Generality of inflation and constraints on scalar-tensor theories of gravity

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The dynamics of a dilaton field and an inflaton field are studied in the FRW(flat, open, closed)models. Because of the interaction between a dilaton and an inflaton the onset of inflationary stage of the universe can be affected considerably even in the spatially flat universe. Assuming an equal probability on the phase space, we find the coupling parameter β (defined in the text) must be $0.76 \gtrsim \beta \gtrsim -0.02$ if more than 50% of initial conditions undergo a period of inflation in the flat universe. We also follow the subsequent evolution of the dilaton to evaluate the corresponding PPN parameters. We find the constraint on the PPN parameter puts a further constraint on the coupling β : $\beta \gtrsim -0.009$.

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I. INTRODUCTION

Recently, inflationary models employing multiple scalar fields have been proposed [1]. They are interesting in producing nonadiabatic density perturbation and different spectrum from Harrison-Zel'dovich one. However, the degree of generality of these inflationary models has not been studied so far. It is necessary to study the generality of these inflationary scenarios in order to verify that they are necessary ingredient of the early universe rather than accidental phenomena. Because of the interaction between each scalar field it is expected that the degree of generality of inflation would be changed. The requirement of generality limits the parameters involved in such inflationary models.

We consider the chaotic inflation in scalar-tensor theories of gravity(chaotic hyperextended inflation) in FRW universes. A lot of works have been done on the dynamics of a single scalar field in FRW universe [2] and it is now well-known that inflation is generic. Although we cannot make a definite conclusion about the generality of inflation in the analysis only in homogeneous isotropic universes, the analysis may give us some insight into the behavior of the evolution in more complicated inhomogeneous universes.

In this paper, in Sec.2 the dynamics of an inflaton coupled to a dilaton in FRW(flat, open, closed)universes are studied. To relate to the post-Newtonian parameter, the subsequent dynamics of dilaton after inflation are followed in Sec.3. We find for the specific choice of dilaton coupling taken by Damour [3]- [5], those theories in which the derivative of the Brans-Dicke coupling function is negative are excluded more strongly than the limit by the binary-pulsar experiment or by the lunar-laser-ranging experiment. These results may seem 'attractive' for the attractor mechanism in scalar-tensor theories.

II. GENERALITY OF SCALAR-TENSOR INFLATION

A. basic equations

We consider an inflaton for the chaotic inflation in scalar-tensor theories of gravity. The action is

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$$S = \int d^4x \sqrt{-\bar{g}} \left[\frac{1}{16\pi} (\Phi \bar{R} - \frac{\omega(\Phi)}{\Phi} \bar{g}^{ab} \Phi_{,a} \Phi_{,b}) - \frac{1}{2} \bar{g}^{ab} \sigma_{,a} \sigma_{,b} - V(\sigma) \right], \quad (2.1)$$

where Φ is the massless Brans-Dicke dilaton and σ is the inflaton and we take $V(\sigma) = \frac{1}{2} m^2 \sigma^2$.

If we consider the conformal transformation

$$\bar{g}_{ab} = e^{2a(\varphi)} g_{ab} \quad (2.2)$$

such that $e^{-2a(\varphi)} = \Phi G$ where G is the gravitational constant, then the action can be written as

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} (R - 2g^{ab} \varphi_{,a} \varphi_{,b}) - \frac{1}{2} e^{2a} g^{ab} \sigma_{,a} \sigma_{,b} - e^{4a} V(\sigma) \right], \quad (2.3)$$

where φ is defined by

$$\alpha(\varphi)^2 \equiv 2 \left(\frac{da(\varphi)}{d\varphi} \right)^2 = \frac{1}{2\omega(\Phi) + 3}, \quad (2.4)$$

and we take $a(\varphi) = \beta \varphi^2 / 2$.

We assume homogeneous cosmological models

$$ds^2 = -dt^2 + R^2(t) \left(\frac{dr^2}{1 - Kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right), \quad (2.5)$$

where $K = -1, 0, 1$ for open, flat, closed models respectively. Then the field equations are

$$\ddot{\varphi} + 3H\dot{\varphi} = -4\pi m_{pl}^{-2} \alpha(\varphi) (-e^{2a} \dot{\sigma}^2 + 4e^{4a} V(\sigma)), \quad (2.6)$$

$$\ddot{\sigma} + 3H\dot{\sigma} = -2\alpha\dot{\varphi}\dot{\sigma} - e^{2a} V'(\sigma), \quad (2.7)$$

$$\dot{H} + H^2 = -\frac{8\pi}{3} m_{pl}^{-2} (e^{2a} \dot{\sigma}^2 - e^{4a} V(\sigma)) - \frac{2}{3} \dot{\varphi}^2, \quad (2.8)$$

$$H^2 + \frac{K}{R^2} = \frac{8\pi}{3} m_{pl}^{-2} \left(\frac{1}{2} e^{2a} \dot{\sigma}^2 + e^{4a} V(\sigma) \right) + \frac{1}{3} \dot{\varphi}^2, \quad (2.9)$$

$$\dot{R} = RH, \quad (2.10)$$

where $m_{pl} = G^{-1/2}$ is the Planck mass. By the following change of variables[†]

$$\sigma = e^{-2a} \left(\frac{3}{4\pi} \right)^{1/2} m_{pl} x, \quad (2.11)$$

$$\dot{\sigma} = e^{-a} \left(\frac{3}{4\pi} \right)^{1/2} m m_{pl} y, \quad (2.12)$$

$$\dot{\varphi} = mv, \quad (2.13)$$

$$H = mz, \quad t = m^{-1} \eta, \quad (2.14)$$

the above equations can be recast in an autonomous form

$$x' = e^a y + 2\alpha v x, \quad (2.15)$$

$$y' = -3zy - \alpha v y - e^a x, \quad (2.16)$$

$$\varphi' = v, \quad (2.17)$$

$$v' = -3zv + \alpha(3y^2 - 6x^2), \quad (2.18)$$

$$z' = x^2 - 2y^2 - z^2 - \frac{2}{3} v^2, \quad (2.19)$$

$$R' = zR, \quad (2.20)$$

$$x^2 + y^2 + \frac{1}{3} v^2 - z^2 = \frac{K}{m^2 R^2}, \quad (2.21)$$

where prime denotes the derivative with respect to η .

[†]Of course, other choice of variables are possible, however, as we will see below the initial phase space structure is clearly seen with this choice of variables.

B. initial conditions

In the chaotic inflationary scenario, the initial condition is the Planck scale(quantum boundary). That is,

$$\frac{1}{2}e^{2a}\dot{\sigma}^2 + e^{4a}V(\sigma) + \frac{1}{8\pi}m_{pl}^2\dot{\varphi}^2 \sim m_{pl}^4. \quad (2.22)$$

In terms of new variables x, y, v the initial conditions are

$$x^2 + y^2 + \frac{1}{3}v^2 = \frac{m_{pl}^2}{m^2}, \quad (2.23)$$

and we take $\varphi = 1$. For flat model the initial condition for R is arbitrary and we take $R = 1$. The initial condition for z is determined by the Hamiltonian constraint Eq.(2.21). For open(closed) model initial z takes $z > m_{pl}/m$ ($0 \leq z < m_{pl}/m$), and the constraint is used to determine R .

For flat space, the structure of initial phase space(the quantum boundary) is 2-sphere of radius m_{pl}/m , for open space the boundary is open, for closed space it is 2-sphere $\times[0, m_{pl}/m]$.

C. numerical results

We solve Eqs.(2.15-2.20) numerically by the fourth-order Runge-Kutta method. Eq.(2.21) is used to check the accuracy. The relative error was less than 1% at most. We take $m = 10^{-2}m_{pl}$ for the save of computation time.

The quantum boundary is the surface of a 3-cylinder($D^2 \times \mathbf{R}$) in the phase space and inside the cylinder is a classical region.

In the spatially flat case the trajectories in the four-dimensional phase space are confined on a cone

$$z^2 = x^2 + y^2 + \frac{1}{3}v^2. \quad (2.24)$$

The quantum boundary is 2-sphere on the cone. Trajectories in (x, y, v) space are shown in Figs.1. In Figs.2, trajectories on $x-y$ plane are plotted. The results for single scalar field is also shown for comparison. In Figs.2, phase space variables are (xe^{-2a}, ye^{-a}) rather than (x, y) in order to make it easier to compare with single scalar results. To solve the horizon problem, the inflationary stage must be persisted until $\ln(R_{final}/R_{initial}) > 65$ [6]. If we assume an equal probability on the initial phase space, the degree of inflation can be calculated. Fig.3 shows the probability of inflation for various β . The grid numbers for the initial conditions (x, y, v) are 100×100 . It is to be noted that as shown in [2] the degree of inflation for single scalar field is $\simeq (1 - m/m_{pl})$. For more than 50% of initial conditions to lead to inflation, it is necessary

$$0.76 \gtrsim \beta \gtrsim -0.02. \quad (2.25)$$

In the spatially open case the trajectories are confined inside the cone

$$z^2 > x^2 + y^2 + \frac{1}{3}v^2. \quad (2.26)$$

We find that the qualitative behaviors of the trajectories is similar to those in the flat model except for negative β (see Figs.4 and Figs.5). As z_{init} increases, the deviations from the single scalar case are suppressed.

In the spatially closed case the trajectories are outside the cone

$$z^2 < x^2 + y^2 + \frac{1}{3}v^2. \quad (2.27)$$

Fig.6 shows the probability of inflation for various β . The grid numbers for the initial conditions (x, y, v, z) are $100 \times 100 \times 100$. Unfortunately, in closed model the probability cannot be greater than 50% for any β examined. Note that the probability for single scalar field is 0.64 for $m = 10^{-2}m_{pl}$ and 0.67 for $m = 10^{-6}m_{pl}$. Again we find that the dilaton coupling modifies the degree of inflation significantly.

The effect of the dilaton on the onset of inflation is understood in the following way. For $\beta \gg 1$ from Eq.(2.6) the effective mass for the dilaton can be massive Hence φ starts moving toward the minimum immediately and thus some of the potential energy of σ is converted into the dilaton kinetic energy. For $\beta \ll -1$ from Eqs.(2.6) and (2.7) we see that the effective mass for the dilaton can be tachyonic and an inflaton becomes effectively massless. Thus the occurrence of inflation in these large $|\beta|$ theories is suppressed. The slow-rolling condition is also required for the dilaton so that the inflaton potential energy can dominate. This condition $|\ddot{\varphi}| < 3H|\dot{\varphi}|$ implies $|\beta| + 4\alpha^2 < 3/2$.

III. PPN PARAMETER AFTER INFLATION

In this section we study the evolution of the dilaton after inflation. We assume for simplicity that inflation is immediately followed by radiation-dominated stage (instant reheating) and then matter-dominated stage follows. Our analysis in this section essentially the same as that of Damour and Nordtvedt [3], although in our analysis the initial conditions for the dilaton at the beginning of radiation dominated era are determined by inflation rather than by hand. Here we repeat their analysis for completeness.

Since we consider the era after inflation, we take a flat universe model. A flat homogeneous cosmological model with perfect fluid matter is described by the following equations

$$\ddot{\varphi} + 3H\dot{\varphi} = -4\pi m_{pl}^{-2}\alpha(\varphi)(\rho - 3p), \quad (3.1)$$

$$\dot{H} + H^2 = -\frac{4\pi}{3}m_{pl}^{-2}(\rho + 3p) - \frac{2}{3}\dot{\varphi}^2, \quad (3.2)$$

$$H^2 = \frac{8\pi}{3}m_{pl}^{-2}\rho + \frac{1}{3}\dot{\varphi}^2, \quad (3.3)$$

$$\dot{R} = RH, \quad (3.4)$$

$$(\rho R^3)^\cdot + p(R^3)^\cdot = (\rho - 3p)R^3\dot{\alpha}(\varphi) \quad (3.5)$$

where ρ , p are the density, the pressure of the perfect fluid respectively.

By introducing the number of e-folding N

$$N = \int H dt, \quad (3.6)$$

Eqs.(3.1)-(3.5) give

$$\frac{2}{3 - \varphi_{,N}^2}\varphi_{,NN} + (1 - \lambda)\varphi_{,N} = -(1 - 3\lambda)\alpha(\varphi), \quad (3.7)$$

where $\varphi_{,N}$ denotes the derivative by N and $\lambda = p/\rho$. In Eq.(3.7) the effect of cosmological expansion is decoupled and thus the evolution of the dilaton is easily studied.

A. evolution of dilaton during radiation-dominated stage

In the radiation-dominated stage ($\lambda = 1/3$), Eq.(3.7) becomes

$$\frac{2}{3 - \varphi_{,N}^2}\varphi_{,NN} + \frac{2}{3}\varphi_{,N} = 0. \quad (3.8)$$

From this equation it is clear that the motion is strongly damped. An exact solution of Eq.(3.8) is known [3](see appendix for its derivation)

$$\varphi(N) = \varphi_\infty - \sqrt{3} \ln \left[\frac{\frac{1}{\sqrt{3}}\varphi_{0,N}e^{-N} + \left(1 + \frac{\varphi_{0,N}^2}{3}\right)(e^{-2N} - 1)^{1/2}}{(1 - \varphi_{0,N}^2/3)^{1/2}} \right], \quad (3.9)$$

where $\varphi_{0,N}$ is the initial ‘velocity’. Therefore the dilaton moves only by a finite amount as $N \rightarrow \infty$

$$\varphi_\infty - \varphi_0 = \frac{\sqrt{3}}{2} \ln \frac{1 + \varphi_{0,N}^2/3}{1 - \varphi_{0,N}^2/3}. \quad (3.10)$$

Thus in the radiation-dominated era, φ quickly comes to rest and moves only by an amount of order unity.

B. evolution of dilaton during matter-dominated stage

In the matter-dominated stage ($\lambda = 0$), Eq.(3.7) is

$$\frac{2}{3 - \varphi_{,N}^2} \varphi_{,NN} + \varphi_{,N} = -\alpha(\varphi). \quad (3.11)$$

φ experiences the external force by the potential $a(\varphi)$ in addition to the damping force $\varphi_{,N}$. In the slow-roll approximation ($\varphi_{,N}^2 \ll 1$) we can see the qualitative behavior of φ .

$\beta > 3/8$ corresponds to a damped oscillation. The solution is written as

$$\varphi(N) \simeq \varphi_{eq} e^{-3N/4} \frac{\sin(\nu N + \theta_0)}{\sin(\theta_0)}, \quad (3.12)$$

where $\nu = \frac{3}{4} \sqrt{\frac{8}{3}\beta - 1}$, $\tan \theta_0 = \sqrt{\frac{8}{3}\beta - 1}$ and φ_{eq} is the value of φ at the radiation and matter equal time. Here we choose the origin of N so that $N = 0$ corresponds to the equal-time.

For $0 < \beta < 3/8$ is overdamped case while for $\beta < 0$ *overamplification* case. For $|\beta| \ll 1$, φ evolves as

$$\varphi(N) \simeq \varphi_{eq} e^{-\beta N}. \quad (3.13)$$

Thus we expect that negative β theories are further constrained by the present values of the PPN parameters.

C. the present values of PPN parameters

To determine the present value of φ , it is necessary to know the number of e-folding from the equal-time to the present. Using Eq.(3.5) it is written as

$$\begin{aligned} N_0 \equiv \ln \frac{R_0}{R_{eq}} &= \ln \frac{\rho_{matter}^0}{\rho_{rad}^0} + a(\varphi_{eq}) - a(\varphi_0) \\ &\simeq 10 + \ln \Omega_0 h^2 + a(\varphi_{eq}) - a(\varphi_0) \\ &\simeq 10, \end{aligned} \quad (3.14)$$

where Ω_0 is the present cosmological matter density and h is defined in term of the Hubble parameter $H_0 = 100h$ km/s/Mpc.

We solve Eq.(3.11) numerically. The initial conditions for $\varphi, \varphi_{,N}$ are given by the following rule: Given $\varphi_0, \varphi_{0,N}$ at the end of inflation defined by $\sigma \simeq m_{pl}$, we simply set considering Eqs.(3.9) and (3.10)

$$\varphi_{eq} = \varphi_\infty, \quad (3.15)$$

$$\varphi_{eq,N} = 0. \quad (3.16)$$

Since we simplify the detailed phase transition of the universe, the above prescription is enough to give numerical estimates for the present PPN parameter α_0 . We take $m = 10^{-1} m_{pl}$ for the save of computation time. In Fig.7, the relation of β to α_0 is shown. The minimum of α_0 for each β is taken there. We find the present limit on α_0 [7]

$$\alpha_0^2 < 1 \times 10^{-3}, \quad (3.17)$$

gives a further constraint on β as

$$\beta \gtrsim -9 \times 10^{-3}, \quad (3.18)$$

independently of α_0 . This constraint is significantly stronger than that obtained by the binary-pulsar experiment on the present PPN parameter β_0 [8]

$$\beta_0 > -5, \quad (3.19)$$

or by the lunar-laser-ranging experiment [9]

$$\frac{\beta_0 \alpha_0^2}{2(1 + \alpha_0^2)} < 6 \times 10^{-4}. \quad (3.20)$$

IV. SUMMARY

We have studied the degree of generality of chaotic inflation in a class of scalar-tensor theories of gravity with a simple dilaton coupling taken by Damour $a(\varphi) = \beta\varphi^2/2$. We also have studied the evolution of the dilaton after the end of inflation to give the corresponding present value of the PPN parameter α_0 .

We have found that in the spatially flat universe β must be $0.76 \lesssim \beta \lesssim -0.02$ so that the probability of inflation can be greater than 50% under the assumption of equal probability on the initial phase space. In the spatially closed universe, for any value of β the probability cannot be greater than 50%.

Following the analysis by Damour and Nordvedt, we have found that the limit on α_0 sets a stronger constraint on β : $\beta \gtrsim -9 \times 10^{-3}$. Although our analysis is limited to a particular choice of dilaton coupling, the constraint is stronger by order-of-magnitude than those by the solar-system experiment or by the binary-pulsar experiment. As is shown in sec.3 $\beta > 0$ scalar-tensor theories tend to toward general relativity ($\alpha = 0$) during matter-dominated era. However, the attractor mechanism does not work if $\beta < 0$ since in this case $a(\varphi)$ is not bounded from below. If β is negative, the nonperturbative strong-field deviations from general relativity can occur [5] [10]. We have shown that those negative β theories are strongly constrained if the universe underwent the inflationary stage.

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APPENDIX A: DERIVATION OF EQ.(3.9)

We derive the solution of Eq.(3.8). By changing the variable

$$3 - \varphi_{,N}^2 = X^{-1}, \quad (\text{A1})$$

Eq.(3.8) is written as

$$X_{,N} + 2X - \frac{2}{3} = 0. \quad (\text{A2})$$

The solution is immediately given

$$X = \frac{1}{3} + \frac{C^2}{3} e^{-2N}, \quad (\text{A3})$$

where C is the integration constant. If we take the sign of C so that it is in accord with the sign of initial $\varphi_{,N}$, then we get

$$\varphi_{,N} = C e^{-N} \sqrt{\frac{3}{C^2 e^{-2N} + 1}}. \quad (\text{A4})$$

By the change of variable $p = e^{-N}$, the solution is easily found, and finally we have

$$\varphi(N) = \varphi_\infty + \ln[Ce^{-N} + \sqrt{1 + C^2 e^{-2N}}], \quad (\text{A5})$$

where C is related to the initial $\varphi_{,N}(N=0) = \varphi_{0,N}$ as

$$C = \frac{\varphi_{0,N}}{\sqrt{3 - \varphi_{0,N}^2}}. \quad (\text{A6})$$

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FIGURE CAPTIONS

- Fig.1. Trajectories in the phase space (x, y, v) in the flat universe are shown. Fig.1(a) is for $\beta = -0.5$, Fig.1(b) for $\beta = 0.5$. $m = 0.1m_{pl}$.
- Fig.2. Trajectories in the phase space (xe^{-2a}, ye^{-a}) in the flat universe are shown. (a)-(e) corresponds to $\beta = -0.5, -0.1, 0, 0.1, 0.5$ respectively. Fig.2(f) is the case for a single scalar field. The initial condition for v is $v = 0$.
- Fig.3. The probability of inflation for various β in the flat universe.
- Fig.4. Trajectories in the phase space (x, y, v) in the open universe are shown. Fig.4(a) is for $\beta = -0.5$, Fig.4(b) for $\beta = 0.5$. The initial condition for z is $z_{init} = 2m_{pl}$. $m = 0.1m_{pl}$.
- Fig.5. Trajectories in the phase space (xe^{-2a}, ye^{-a}) in the open universe are shown. (a)-(e) corresponds to $\beta = -0.5, -0.1, 0, 0.1, 0.5$ respectively. Fig.2(f) is the case for a single scalar field. The initial conditions for z and v are $z_{init} = 2m_{pl}$, $v = 0$.
- Fig.6. The probability of inflation for various β in the closed universe.
- Fig.7. The present value of α is shown as a function of $\beta (\neq 0)$. The dotted line is the limit on α_0 by the solar system experiment.

Fig.1-(a)

Fig.1-(b)

Fig2-(c)

Fig2-(d)

Fig2-(f)

Fig2-(e)

FIGURES

Fig.3

Fig.4-(a)

Fig.4-(b)

Fig5-(a)

Fig5-(b)

Fig5-(c)

Fig5-(d)

Fig5-(e)

Fig5-(f)

Fig.6

Fig.7