How to falsify scenarios with primordial fluctuations from inflation\textsuperscript{1}

1 Introduction

In the 15 years since Alan Guth’s famous paper\cite{1} the idea of cosmic inflation has established itself as a central feature of modern cosmology. At this point the inflationary cosmologies are the only theories which can claim to explain the initial conditions of the Standard Big Bang Model, although people still debate the extent to which this claim is legitimate. One of the main difficulties currently facing the field is the absence of a compelling microscopic theory on which to base an inflationary cosmology. In order to achieve the desired period of cosmic inflation, the underlying fundamental field theory must contain a special “flat direction” (the so called inflaton) with very special properties. A great many such field theories have been proposed, but our understanding of very high energy physics remains sufficiently ambiguous that there are no really strong reasons to expect the microphysics to accommodate inflation other than our desire to explain the “initial conditions” of the Universe.

Perhaps a more serious problem for inflation is that although a given inflationary model makes a host of very specific and calculable predictions, a great many different models have been proposed embodying a wide range of possible predictions. As a result there is a widespread perception that inflation can “predict anything you want” and therefore is impossible to falsify in general, even though specific models of inflation can be falsified.

My goal in this contribution is to mount a defence of inflation on this score. Specifically, I will concentrate on the aspects of inflation which pro-

\textsuperscript{1}To be published in Critical Dialogues in Cosmology (proceedings of the Princeton 250th Anniversary conference, June 1996), ed. N. Turok (World Scientific)
duce “primordial inhomogeneities” which lead to CMB anisotropies. The
main focus of my argument will be the predictions which emerge from the
Gaussianity and “passivity” of the inflationary perturbations. By way of
contrast, I briefly mention the flatness of the Universe and near scale invari-
ance of the perturbation spectra as other feature which are often discusses
as “predictions” of inflation.

The organization of this article is as follows: Section 2 briefly comments
on the question of whether flatness and near scale invariance are predictions
of inflation. Section 3 discusses Gaussianity, and Section 4 presents the spe-
cial predictions associated with the “passivity” of inflationary perturbations.
Conclusions are presented in Section 5.

The basic point of this article is that when it comes to the primordial
perturbations, inflation actually commits itself to serious constraints on the
nature of these perturbations. Thus there are many realistic experiments
which can falsify the proposition that these perturbations are predominantly
inflationary despite the enormous flexibility in the inflationary picture. While
these do not offer the opportunity to falsify inflation altogether, such obser-
vations would still have an enormous impact.

2 Predictions of \( \Omega \) and scale invariance

It is often hotly debated whether inflation makes any concrete predictions
for \( \Omega \) and for the near scale invariance of the power spectra (for both scalar
and tensor perturbations). On one hand inflationary models have been built
which give just about any power spectrum and \( \Omega \) \cite{2, 3}. On the other hand
certain cosmologists have steadfastly held out for \( \Omega = 1 \) and a nearly scale
invariant power spectrum as strong predictions of inflation.

I believe the intransigence on the part of those who hold out for \( \Omega = 1 \)
and near scale invariance as predictions, despite obvious evidence to the
contrary has conveyed an air of desperation to the wider community. This
has only enhanced the general dissatisfaction with the predictive power of
inflation. The resulting debate has actually made inflation appear to be in
a much weaker position than is actually the case. For one, some aspects of
inflation are falsifyable on the basis of much more concrete predictions (as
I will discuss below). Furthermore, creating a rather artificial debate about
whether \( \Omega \neq 1 \) or non-scale invariance would falsify inflation actually draws
attention away from the very substantial impact that either of these results would have.

A good analogy can be made with grand unification. In the early days of grand unification there was basically only one game in town: The standard SU(5) grand unified theory. (In fact it was this model on which the early inflationary models were built.) When proton decay experiments ruled out standard SU(5), grand unification was not falsified, but none the less the impact on the field was enormous. It is fair to say that the field of grand unification, with its multitude of under-motivated models has never has regained the focus it had in the days of standard SU(5).

Observations that show $\Omega \neq 1$ or a not non-scale invariant perturbations spectrum would not rule out inflation, but they would have an impact no less great than the proton decay experiments had on grand unification. The whole picture of how inflation can come about would be severely and (by current perspective) very awkwardly constrained. Observers should not let philosophical discussion about what it actually takes to falsify inflation distract them from the importance of these observations.

### 3 Gaussianity

Inflation generates primordial perturbations by amplifying zero-point quantum fluctuations to macroscopic classical scales. Furthermore, inflationary models which have the realistic perturbation amplitudes have an extremely weakly coupled inflaton. This results in a predicted distribution of perturbations which is Gaussian to an excellent approximation.

This is in fact an extremely strong prediction, and one which is easily falsified. A host of alternative models predict non-gaussian features in both the matter field and the CMB anisotropies. (Eg the cosmic string model depicted in Figure 1.) A clear measurement of any one of these characteristic signals would be strong evidence against an inflationary origin for the perturbations. This would reflect a failure of the inflationary model which could NOT be circumvented by fiddling around with the inflaton potential. Although for certain exotic cases non-gaussianity may occur[4], these should give a very different signal which could not be confused with the sort of signals predicted by the defect models.

Given the importance of the question of Gaussianity, this is a rather
Figure 1: A temperature map produced by calculating the Kaiser-Stebbins effect for a network of cosmic strings (from [5]). The non-gaussian nature is clear in this approximation.

poorly developed area of research. There are two main difficulties which must be addressed:

One is “creeping Gaussianity”. The Central Limit Theorem teaches us that the accumulated result of a lot of non-gaussian process can be very Gaussian. Also, a lot of practical factors in realistic experiments (eg the presence of noise and the need for foreground subtraction) can greatly reduce the sensitivity of an experiment to a primordial non-Gaussian signal. A new paper by Ferreira and Magueijo[5] represents welcome progress in addressing these issues, and suggests that despite the obstacles one can expect to make realistic tests of non-Gaussianity in the CMB anisotropies.

The second difficulty is that the term “non-Gaussianity” taken alone does not mean very much. There is a huge space of non-Gaussian distributions, and a randomly chosen “non-Gaussian statistic” is unlikely to be sensitive to a realistic signal. (This has been shown in [5], where a number
of standard “Non-Gaussian” statistics we unable to detect a pretty simple
cosmic string signal.) So the task is to determine what are “good” measures
of non-Gaussianity which have a chance at leading to interesting signals.
While there has been reasonable progress with (for example) the statistics
of extreme peaks (“hot spots”)[6] I feel that this is an extremely worthwhile
field of investigation which deserves more attention. The fact that models
in which the perturbations originate from inflation can be falsified by this
kind of investigation adds motivation to this effort, and represents a great
strength of the inflationary picture.

4 Passivity

4.1 Overview

Another very distinctive feature of inflationary perturbations is the “passiv-
ity” of these perturbations. “Passivity” refers to the fact that perturbations
are set up during the inflationary epoch and evolve in a linear (or “passive”)
manner until gravitational collapse goes non-linear much later in the history of
the Universe. This is in strong contrast to “active” models such as defects or
explosions, which involve non-linear processes in a much more fundamental
way. Another type of passive model is a simple standard Big Bang model
where the perturbations are put it “by hand” in the initial conditions. The
evolution of perturbations in cosmologies with the standard matter content
is highly linear until late times.

The passivity of the inflationary perturbations is another feature that
cannot be “adjusted away” despite the great flexibility in inflationary model
building. Furthermore, passive models give rise to very striking signals in
the angular power spectrum of CMB anisotropies (namely the “secondary
Doppler peaks” or “Sakharov oscillations”). Over the next ten years the
CMB anisotropies will be mapped in ever-increasing detail, and the presence
or absence of Sakharov oscillations will be clearly revealed. If the pertur-
bations are determined not to be passive, all inflationary models for the origin
of these perturbations will be falsified.

The differences between active and passive are quite fundamental, and I
illustrate some key aspects of these difference in the rest of this discussion.
4.2 The evolution of perturbations and the CMB

Most models of structure formation consider perturbations which originate at an extremely early time (e.g., the GUT era or even the Planck era) and which have very small amplitudes (of order \(10^{-6}\)) until well into the matter era. Perturbations of inflationary origin start as short wavelength quantum fluctuations which evolve (during the inflationary period) into classical perturbations on scales of astronomical interest. Defect based models undergo a phase transition (typically at around GUT temperatures, e.g., \(T \approx 10^{16}\) GeV) forming defects which generate inhomogeneities on all scales.

For all these models, once the inflationary period and/or phase transition is over, the Universe enters an epoch where all the matter components obey linear equations except for the defects (if they are present). This “Standard Big Bang” epoch can be divided into three distinct periods. The first of these is the “tight coupling” period where radiation and baryonic matter are tightly coupled and behave as a single perfect fluid. When the optical depth grows sufficiently the coupling becomes imperfect and the “damping period” is entered. Finally there is the “free streaming” period, where the CMB photons only interact with the other matter via gravity. While the second and third periods can have a significant impact on the overall shape of the angular power spectrum, all the physics which produces the Sakharov oscillations takes place in the tight coupling regime, which is the focus of the rest of this section.

Working in in synchronous gauge, and following the conventions and definitions used in [7], the Fourier space perturbation equations are:

\[
\dot{\tau}_{00} = \Theta_D + \frac{1}{2\pi G} \left( \frac{\dot{a}}{a} \right)^2 \Omega_r \dot{s} \left[ 1 + \frac{s}{a} \right]
\]

\[
\dot{\delta}_c = 4\pi G \frac{a}{\dot{a}} (\tau_{00} - \Theta_{00}) - \frac{3a}{2a} \left( \Omega_c + 2 \left[ 1 + \frac{s}{a} \right] \Omega_r \right) \delta_c
\]

\[
\ddot{s} = -\frac{\dot{R}}{1 + \frac{\dot{s}}{s}} - c_s^2 k^2 (s + \delta_c)
\]

Here \(\tau_{\mu\nu}\) is the pseudo-stress tensor, \(\Theta_D \equiv \partial_i \Theta_0^i\), \(\Theta_{\mu\nu}\) is the defect stress energy, \(a\) is the cosmic scale factor, \(G\) is Newton’s constant, \(\delta_X\) is the density contrast and \(\Omega_X\) is the mean energy density over critical density of species.
\( X \) (\( X = r \) for relativistic matter, \( c \) for cold matter, \( B \) for baryonic matter), 
\[ s \equiv \frac{3}{4} \delta_r - \delta_c, \quad R = \frac{3}{4} \frac{\rho_B}{\rho_r}, \quad \rho_B \text{ and } \rho_r \text{ are the mean densities in baryonic and relativistic matter respectively, } c_s \text{ is the speed of sound and } k \text{ is the comoving wavenumber.} \]

The dot denotes the conformal time derivative \( \partial_\eta \).

In the inflationary case there are no defects and \( \Theta_{\mu \nu} = 0 \). With suitable initial conditions these linear equations completely describe the evolution of the perturbations. In the defect case \( \Theta_{\mu \nu} \neq 0 \), and certain components of \( \Theta_{\mu \nu}(\eta) \) are required as input. (Conservation of stress energy allows the equations to be manipulated so that different components of \( \Theta_{\mu \nu} \) are required as input, a matter mainly of numerical convenience [8, 9, 10, 11, 12, 13, 14].) Cosmic defects are “stiff”, which means \( \Theta_{\mu \nu}(\eta) \) can be viewed as an external source for these equations. The additional equations from which one determines \( \Theta_{\mu \nu}(\eta) \) are highly non-linear, although the solutions tend to have certain scaling properties which allow \( \Theta_{\mu \nu}(\eta) \) to be modelled using a variety of techniques [11, 10, 7, 15].

### 4.3 The passive case: Squeezing and phase coherence

Quite generically, for wavelengths larger than the Hubble radius (\( R_H \equiv a/\dot{a} \)), Eqns [1-3] have decaying and growing solutions. The growing solutions reflect the gravitational instability, and, as required of any system which conserves phase space volume, there is a corresponding decaying solution. For example, in the radiation dominated epoch, two long wavelength solutions for the radiation perturbation \( \delta_r \) are \( \delta_r \propto \eta^2 \) and \( \delta_r \propto \eta^{-2} \) (the “adiabatic” modes). The adiabatic growing solution, if present, eventually comes to dominate over any other component. Over time, \( R_H \) grows compared with a comoving wavelength so in the Standard Big Bang epoch a given mode starts with wavelength \( \lambda >> R_H \) but eventually crosses into the \( \lambda < R_H \) regime. In the period of tight coupling modes with \( \lambda < R_H \) undergo oscillatory behavior since the radiation pressure stabilizes the fluid against gravitational collapse. This process of first undergoing unstable behavior which eventually converts to oscillatory behavior is the key to the formation of Sakharov oscillations.

This effect is illustrated in Figure 2, where different solutions for \( \delta_r \) are shown. The key point is that the domination of the growing solution at early times guarantees that each member of the ensemble will match onto an oscillating solution at late times with the same temporal phase (for an intuitive discussion of this effect based on simple harmonic oscillators see
Figure 2: Passive perturbations: Evolution of two different modes during the tight coupling era. While in (a) elements of the ensemble have non-zero values at $\eta_\star$, in (b), all members of the ensemble will go to zero at the final time ($\eta_\star$), due to the fixed phase of oscillation set by the domination of the growing solution (or squeezing) which occurs before the onset of the oscillatory phase. The $y$-axis is in arbitrary units. (From [15]).

Figure 3 shows the ensemble averaged values of $\delta$ at a fixed time as a function of wavenumber (the power spectrum). The zeros correspond to modes which have been caught at the “zero-point” of their oscillations. The phase focusing across the entire ensemble guarantees that there will always be some wavenumbers where the power is zero. It is these zeros which are at the root of the oscillations in the angular power spectrum.

An important point is that these zeros are an absolutely fundamental feature in any passive theory. No amount of tinkering with the details can counteract that fact that an extended period of linear evolution will lead to growing mode domination, which in turn fixes the phase of the oscillations.
in the tight coupling era. If one were to require oscillation in a passive model to be out of phase from the prescribed value, one would imply domination by the decaying mode outside the Hubble radius – in other words a Universe which is not at all Robertson-Walker on scales greater than $R_H$.

4.4 The active case: Coherence lost

As already discussed, the active case is very different from the passive case, due to the presence of what is effectively a source term in Eqns [1-3]. One consequence is that the whole notion of the ensemble average is changed. In the passive case any model with Gaussian initial conditions can be solved by solving Eqns [1-3] with the initial values for all quantities given by their initial RMS values. The properties of linearly evolved Gaussian distributions guarantee that this solution will always give the RMS values at any time.
Figure 4: Active perturbations: Evolution of $\delta_r(k)$ and the corresponding source $\Theta_{00}$ during the tight coupling era ($\Theta_D$ is not shown). Two members of the ensemble are shown, with matching line types. Due to the randomness of the source, the ensemble includes solutions with a wide range of values at $\eta_*$. Unlike the inflationary case (Figure 2) the phase of the temporal oscillations is not fixed. (From [15].)

Thus the entire ensemble is represented by one solution.

In the active case this is not in general possible. One has to average over an ensemble of possible source histories, which is a much more involved calculation. In [15] we “square” the evolution equations to write the power spectrum as convolution of two-point functions of the sources, but there the added complexity requires the use of the full unequal time correlation functions.

In general, the source term will “drive” the other matter components, and temporal phase coherence will be only as strong as it is within the ensemble of source terms. An illustration of this appears in Figure 4. In many active models the sources are sufficiently decoherent that no oscillations appear in
the power spectrum (see for example the dashed curve in Figure 3.)

4.5 Coherence regained

The source evolution is a highly non-linear process, so from the point view of a single wavenumber the source may be viewed as a “random” force term. At first glance it may seem impossible for such random force term could induce \textit{any} temporal coherence, but here is how temporal coherence can occur: The source term only plays a significant role in Eqns [1-3] for a \textit{finite} period of time. This is somewhat apparent in Figure 4. (The y-axis of Figure 4 shows a quantity specially chosen to indicate the significance of the source in Eqns [1-3].) In the limit where this period of significance is short compared to the natural oscillation time of $\delta_r$ (and happens at the same time across the entire ensemble) the ensemble of source histories \textit{can} be phase coherent. Thus one way an active model can produce oscillations in the power spectrum is by having a source term which is “sharply peaked” in time.

It is also the case that on scales larger than $R_H$ there are “squeezing” mechanisms at work, even for active perturbations. (The gravitational instability is, after all, still present.) In [17] a Green function method is developed which clearly illustrates how the active case involves a competition between squeezing effects, which tend to produce oscillations in the power spectrum, and the randomizing effect of the nonlinear source evolution. In extreme cases, where the “randomizing” effects are minimized it is even possible to have active models with mimic an inflationary signal[12, 13, 14].

5 Conclusions

I have outlined the fundamental ways in which inflationary models commit themselves to a constrained range of possible predictions for the primordial fluctuations. The passivity and Gaussianity of inflationary models leads to strong predictions which can be falsified by experiments which will be completed within a decade. Thus, despite the numerous uncertainties in inflationary model building, the inflationary scenarios present an opportunity to do absolutely first rate science.

What if an inflationary origin for the perturbations is ruled out? The idea of inflation as a solution for other cosmological problems will still be
attractive. In fact, it has been argued that perhaps the most ideal role for inflation is to solve the horizon and flatness problems while producing negligible perturbations\[18, 19\], leaving the origin of perturbations to some alternative like cosmic defects, which have their own unique signals. Either way, the new data is certain to have a profound impact on the world of theoretical cosmology.

**Bibliographic Notes**

**References**


