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ASC Pittsburgh '96

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Geneva, 6/11/96

Calculation of Minimum Quench Energies in Rutherford Cables

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Abstract - The Minimum Quench Energy MQE of a conductor may give some indication about the likelihood of training in magnets. We have used a numerical solution of the heat flow equation to calculate the MQE of a single superconducting wire and have found the results to be in good agreement with experiment. This model was then extended to an approximate representation of Rutherford cable by including current and heat transfer between strands. Reasonable agreement with experiment has been found, although in some cases it appears that the effective thermal contact between strands is greater than expected from electrical resistance measurements.

1. INTRODUCTION

Training and degraded performance in superconducting magnets are thought to be caused by sudden local releases of energy within the winding. The main source of this energy is believed to be mechanical and, in the continuing effort to reduce training, most of the effort so far has been devoted to reducing the occurrence of mechanical energy releases, via better support of the electromagnetic forces. The work described here pursues a complementary approach - given that some release of energy will always occur in a winding, can we improve the stability of the cable so that it can absorb the energy without quenching? This paper should preferably be read together with the associated experimental paper [1].

We have chosen to define stability in terms of the minimum quench energy MQE of the cable, ie. that energy input, of short duration applied to a small volume of one wire in the cable, which is just sufficient to trigger a quench. Of course this may not be the only kind of disturbance to cause training; pulses of long duration affecting large volumes could also be a problem. Unfortunately the experimental

evidence is somewhat weak in this area, but there are some indications. Fig. 1 shows the quench precursor in a LHC test dipole. A spike, thought to be movement of the conductor, is followed by a resistance which grows, starts to recover and then grows again. Duration of the spike is $\sim 1/2$ msec. and the resistive voltage at the first peak corresponds to a normal zone length of ~ 60 mm. Thus this quench seems to have been triggered by a short pulse affecting a small volume.

MQE also has the pragmatic advantage that it is uniquely defined as the smallest number of Joules; pulses of longer duration or larger size always need more energy.

II. SINGLE WIRES

To check out some of the functions used before proceeding to the complexity of a cable, we first look at the stability of single wires. Assuming uniform temperatures over any cross section of the wire, the heat flow equation is:

$$\frac{d}{dx} \left(k(\theta) \frac{d\theta}{dx} \right) + \frac{I^2 R(\theta)}{A} + Q_{ini}(x,t) - H(\theta, Q) \frac{P}{A} - C(\theta) \frac{d\theta}{dt} = 0 \quad (1)$$

where x is distance along the wire, $k(\theta)$ thermal conductivity along the wire, θ temperature, I current, $R(\theta)$ is the effective resistance, A area of cross section, Q_{ini} is the initiating energy pulse ($W.m^{-3}$), $H(\theta, Q)$ heat transfer rate to the liquid helium ($W.m^{-2}$), Q total heat transferred to helium ($J.m^{-2}$), P wetted perimeter, $C(\theta)$ specific heat per unit volume and t is time. We first discuss two of the functions used in (1).

Heat transfer rates involved here are much greater than can be sustained in the steady state but, during the short times involved, transient heat transfer can play an important role. We assume that the wire surface is initially wetted by liquid helium and the heat transfer is given by Kapitza resistance, with measured values as reported in [2]. After a certain time, the heat transfer switches to film boiling with a heat transfer coefficient of only $250 W.m^{-2}K^{-1}$. This switch occurs according to either of two criteria, depending on the cumulative heat transferred from the surface Q ($J.m^{-2}$), as follows:

- when Q exceeds the cooling capacity of the helium contained in the channel (we assume no movement of helium during the short pulses involved). For helium boiling at $\sim 4K$, we take this cooling capacity to be $\sim 1/7$ the latent heat [3]. For subcooled superfluid, we take it to be the energy needed to raise the channel volume to the lambda point [4].
- when the heat flux is too high for diffusion into the liquid. For helium boiling at $\sim 4K$, we use the 'take off' time τ

Fig. 1 Quench precursor in a 1m LHC model dipole, scale 2 msec./div. horizontal and 2 mV./div. vertical. [A. Siemko: private communication]

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reported in [5] and make the switch to film boiling when $Q = \tau H$. For superfluid we use the take off times from [6] to derive two criteria: at short times before the onset of vortex turbulence and at long times in the Gorter-Mellink regime.

In order to calculate $H(\theta)$ in the numerical solution of (1), we must also solve for Q at each point in time and space.

$$\frac{dQ}{dt} = H(\theta) \quad (2)$$

The usual assumption about the temperature variation of resistance in a superconducting composite is that, above $J_c(\theta)$, the superconductor carries its critical current and the remaining current flows in the copper, giving a curve like the solid line in Fig 2. With this assumption, we compute zero MQE at the critical current - strongly at variance with experiment. However, it is well known that superconducting composites do not have a sharp transition at $J_c(\theta)$, but have a progressive transition in which resistivity varies as:

$$\rho(J, \theta) = \rho_o \left(\frac{J}{J_o(\theta)} \right)^n \quad (3)$$

where ρ_o is some arbitrary resistivity (usually $10^{-14} \Omega.m$) defined at current $J_o(\theta)$ and n is an empirically determined index, usually $\sim 20-50$. To our knowledge, there are no experimental data on the form of (3) at high resistivities - samples usually quench at $\rho \sim 10^{-13} \Omega.m$. We therefore make a guess that ρ follows (3) at low values and joins smoothly onto the copper line at high currents, as shown in Fig 2.

Fig 2: Models of effective resistivity in a composite with a current sharing temperature of 4.5K: solid shows usual model with abrupt transition in the superconductor, dashed lines shows progressive transitions with 'n' = 25 & 50.

The coupled equations (1) and (2) have been solved numerically by the method of lines using the NAG Fortran routine D03PHF [7]. MQE is found by solving repeatedly with different Q_{ini} and finally taking the mean between two pulses $\sim 1\%$ apart which either just quench or just allow recovery. Fig. 3 shows the MQE calculated for a 1.065mm dia. NbTi composite wire in boiling helium at 4.2K and in boiling superfluid at 2.0K. Also shown are the experimental measurements from [1]. It must be admitted that the agreement in this case is fortuitously good but, in general, we

are able to calculate MQEs for single wires to within a factor 2, showing that the model and functions are basically sound.

Fig.3 Calculated MQEs for a single wire in HeI and HeII compared with experimental measurements. Heater pulse is 45 μs duration over 1 mm length.

III EQUATIONS FOR CABLES

We now extend the equations to a cable, by including the possibility of current and heat transfer between strands. This possibility increases the complexity enormously and we have therefore used the simplified model, sketched in Fig. 4, where the heated strand is in contact with two neighboring groups of 2 wires, which themselves are in contact with two groups representing the remainder of the cable.

Fig.4 Simplified model used to calculate the cable

By symmetry, Fig.4 may be reduced to a problem of three lumped groups, resulting in three equations each for I, θ and Q . Similar calculations for 3 wires in 2 groups were presented at the last ASC [8]. In the equations that follow, r_{12} etc. are the interstrand electrical contact resistances per unit length and are used to calculate the interstrand thermal contact conductances Hk_{12} via the Weidemann Franz law. In some calculations, thermal conductance is taken to be an adjustable parameter γHk via the multiplier γ . The inductances between groups L_{12} etc are defined per unit length in x as:

$$L_{12} = L_1 + L_2 - 2M_{12} \quad (4)$$

ie. the inductance of a loop having 1 as 'go' and 2 as 'return'. The inductances L_{1e} etc. are defined by:

$$\begin{aligned} 2L_{1e} &= L_{12} + L_{13} - L_{23} & 2L_{2e} &= L_{21} + L_{23} - L_{13} \\ 2L_{3e} &= L_{31} + L_{32} - L_{12} \end{aligned} \quad (5)$$

With constant total current I_{cable} we find the equations:

$$I_{cable} = I_1 + I_2 + I_3 \quad (6)$$

$$-I_1 R_1(I_1, \theta_1) - L_{4e} \frac{dI_1}{dt} + \frac{d}{dx} \left(r_{21} \frac{dI_1}{dx} \right) + I_2 R_2(I_2, \theta_2) + L_{2e} \frac{dI_2}{dt} = 0 \quad (7)$$

$$-I_2 R_2(I_2, \theta_2) - L_{23} \frac{dI_2}{dt} + \frac{d}{dx} \left(r_{32} \frac{dI_2}{dx} \right) + \frac{d}{dx} \left(r_{32} \frac{dI_1}{dx} \right) + I_3 R_3(I_3, \theta_3) - L_{3e} \frac{dI_3}{dt} = 0 \quad (8)$$

$$\frac{d}{dx} \left(k(\theta_1) \frac{d\theta_1}{dx} \right) + \frac{r_{21}}{2A_1} \left(\frac{dI_1}{dx} \right)^2 + I_1^2 \frac{R_1(I_1, \theta_1)}{A_1} + Q_{ini}(x, t) - H(\theta_1, Q_1) \frac{P_1}{A_1} - C(\theta_1) \frac{d\theta_1}{dt} = 0 \quad (9)$$

$$\frac{d}{dx} \left(k(\theta_2) \frac{d\theta_2}{dx} \right) + \frac{(r_{21} + r_{32})}{2A_2} \left(\frac{dI_1}{dx} \right)^2 + \frac{r_{32}}{A_2} \frac{dI_1}{dx} \frac{dI_2}{dx} + \frac{r_{32}}{2A_2} \left(\frac{dI_2}{dx} \right)^2 + I_2^2 \frac{R_2(I_2, \theta_2)}{A_2} - H(\theta_2, Q_2) \frac{P_2}{A_2} - \frac{Hk_{23}(\theta_2, \theta_3)}{A_2} + \frac{Hk_{12}(\theta_1, \theta_2)}{A_2} - C(\theta_2) \frac{d\theta_2}{dt} = 0 \quad (10)$$

$$\frac{d}{dx} \left(k(\theta_3) \frac{d\theta_3}{dx} \right) + \frac{r_{23}}{2A_3} \left(\frac{dI_1}{dx} \right)^2 + \frac{r_{32}}{A_3} \frac{dI_1}{dx} \frac{dI_2}{dx} + \frac{r_{32}}{2A_3} \left(\frac{dI_2}{dx} \right)^2 + I_3^2 \frac{R_3(I_3, \theta_3)}{A_3} - H(\theta_3, Q_3) \frac{P_3}{A_3} - \frac{Hk_{23}(\theta_2, \theta_3)}{A_3} - C(\theta_3) \frac{d\theta_3}{dt} = 0 \quad (11)$$

In addition, there are 3 equations like (3) on Q_1 , Q_2 and Q_3 . These 9 coupled equations have been solved numerically on a Pentium PC, again using the NAG routine D03PHF and testing repeatedly until the MQE is found. The cable is that of the LHC dipole inner coil (28 strands of 1.065mm dia wire, NbTi:Cu ratio 1.6:1) and the conditions are those used in our experiments at Brookhaven [1], ie boiling helium at 4.46K and a background field of 7T. For comparability with other work, we define r_{12} etc. in terms of the resistance at a crossover R_c , which in this cable occupies strand a length of 2.1mm. We calculate for three different types of cable:

- bare copper wires with some oxide, $R_c = 100\mu\Omega$.
- partially soldered but still permeable to liquid helium, with $R_c = 0.1\mu\Omega$.
- cables filled with a porous metal sponge, which provides an extended surface for heat exchange to the liquid helium, with $R_c = 3\mu\Omega$.

For each cable, we define three parameters related to heat transfer: f is the fraction of each strand perimeter wetted by liquid helium, v is the volume fraction of liquid in the cable and γ is the multiplying factor on thermal contact noted above.

Fig 5 Calculated MQE for a bare cable type (a). with $f=0.15$, $v=0.04$, $\gamma=1$ or 20, dashed line is calculated for single wire, points are experimental data.

Fig. 5 shows calculations for cable type (a) with $\gamma = 1$ or 20. It seems that the latter assumption of enhanced thermal contact gives a better match to experiment. At high currents, the MQE is small and corresponds to that calculated for a single wire, as shown by the dashed curve. In this regime, if the heat pulse is sufficient to quench wire 1, the whole cable quenches. At lower currents, below the first 'kink' we enter a second regime where wire 1 can quench without quenching the cable. Recovery then occurs after the current in wire 1 has transferred to the rest of the cable. At lower currents still, there is a second 'kink' below which wire 1 and wires in group 2 can all quench without quenching the other wires.

Many experiments show the kink at high current. The low current kink is probably an artefact of our model, coming from lumping the wires into groups. However some bare cables, usually those with lower filling factor, show no kink at all. We believe this to be an effect of greater heat transfer. Fig 6 shows calculations for a cable with $f=0.3$ and $v=0.08$. It may be seen that the first kink has been pushed above I_c , where it could not be observed experimentally (note that, with the generation model shown in Fig. 2, cables are stable

some way above I_c). In order to compute the curve shape shown in Fig. 6 however, we have had to set $\gamma = 20$.

Fig 6 Computed MQE for a bare cable with greater heat transfer and thermal contact between wires ($f=0.3$, $v=0.08$ $\gamma=20$) points are experimental data.

Experimental measurements on soldered cables type (b) never show any trace of a kink. Computed curves show a slight kink, which gets smaller as the contact resistance is reduced. The good contact between strands seems to make them behave as a single unit. Incidentally, it would appear that thermal contact is always the more important factor; increasing electrical contact resistance for the same thermal contact makes the current transfer over larger distances, so that normal zone grows larger, but it does not significantly change the MQE. As shown in Fig. 7, the computed curve fits the experimental data quite well without needing to adjust γ .

Fig. 7 Computed MQE for a soldered cable type (b) with $f=0.25$, $v=0.06$ and $\gamma = 1$, points are experimental data.

Finally, we look at cables filled with porous metal which, in experiments, has been shown to increase stability [9] [1].

We model the increased heat transfer of porous metal simply by increasing the wetted perimeter, neglecting any temperature drop through the metal sponge. For a cable filled with

Fig 8 Computed MQE for porous metal cable with $f=2$, $v=0.07$ $\gamma=1$ (note factor 10x vertical scale) points are experimental data.

porous metal of $\sim 25 \mu\text{m}$ particle size we have $f \sim 2$. Fig. 8 shows the results of this calculation; it may be seen that the strong cooling has pushed the MQE up by an order of magnitude and pushed the kink almost to I_c . Fairly good agreement with experiment is again found with $\gamma=1$.

IV. CONCLUSIONS

Reasonable agreement between theory and experiment has been seen for single wires, showing that the experimental models for transient heat transfer, both in HeI and HeII, may be applied here. Although never observed experimentally, our assumption about the resistive transition of composite wires at high currents also seems to fit the data quite well.

For cables, the behaviour is much more complicated, both experimentally and theoretically. It is dominated by the 'kink', ie the point separating regions where quench is triggered by a single wire quenching or by many wires quenching. Very small MQEs, similar to single wires, are seen above the kink. For some unsoldered cables, this region is accessible at high currents, making the cable very sensitive to quenching. For bare unsoldered cables, a better approximation to experiment is obtained by assuming that the thermal contact between strands is higher than predicted by Weidemann Franz law. This may be physically correct in a situation where oxide layers account for most of the contact resistance, or it may be an error in the model. Good heat transfer to the liquid helium is the most effective way of increasing MQE, not only does it increase the general level of quench energy, but it also pushes the kink out to currents at or even above I_c . An extreme case of good heat transfer is

demonstrated by cables filled with porous metal, where the MQEs are increased by an order of magnitude.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the help of many colleagues at CERN, D.E Bayham at Rutherford Laboratory and the staff at NAG, whose interest often went well beyond that normally expected from a software helpline.

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