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Fermion helicity flip in higher-derivative electromagnetism

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Abstract. It is shown that massive fermions have their helicity flipped on account of their interaction with an electromagnetic field described by Podolsky's generalized electrodynamics. Massless fermions, in turn, seem to be unaffected by the electromagnetic field as far as their helicity is concerned.

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1 Introduction

In a series of papers by Podolsky and co-authors [1 – 4] a higher-order electrodynamics was developed which leads to results free of infinities usually associated with a point charge. Recently, the classical self-force acting on a point charge in the Podolsky model was evaluated and it was shown that, contrary to what happens in Maxwell's theory, in this model the electromagnetic mass is finite and enters the particle's equation of motion in a form consistent with special relativity [5]. Accordingly, the idea of an electron as a point charge is perfectly compatible with Podolsky's theory. Incidentally, experience supports this view [6].

In generalizing the equations of electrodynamics Podolsky made every effort to leave the usual assumptions of Maxwell's theory as nearly unaltered as possible. Therefore, lagrangians of both particles and interactions are left unchanged. Moreover, the field equations are still linear in the field quantities. The lagrangian of the field, in turn, is generalized by allowing a dependence upon the second-order derivatives of the electromagnetic potentials, namely,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}a^2(\partial_\nu F^{\mu\nu})^2, \quad (1)$$

whereupon $F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}$, where a is a real constant with dimension of length which in practice should be small in order not to conflict with well established results of QED.

Our aim here is to analyse the behaviour of the helicity of a massive spin-1/2 Dirac particle in the framework of Podolsky's generalized electrodynamics.

In the next section we compute the effective nonrelativistic potential for the interaction of two spinless charged bosons of equal mass via a “Podolskian photon” exchange. In section 3 we evaluate the polarization of a beam of massive spin-1/2 Dirac particles scattered by an external electromagnetic field described by Podolsky's electrodynamics using the potential found in section 2. In section 4 we consider a polarized beam of massive fermions interacting with a charged scalar particle via a “Podolskian photon” exchange and calculate at the tree

level the polarization of the scattered fermions. We present our conclusions in section 5.

2 The effective nonrelativistic potential for Podolsky's generalized electrodynamics

To begin with we compute the effective nonrelativistic potential for the interaction of two spinless charged bosons of equal mass via a "Podolskian photon" exchange. The expression for the potential is

$$U(\vec{r}) = \frac{1}{4m^2} \frac{1}{(2\pi)^3} \int d^3\vec{k} \mathcal{F}_{\text{N.R.}} e^{-i\vec{k}\cdot\vec{r}} , \quad (2)$$

whereupon $\mathcal{F}_{\text{N.R.}} = i\mathcal{M}_{\text{N.R.}}$, where $\mathcal{M}_{\text{N.R.}}$ is the nonrelativistic limit of the Feynman amplitude for the process $S + S \rightarrow S + S$; here S stands for a spinless boson of mass m and charge Q . The corresponding Feynman diagram is shown in Fig. 1.

Fig. 1 Lowest order contribution to the reaction $S + S \rightarrow S + S$, where S stands for a spinless charged boson.

In the Lorentz gauge Podolsky's scalar QED is described by the lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{a^2}{2} \partial_\mu F^{\mu\nu} \partial^\alpha F_{\mu\alpha} - \frac{1}{2\lambda} (\partial_\nu A^\nu)^2 + (D_\mu \phi)^* D^\mu \phi - m^2 \phi^* \phi , \quad (3)$$

where

$$D_\mu \phi = \partial_\mu \phi + iQ A_\mu \phi .$$

The free propagator for the field A^μ is fixed by the quadratic part of the above lagrangian.

In momentum space we have

$$\Delta_{\mu\nu}(k) = \frac{iM^2}{k^2(k^2 - M^2 + i\epsilon)} \left[\eta_{\mu\nu} - \frac{1 - \lambda \left(1 - \frac{k^2}{M^2}\right)}{k^2} k_\mu k_\nu \right] , \quad (4)$$

with $M^2 \equiv 1/a^2$.

Note that the gauge-independent term of (4) has a good ultraviolet behaviour ($\approx k^{-4}$) at the expense of a negative massive ghost which, contrary to what is currently assumed in the literature [7-9], is a non-tachyonic ghost.

The non-quadratic part of the lagrangian (3) defines the interaction vertices. However, since our calculation will be restricted to trilinear vertices, the interaction lagrangian reduces to

$$\mathcal{L}_{\text{inter.}} = iQ A^\mu (\phi \partial_\mu \phi^* - \phi^* \partial_\mu \phi) .$$

From the previous expression the Feynman rule for the elementary vertex may readily be deduced. This is shown in Fig. 2.

Fig. 2 Elementary vertice for Podolsky's scalar electrodynamics of a spinless boson with charge Q .

Accordingly, the invariant amplitude for the process shown in Fig. 1 is

$$\mathcal{M} = \frac{-iM^2 Q^2}{k^2(k^2 - M^2)} (2p - k)(2q + k) ,$$

which in the nonrelativistic limit becomes

$$\mathcal{M}_{\text{N.R.}} = -4i \frac{M^2 Q^2 m^2}{\vec{k}^2 (\vec{k}^2 + M^2)} \quad (5)$$

Substituting (5) into (2) and performing the integration we obtain the expression for the nonrelativistic potential

$$U(\vec{r}) = \frac{Q^2}{4\pi} \left(\frac{1 - e^{-Mr}}{r} \right),$$

from which it follows immediately the expression for "Coulomb's generalized potential"

$$V(\vec{r}) = \frac{Q}{4\pi} \left(\frac{1 - e^{-r/a}}{r} \right) \quad (6)$$

Notice that this potential approaches $\frac{Q}{4\pi a}$ as r approaches zero.

3 Coulomb's generalized scattering

Next we work out a version of the Coulomb scattering problem. Consider for this purpose the scattering of a negative charged spin-1/2 Dirac particle with mass m_F by the "generalized Coulomb field" of a heavy nucleus, treated as a point charge. In the Coulomb gauge, the potential is given by

$$A_{\text{ext}}^\mu(\vec{x}) = \left(\frac{Ze \left(1 - e^{-|\vec{x}|/a} \right)}{4\pi |\vec{x}|}, 0, 0, 0 \right),$$

with the momentum space potential

$$A_{\text{ext}}^\mu(\vec{k}) = \left(\frac{Ze}{|\vec{k}|^2 (1 + a^2 |\vec{k}|^2)}, 0, 0, 0 \right).$$

Here the charge of the electron is $-e < 0$.

The Lagrange function of the system is

$$\mathcal{L} = \bar{\psi} (i \not{\partial} - m_F) \psi + e \bar{\psi} A_{\text{ext}} \psi.$$

The corresponding fermion-external-field vertex is shown in Fig. 3.

Fig. 3 The fermion-external-field vertex. Here $\vec{k} = \vec{p}' - \vec{p}$ and $|\vec{p}| = |\vec{p}'|$.

Suppose now that in the initial state we have a beam of fermions with momentum $p = (E, \vec{p})$ in the spin state $u(\vec{p}, S_L(\vec{p}))$, namely, the incident fermions are left-handed (L).

The spinors u and $\bar{u}$ are such that

$$\begin{aligned} \frac{(1 + \gamma_5 \not{p})}{2} u(\vec{p}, S_i) &= u(\vec{p}, S_i), \\ \frac{(1 + \gamma_5 \not{p})}{2} u(\vec{p}, -S_i) &= 0, \\ u_\alpha(\vec{p}, S_i(\vec{p})) \bar{u}_\beta(\vec{p}, S_i(\vec{p})) &= \left[\frac{\not{p} + m_F}{2m_F} \frac{1 + \gamma_5 \not{p}}{2} \right]_{\alpha\beta}, \\ i &= L, R, \end{aligned}$$

whereupon the polarization vectors are given by

$$\begin{aligned} S_R^\alpha(\vec{p}) &= \left(\frac{|\vec{p}|}{m_F}, \frac{p_\alpha \vec{p}}{m_F} \right), \\ S_L^\alpha(\vec{p}) &= \left(\frac{-|\vec{p}|}{m_F}, \frac{-p_\alpha \vec{p}}{m_F} \right), \end{aligned}$$

where R stands for right-handed fermions and $\hat{p} \equiv \frac{\vec{p}}{|\vec{p}|}$.

The polarization of the scattered fermions is measured by

$$P = 1 - \frac{2N_R}{N_L + N_R},$$

where $N_L(N_R)$ denotes the number of fermions emerging with negative(positive) helicity.

For the process in hand,

$$N_R = \left| \bar{u}(\vec{p}', S_R(\vec{p}')) A_{\text{ext.}}(\vec{k}) u(\vec{p}, S_L(\vec{p})) \right|^2$$

and

$$N_L = \left| \bar{u}(\vec{p}', S_L(\vec{p}')) A_{\text{ext.}}(\vec{k}) u(\vec{p}, S_L(\vec{p})) \right|^2,$$

where p' is the momentum of the emerging fermion. Note that $P = 1$ indicates no depolarization of the initial fermions, while $P = -1$ says that all the left-handed initial fermions have their helicity flipped.

Some more algebra and we arrive at the following expressions for N_R and N_L :

$$N_R = \frac{1}{4m_F^2} \left\{ (2A \cdot p' A \cdot p - A^2 p' \cdot p + A^2 m_F^2) (1 - S_R(\vec{p}') \cdot S_L(\vec{p})) \right. \\ \left. + 2(m_F^2 - p \cdot p') A \cdot S_R(\vec{p}') A \cdot S_L(\vec{p}) + 2p' \cdot S_L(\vec{p}) S_R(\vec{p}') \cdot A p \cdot A \right. \\ \left. + (2p' \cdot A A \cdot S_L(\vec{p}) - A^2 p' \cdot S_L(\vec{p})) S_R(\vec{p}') \cdot p \right\},$$

$$N_L = \frac{1}{4m_F^2} \left\{ (2A \cdot p' A \cdot p - A^2 p' \cdot p + m_F^2 A^2) (1 - S_L(\vec{p}') \cdot S_L(\vec{p})) \right. \\ \left. + 2(m_F^2 - p \cdot p') A \cdot S_L(\vec{p}) S_L(\vec{p}') \cdot A + 2A \cdot S_L(\vec{p}') p' \cdot S_L(\vec{p}) A \cdot p \right. \\ \left. + (2A \cdot p' A \cdot S_L(\vec{p}) - A^2 p' \cdot S_L(\vec{p})) S_L(\vec{p}') \cdot p \right\},$$

where $A \equiv A_{\text{ext.}}^\mu(\vec{k})$ and $A^2 \equiv A \cdot A$.

Therefore, the polarization of the scattered fermions is given by:

$$P = 1 - \left[\frac{2m_F^2 \tan^2 \theta / 2}{E^2 + m_F^2 \tan^2 \theta / 2} \right], \quad (7)$$

where θ denotes the scattering angle. We shall discuss this result in section 5.

4 Tree-level behaviour of the helicity of massive fermions in Podolsky's electrodynamics

Consider now the reaction

$$F^- + S^+ \rightarrow F^- + S^+,$$

where F^- denotes a negative charged fermion and S^+ stands for a positive charged scalar particle (see Fig. 4).

Fig. 4 Lowest order contribution to the reaction $F^- + S^+ \rightarrow F^- + S^+$ (F^- and S^+ are defined as particles)

The corresponding lagrangian in the Lorentz gauge is given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{a^2}{2} (\partial_\lambda F^{\mu\lambda})^2 - \frac{1}{2\lambda} (\partial_\mu A^\mu)^2 + \bar{\psi} (i \not{\partial} - m_F) \psi + e \bar{\psi} \not{A} \psi + \partial_\alpha \phi^* \partial^\alpha \phi \\ - m_S^2 \phi^* \phi + i e (A^\mu \phi \partial_\mu \phi^* - A^\mu \phi^* \partial_\mu \phi) + e^2 A_\mu \phi^* A^\mu \phi,$$

where m_F and m_S are the masses of the fermion and scalar particle, respectively.

The pertinent Feynman rules for the process shown in Fig. 4 are displayed in Fig. 5.

Fig. 5 The relevant Feynman rules for fermion-scalar-field interaction.

Suppose then that in the initial state we have a scalar particle with momentum q and a left-handed fermion with momentum $p = (E, \vec{p})$.

For the process in hand,

$$N_R = \left| \bar{u}(\vec{p}', S_R(\vec{p}')) V_1^\alpha u(\vec{p}, S_L(\vec{p})) \Delta_{\alpha\beta}(k) V_2^\beta(q, q') \right|^2$$

and

$$N_L = \left| \bar{u}(\vec{p}', S_L(\vec{p}')) V_1^\alpha u(\vec{p}, S_L(\vec{p})) \Delta_{\alpha\beta}(k) V_2^\beta(q, q') \right|^2,$$

where $p'(q')$ is the momentum of the emerging fermion (scalar particle).

Using much algebra we arrive at the following expressions for N_R and N_L :

$$N_R = \frac{e^4 M^4}{4m_F^2 t^2 (t - M^2)^2} \left\{ [1 - S_L(\vec{p}) \cdot S_R(\vec{p}')] \left[\frac{(s-u)^2}{2} + \frac{t}{2} (4m_S^2 - t) \right] + t (S_R(\vec{p}') \cdot Q) (S_L(\vec{p}) \cdot Q) \right. \\ \left. + (s-u) [(S_L(\vec{p}) \cdot p') (S_R(\vec{p}') \cdot Q) + (S_L(\vec{p}) \cdot Q) (S_R(\vec{p}') \cdot p)] \right. \\ \left. - (4m_S^2 - t) (S_L(\vec{p}) \cdot p') (S_R(\vec{p}') \cdot p) \right\},$$

$$N_L = \frac{e^4 M^4}{4m_F^2 t^2 (t - M^2)^2} \left\{ [1 + S_L(\vec{p}) \cdot S_R(\vec{p}')] \left[\frac{(s-u)^2}{2} + \frac{t}{2} (4m_S^2 - t) \right] - t (S_R(\vec{p}') \cdot Q) (S_L(\vec{p}) \cdot Q) \right. \\ \left. - (s-u) [(S_R(\vec{p}') \cdot p) (S_L(\vec{p}) \cdot Q) + (S_R(\vec{p}') \cdot Q) (S_L(\vec{p}) \cdot p)] \right. \\ \left. + (4m_S^2 - t) (S_L(\vec{p}) \cdot p') (S_R(\vec{p}') \cdot p) \right\},$$

where $Q \equiv q' + q$ and $s = (p+q)^2$, $t = (p-p')^2$, and $u = (p-q')^2$ are the usual Mandelstam variables.

Next, we evaluate the polarization P in the scalar particle rest system [$q^\mu = (m_S, \vec{0})$] (see Fig. 6).

Fig. 6 The definition of the F^- scattering angle in $F^- S^+$ scattering.

In this frame

$$s = m_F^2 + m_S^2 + 2Em_S,$$

$$t = 2m_S(E' - E),$$

$$u = 2(m_F^2 + m_S^2) - s - t,$$

whereupon

$$E' = \frac{(m_S E + m_F^2)(E + m_S) + (E^2 - m_F^2) \cos \theta \sqrt{m_S^2 - m_F^2} \sin^2 \theta}{(E + m_S)^2 - (E^2 - m_F^2) \cos^2 \theta},$$

where $E(E')$ is the initial(final) energy of the fermion.

The final expression for P is then given by

$$P = 1 - 2 \left\{ \left[1 + \frac{\sqrt{E^2 - m_F^2} \sqrt{E'^2 - m_F^2} - E E' \cos \theta}{m_F^2} \right] \left[\frac{(s-u)^2}{2} + \frac{t}{2} (4m_S^2 - t) \right] \right. \\ \left. - t \left[\frac{\sqrt{E'^2 - m_F^2} (2m_S + E) - E' \sqrt{E^2 - m_F^2} \cos \theta}{m_F} \right] \right. \\ \times \left[\frac{\sqrt{E^2 - m_F^2} (2m_S - E') + E \sqrt{E'^2 - m_F^2} \cos \theta}{m_F} \right] \\ \left. - (s-u) \left[\left(\frac{E' \sqrt{E^2 - m_F^2} - E \sqrt{E'^2 - m_F^2} \cos \theta}{m_F} \right) \right. \right. \\ \times \left(\frac{\sqrt{E'^2 - m_F^2} (2m_S + E) - E' \sqrt{E^2 - m_F^2} \cos \theta}{m_F} \right) \\ \left. \left. + \left(\frac{\sqrt{E^2 - m_F^2} (2m_S - E') + E \sqrt{E'^2 - m_F^2} \cos \theta}{m_F} \right) \right. \right. \\ \times \left. \left. \left(\frac{E \sqrt{E'^2 - m_F^2} - E' \sqrt{E^2 - m_F^2} \cos \theta}{m_F} \right) \right] \right. \\ \left. + (4m_S^2 - t) \left(\frac{E' \sqrt{E^2 - m_F^2} - E \sqrt{E'^2 - m_F^2} \cos \theta}{m_F} \right) \right. \\ \times \left. \left(\frac{E \sqrt{E'^2 - m_F^2} - E' \sqrt{E^2 - m_F^2} \cos \theta}{m_F} \right) \right\} / \\ \left\{ (s-u)^2 + t(4m_S^2 - t) \right\}. \quad (8)$$

In the next section we present a discussion concerning the previous result.

5 Conclusion

In Fig. 7 it is shown the behaviour of P (Eq. (7)) as a function of both θ and m_F , for fixed E ; whereas in Fig. 8 it is displayed P (Eq. (8)) as a function of both θ and m_F , for fixed E and m_S . As may be seen, the larger the scattering angle, the larger the helicity flip. Figs. 7 and 8 tell us also that $P \rightarrow 1$ as $m_F \rightarrow 0$.

In summary, we have shown that at the tree level all the massive fermions have their helicity flipped on account of their interaction with Podolsky's electromagnetic field. Massless fermions, in turn, seem to be unaffected by this field as far as their helicity is concerned.

On the other hand, as is well know, if one starts from a renormalizable theory and calculates the effective action one obtains typically several operators of dimension 6(or higher) and not just Podolsky's term [10]. This raises the question: Would the above results change significantly if one takes all these terms into account or are they fairly generic? Since the amount of spin-flip is determined by the vector nature of the electromagnetic field but not by its spatial distribution, the aforementioned results will not change at all. What about the finiteness of the electromagnetic mass? Since the evaluation of the classical self-force acting on point charge depends on the Podolsky length a [5], one is led to conjecture that the finiteness of the electromagnetic mass is a very specific property of Podolsky's electrodynamics and not a general feature of higher-derivative theories.

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Fig. 7 P as function of both θ and m_F , for $E = 100 \text{ MeV}$.

Fig. 8 P as function of both θ and m_F , for $E = 100\text{MeV}$ and $m_S = 110\text{MeV}$.

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