^S_{5\mu} = \pi\gamma_\mu\gamma_5s$ are parametrized in terms of the axial form factors $G_A$: and $G_P$:

$$
\langle N(P')|J^S_{5\mu}(0)|N(P)\rangle = \pi(P') \left( \gamma_\mu \gamma_5 G^*_A(Q^2) + \frac{q_\mu}{M} \gamma_5 G^*_P(Q^2) \right) u(P).
$$

(21)

The strange pseudoscalar form factor $G^*_P$ is not observable in semi-leptonic neutral current reactions [16], and is therefore not considered here. Because the kaon has spin 0, the strange axial vector form factor $G^*_A$ receives contributions only from the $\gamma^*\Lambda$ coupling in Fig.1(a). In this case one has:

$$
G^*_A(Q^2) = Q_A \int_0^1 dy \Delta f^{(A)}(y, Q^2) \ H_\Lambda(Q^2),
$$

(22)

where the light-cone axial distribution function $\Delta f^{(A)}$ is given by:

$$
\Delta f^{(A)}(y, Q^2) = \frac{g_{KNA}^2}{16\pi^3} \int \frac{d^2k_\perp}{y^2(1-y)} \frac{\mathcal{F}(M^2_{\Lambda K,i}) \mathcal{F}(M^2_{\Lambda K,f})}{(M^2_{\Lambda K,i} - M^2)(M^2_{\Lambda K,f} - M^2)}
$$
\[ \times \left( -k_\perp^2 + (M_A - yM)^2 + (1 - y)^2 \frac{Q^2}{4} \right). \]  

The $Q^2$ dependence of the axial $G_A$ form factor is shown in Fig. 5 for three values of the $K\Lambda A$ vertex function cut-off, $\Lambda_{KA} = 0.7, 1$ and 1.3 GeV. At zero transferred momentum the ratio

\[ \eta_s = \frac{G_A^s(0)}{g_A}, \]

where $g_A = 1.26$ from nucleon $\beta$-decays, measures the strange isovector axial charge of the proton. For $K\Lambda A$ vertex function factor cut-offs between $\Lambda_{KA} = 0.7$ and 1.3 GeV, $\eta_s$ is found to be very small, ranging between $\eta_s \approx 0.0017$ and 0.002. Note that the sign of $\eta_s$ (and $G_A^s$) is positive, becoming negative only for $\Lambda_{KA} \gtrsim 1.6$ GeV, as seen in Fig. 5. As we discuss in more detail in Sec. IV B, this fact illustrates that kaon loops alone cannot explain the apparent large strange component of the proton spin.

### IV. DEEP-INELASTIC SCATTERING

The meson cloud model of the nucleon has been successful in providing understanding of the origin of some of the symmetry breaking amongst the proton’s sea quark distributions observed in recent experiments. A pion cloud, for example, naturally allows one to account for the excess of $\bar{d}$ quarks over $\bar{u}$ in the proton [39]. In a similar vein, the kaon cloud of the nucleon gives rise to the observed SU(3) flavor symmetry breaking in the proton sea [19]. Furthermore, it also leads to different strange and antistrange quark distributions in the nucleon, as first pointed out by Signal and Thomas [20].

This question has recently come to prominence again with the availability of new neutrino and antineutrino deep-inelastic scattering data, which were analyzed for a possible non-zero $s-\bar{s}$ difference [9]. Such an asymmetry arises naturally in a kaon cloud picture of the nucleon, since the $s$ and $\bar{s}$ quarks have quite different origins in this model. Indeed, it has been argued [11,20] that because the $s$ quark comes from the $\Lambda$, its distribution should be valence-like, while the $\bar{s}$, originating in the lighter kaon, should be much softer and resemble a typical sea distribution. On the other hand, the experimental $s/\bar{s}$ ratio was found to be consistent, within large errors, with unity, $s/\bar{s} \propto (1 - x)^{-0.46\pm0.085\pm0.17}$, prompting suggestions [11] that the meson cloud model is ruled out by these data.

In this Section we discuss how the above argument is modified when one takes into account the different $K$ and $\Lambda$ distribution functions in the nucleon. The difference $s-\bar{s}$ turns out to be very sensitive to the details of the hadronic vertex functions used in calculating these distributions. When calculated consistently in terms of the light-cone vertex functions discussed in the previous Sections, the kaon cloud model predicts a very small excess of $\bar{s}$ over $s$ at large $x$, which is not in contradiction with the neutrino deep-inelastic data [9].

#### A. $s-\bar{s}$ Asymmetry

Within the same impulse approximation in which the form factors in Sec. III are calculated, the deep-inelastic quark distribution (at a scale $\mu^2$) in the meson cloud model can be
written as a one-dimensional convolution of the meson or hyperon light-cone distribution function and the intrinsic quark distribution in the meson or hyperon. The $s$ and $\bar{s}$ quark distributions, generated from Figs.1(a) and (b), respectively, can be written [20,21]:

$$s(x, \mu^2) = \int_x^1 dy \frac{dy}{y} f_{\Lambda K}(y) s^\Lambda \left(\frac{x}{y}, \mu^2\right),$$  \hspace{1cm} (25a)$$

$$\bar{s}(x, \mu^2) = \int_0^{1-x} \frac{dy}{1-y} f_{K\Lambda}(1-y) \bar{s}^{K^+} \left(\frac{x}{1-y}, \mu^2\right),$$  \hspace{1cm} (25b)$$

where the $N \to K\Lambda$ splitting functions are related to the $\Lambda$ and $K$ distribution functions in Eqs.(14a) and (17a) by:

$$f_{\Lambda K}(y) = f_{1}^{(A)}(y, Q^2 = 0),$$  \hspace{1cm} (26a)$$

$$f_{K\Lambda}(y) = f_{1}^{(K)}(y, Q^2 = 0).$$  \hspace{1cm} (26b)$$

The net strangeness in the nucleon being zero implies that the distributions must be normalized such that:

$$\int_0^1 dx \left(s(x, \mu^2) - \bar{s}(x, \mu^2)\right) = 0,$$  \hspace{1cm} (27)$$

as can be explicitly verified from Eqs.(25).

As for the intrinsic form factors of the virtual $K$ and $\Lambda$, the intrinsic $K$ and $\Lambda$ structure functions are essentially unknown. However, the advantage of the light-cone approach is that the intermediate state particles are on-mass-shell, thus allowing the on-mass-shell structure functions of the kaon and $\Lambda$ to be used [21]. In a covariant instant-form formulation where the $K$ and $\Lambda$ are off their mass shells, one needs to make additional assumptions about the extrapolation of the structure functions into the off-shell region, and indeed about the definition of the structure function of an off-shell $K$ or $\Lambda$ itself [27]. According to the usual prescription adopted in the literature, in the instant-form calculations it is usually simply assumed that the off-mass-shell structure function is the same as that on-shell. While possibly a reasonable approximation for heavy baryons, the justification for such an ansatz is certainly not clear for the kaon, which is typically much further off-mass-shell.

The $\bar{s}$ distribution in kaons is obtained from measurements of final states in inclusive $K^+$ target $\to VX$ reactions, where $V = \mu^+\mu^-$ in Drell-Yan production [40], or $V = \rho, \phi, \cdots$ in inclusive meson production [41]. One finds that the ratio of the $K$ structure function to the much better determined $\pi$ structure function [42] is consistent with unity over most of the range of $x$, dropping slightly at large $x$, $q^K/q^\pi \sim (1-x)^{0.18\pm0.07}$ [40]. For the $s$ quark distributions in $\Lambda$ and $\Sigma$ one can expect the $SU(3)$ symmetric relations $s^{\Sigma^+} \sim d$ and $s^{\Sigma^0} \sim s^\Lambda \sim u/2$ to be reasonable approximations.

With these distribution functions, and the splitting functions in Eqs.(14), (17) and (26), the resulting $s-\bar{s}$ asymmetry is plotted in Fig.6 at $\mu^2 = 10$ GeV$^2$, for different values of the $K\Lambda$ vertex function cut-off, $\Lambda_{KA} = 0.7, 1$ and 1.3 GeV. (Note that it would not be meaningful to compare the calculated $s$ and $\bar{s}$ distributions in Eq.(25) separately with the structure function data, since the distributions calculated in the kaon cloud model do not contain any contributions from the perturbative process $g \to s\bar{s}$.) The asymmetry in the kaon cloud model turns out to be very small, and for not too large values of $\Lambda_{KA}$, broadly
consistent with the CCFR experiment within the given errors [9]. To obtain the difference \( s - \bar{s} \) we have used the absolute values of \( s + \bar{s} \) from the parametrizations of Refs. [43]. We should point out, however, that there exists some controversy regarding the overall normalization of the deep-inelastic neutrino data from which the strange quark distribution was extracted, resulting from an apparent inconsistency between the neutrino data and data on inclusive charm production [44,45]. In addition, the CCFR data were collected with Fe nuclei targets, so that one needs to consider possible nuclear EMC corrections in the data analysis [45] before making any definitive conclusions about the \( s \) and \( \bar{s} \) distributions. In view of these uncertainties in the data themselves, one can certainly not conclude that the meson cloud model is inconsistent with the data from deep-inelastic scattering experiments.

The sign of the difference \( s - \bar{s} \) is very sensitive to the vertex function used at the \( K\Lambda \) vertex. With a vertex function that depends only on the virtuality of the off-mass-shell kaon, such as in Eq.(20), the covariant perturbation theory calculation [11,20] leads to \( s - \bar{s} > 0 \) at large \( x \). The origin of this difference can be traced back to the fact that the light-cone \( K \) distribution function peaks at larger values of \( y \) compared with the instant-form distribution with the vertex function (20), which is somewhat less symmetric about \( y = 1/2 \), Fig.7. Upon convoluting the more symmetric light-cone distribution with the \( s^\Lambda \) and \( \bar{s}^K \) distributions, the \( \sim (1 - x)^3 \) behavior of \( s^\Lambda \) and the harder \( \sim (1 - x) \) behavior of \( \bar{s}^K \) are translated into a harder overall \( \bar{s} \) distribution compared with the \( s \). On the other hand, since at small \( y \) the instant-form distribution \( f_{K\Lambda}^{(IF)}(y) \gg f_{K\Lambda}^{(LC)}(y) \), the original \( x \) dependence in \( s^\Lambda \) and \( \bar{s}^K \) is skewed to such an extent that the resulting \( s \) distribution actually becomes dominant at large \( x \). As discussed in Sec.II, however, the instant-form function \( f_{K\Lambda}^{(IF)}(y) \), evaluated with the \( t \)-dependent vertex function (20), does not satisfy the probability conservation relation in Eq.(3). Hence we believe that the results for the \( s - \bar{s} \) difference in Fig.6 are more realistic for the light-cone distribution functions.

### B. Polarized Strangeness

Given the simplicity with which one can describe symmetry breaking in the nucleon’s sea quark distributions with the meson cloud model, it is tempting to attribute the large violation of the Ellis-Jaffe sum rule [2] to a kaon cloud of the nucleon. By carrying away some orbital angular momentum of the nucleon, a kaon cloud could naively be expected to reduce the intrinsic spin carried by quarks, and give rise to a negatively polarized strange quark distribution, \( \Delta s(x, \mu^2) \). The effect of the meson cloud on the polarized nucleon quark distributions has been addressed by several authors [15,22,46]. Although the kaon cloud picture can provide a simple framework within which the non-perturbative sea could be understood qualitatively, it is clearly important to determined whether such a picture can provide, or even be consistent with, a more quantitative description. In view of the sensitivity of the sign and magnitude of the strange form factors and the unpolarized \( s - \bar{s} \) difference to the hadronic vertex functions, we shall use the light-cone framework to re-examine the question of what role kaons play in the proton spin problem.

The contribution to the polarized strange quark distribution in the nucleon from kaon loops can be written:
\[ \Delta s(x, \mu^2) = \int_x^1 \frac{dy}{y} \Delta f_{\Lambda K}(y) \Delta s^A \left( \frac{x}{y}, \mu^2 \right), \]  
(28a)

\[ \Delta s(x, \mu^2) = 0, \]  
(28b)

where \( \Delta s \) is zero for the same reason that the \( G_A^s \) form factor receives contributions only from the \( \gamma^* \Lambda \) interaction diagram in Fig.1(a). The helicity-dependent \( N \to K \Lambda \) splitting function is given by:

\[ \Delta f_{\Lambda K}(y) = \Delta f^{(A)}(y, Q^2 = 0), \]  
(29)

with \( \Delta f^{(A)}(y, Q^2) \) defined in Eq.(23).

If one had available information about the spin-dependent \( x \)-distribution of the strange quark in the bare \( \Lambda \), one could predict the resulting \( x \)-dependence of \( \Delta s(x, \mu^2) \) arising from kaon loops. Unfortunately, there is no information about polarized quark distributions in any hadron other than the nucleon. Nevertheless, one can still get an estimate of the size of the kaon loop contribution by considering the first moment of \( \Delta s(x, \mu^2) \). In the SU(6) symmetric model the spin of the \( \Lambda \) is carried entirely by the \( s \) quark. It may seem reasonable therefore to expect that

\[ \Delta s^A(\mu^2) \equiv \int_0^1 dx \Delta s^A(x, \mu^2) \sim \mathcal{O}(1) \]  

at some low scale \( \mu^2 \), even taking into account SU(6) symmetry-breaking effects, or some of the spin residing on gluons or in the form of angular momentum. Evolution to larger values of \( \mu^2 \) would imply that \( \Delta s^A(\mu^2) \lesssim \mathcal{O}(1) \) [10]. In this case the total strange quark contribution to the proton spin would be:

\[ \Delta s(\mu^2) \equiv \int_0^1 dx \Delta s(x, \mu^2) \lesssim \int_0^1 dy \Delta f_{\Lambda K}(y) = G_A^s(0). \]  
(30)

For cut-offs \( \Lambda_{K\Lambda} = 0.7-1.3 \) GeV this would give \( \Delta s \lesssim 0.002-0.003 \). This is to be compared with values \( \Delta s \sim -0.1 \) quoted in connection with determinations of \( \Delta s \) from the spin-dependent structure functions in Ref. [2]. These results therefore suggest that a kaon cloud alone cannot be expected to reproduce the observed deviation from the Ellis-Jaffe sum rule. Other mechanisms for generating a non-zero \( \Delta s \), such as ones based on the gluon axial U(1) anomaly, need to be invoked to account for the data.

V. CONCLUSION

We have presented a framework for studying strange form factors and quark distributions of the nucleon consistently in terms of the meson cloud model. Working on the light-cone, one avoids many of the problems and ambiguities associated with the choice of momentum dependent hadronic vertex functions encountered in instant-form calculations.

Within the uncertainty range of the input parameters, the strangeness (Sachs) radius is found to be very small and negative, in the vicinity \( r_s^2 \approx -0.004 \to -0.008 \) fm\(^2\) for \( K \Lambda \) vertex function cut-offs of \( \Lambda_{K\Lambda} = 0.7-1.3 \) GeV. The strange Dirac radius is somewhat smaller, \( r_s^{Dirac} \approx -0.0007 \to -0.0012 \) fm\(^2\). The strange magnetic moment is found to be \( \mu_s \approx -0.04 \to -0.1 \), which is two to three times smaller than in previous estimates. The strange axial vector charge is also small, \( \eta_s \approx 0.0017 \to 0.002 \), but in addition comes with the opposite sign compared with instant-form calculations with the meson cloud model.
Combined, these results suggest that the strangeness content of the nucleon at low $Q^2$ is indeed very small, and may represent a challenge to experimentalists seeking to verify its presence.

Using the same light-cone framework, we have estimated the asymmetry between the $s$ and $\bar{s}$ quark distributions in the nucleon, which has been the subject of recent experimental investigation [9,44]. We find the magnitude of the $s-\bar{s}$ difference to be very small, with the $\bar{s}$ distribution slightly harder than the $s$, but somewhat sensitive to the shape of the hadronic $K\Lambda$ vertex function. Within the current experimental errors this is consistent with the recent experimental determination of the asymmetry from neutrino deep-inelastic scattering, if moderately soft $K\Lambda$ vertex functions are used. For very hard vertex functions, the predicted asymmetry seems to be somewhat large in comparison with the data of Ref. [9]. However, to be more definitive, more statistics on the charm production data are needed, and the apparent discrepancy between the inclusive deep-inelastic muon and neutrino data and those on $c\bar{c}$ production must be resolved [9,44,45]. Finally, the contribution of the kaon cloud to the polarized $\Delta s$ quark distribution is very small and positive, $\Delta s \lesssim 0.002 \rightarrow 0.003$, and therefore is unlikely to feature prominently in any final explanation of the proton spin puzzle.

Aside from the experimental uncertainties, the small values for both the slope of the strangeness form factor and the $s-\bar{s}$ difference within the simple model considered here gives some support to the suggestion of Ref. [11] that the coordinate and momentum space strange quark distributions are correlated. This relationship will be clarified when new, more precise data from deep-inelastic neutrino scattering become available. Measurement of the $Q^2$ dependence of strange matrix elements at low and intermediate values of $Q^2$ at MIT-Bates, Jefferson Lab and elsewhere will also be very valuable [7,8]. The dynamical source of intrinsic strangeness can also be explored in semi-inclusive experiments involving coincidence measurement of scattered leptons and specific hadronic final states [47]. Measurement of the spin transfer from target protons to recoiling $\Lambda$ hyperons in the target fragmentation region could discriminate between kaon cloud models, in which the spins are highly correlated, and parton fragmentation models, for which the correlations are very weak [47]. Similar effects have also been discussed in hadronic reactions, $\bar{p}p \rightarrow \Xi\Lambda$ [48]. While the upcoming experiments will be difficult, if successful they should provide quite important information on the role of strangeness in the nucleon, as well as its dynamical origin.

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FIG. 1. Kaon cloud model of the nucleon: interaction of the probe with the (a) hyperon ("Λ"), (b) kaon.

FIG. 2. \( Q^2 \) dependence of the Sachs electric strange form factor of the nucleon, \( G_E^s \), for various \( K\Lambda \) vertex function momentum cut-offs. Contributions from \( K\Sigma \) components are also included.
FIG. 3. $Q^2$ dependence of the Sachs magnetic strange form factor of the nucleon, $G_M^S$. 
FIG. 4. Cut-off dependence of the strangeness radii, $r_s^2$ and $r_s^2,\text{Dirac}$ (in fm$^2$), strange magnetic moment, $\mu_s$ (scaled by 1/10), and strange axial charge, $\eta_s$, of the nucleon.
FIG. 5. $Q^2$ dependence of the strange axial vector form factor of the nucleon, $G_A^s$. 
FIG. 6. Strange–antistrange quark distribution asymmetry in the nucleon. The solid lines correspond to the asymmetry calculated for $\Lambda_{K\Lambda} = 0.7$ GeV (smallest asymmetry), 1 GeV and 1.3 GeV (largest asymmetry), while the shaded region represents the uncertainty range of the data [9].
FIG. 7. Splitting function for $N \to KA$, $f_{KA}(y)$. The solid curve is the light-cone distribution function, for a vertex function cut-off mass $\Lambda_{KA} = 1$ GeV; dashed is the instant-form result with the $t$-dependent form factor in Eq.(20), which does not satisfy Eq.(3), normalized to give the same value for $\langle n \rangle_{KA}$. 