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THE AKM THEOREM AND OSCILLATIONS IN THE HADRON SCATTERING AMPLITUDE AT HIGH ENERGY AND SMALL MOMENTUM TRANSFER

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Abstract

We show that the high precision dN/dt UA4/2 data at $\sqrt{s} = 541 \text{ GeV}$ are compatible with the presence of Auberson - Kinoshita - Martin (AKM) type of oscillations at very small momentum transfers. These oscillations seem to be periodic in $\sqrt{|t|}$, the corresponding period being $\simeq 2 \cdot 10^{-2} \text{ GeV}$. The existence of such visible oscillations suggests a general mechanism of saturation of axiomatic bounds. As an illustration the consequences for extracting the parameter $\rho = \text{Re}F/\text{Im}F$ from dN/dt data are also discussed.

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It is well known that the very precise measurement of the hadron-hadron differential cross-sections at small momentum transfer and high energy is of outmost importance for the understanding of strong interactions. In this kinematical domain perturbative QCD cannot be directly operative; however, there are numerous results derived in the framework of axiomatic field theory which lead to interesting suggestions both on theoretical and experimental levels. For example, on the one hand, the theoretical corpus of asymptotic theorems [1], derived from general principles, has indicated, already thirty years ago, the possibility of oscillations in this kinematical region [2]. On the other hand, a substantial real part of the odd-under-crossing scattering amplitude could lead to the fascinating possibility of a difference between hadron-hadron and antihadron-hadron scattering at very high energies [3]. Such a possibility was already confirmed by rigorous calculations, exploring the $C = -1$ reggeized gluons sector in perturbative QCD [4]. but, of course, it is not yet possible to predict if such a result will persist in the confinement region.

A very precise $\bar{p}p$ experiment at small t and high s was performed three years ago by the UA4/2 Collaboration [5]. The analysis of these data, based on exponential fits [5, 6], led the experimentalists of the UA4/2 group to the conclusion that no new phenomena, like the above mentioned ones, occur at this energy $\sqrt{s} = 541 \text{ GeV}$. However, on a theoretical level, the situation is still ambiguous because one has, at every fixed s and t , only one observable $(dN/dt)(s, t)$ for two unknowns $ReF_N(s, t)$ and $ImF_N(s, t)$: some additional experimental information is needed in order to extract, in a model-independent way, the nuclear amplitude $F_N(s, t)$ from the data [7]. In the absence of such additional experimental information, one has to introduce a theoretical input and therefore the definition of the nuclear amplitude is model dependent. Indeed the experimental analysis is plagued by a *theoretical input*: the exponential form of the hadron scattering amplitude in the diffraction peak. Such a form works so well at lower energies that one has the tendency to forget that it has no deep theoretical justification. It is therefore understandable

why, in the present paper, we conclude that the extraction of the real part of the forward scattering amplitude by UA4/2 group is doubtful and possibly wrong, even if their dN/dt raw data are perfectly correct.

The detection of oscillations or the extraction of the semi-theoretical parameter $\rho = \text{Re}F_N(s, t=0)/\text{Im}F_N(s, t=0)$ - containing the crucial information on the phase of the hadron amplitude - are obviously very sensitive to the theoretical input [8, 9]. One has also to note that the *continuity* of the physical phenomena in going from low energies to high energies requires that the new phenomena will first occur as *small effects* on a large background of old physics. The beginnings of the theory of relativity and quantum mechanics illustrate very vividly such a situation.

We need therefore, in analysing the UA4/2 data, to find a way of avoiding the logical vicious circle consisting in inferring, from a more or less good χ^2 - fit of the raw dN/dt data [5], that there are no new effects just because the theoretical input excludes them.

In this work we test for the existence of oscillations in the nuclear scattering amplitude $F_N(s, t)$ at very small t , possibly connected with the Auberson-Kinoshita-Martin (AKM) theorem [2].

Namely, the AKM theorem [2, 10] states that, in the case of the regime of maximal $\ln^2 s$ axiomatic growth of the nuclear scattering amplitude allowed at $s \rightarrow \infty$, the ratio between the even(odd)-under-crossing amplitude and its value at $t = 0$ is not a function separately of s and t but obeys a *scaling* property:

$$F_{\pm}(s, t)/F_{\pm}(s, t=0) = g_{\pm}(\tau), \quad (1)$$

where $g_{\pm}(\tau)$ are entire functions of order $1/2$ of τ^2 , satisfying the normalization condition

$$g_{\pm}(\tau=0) = 1 \quad (2)$$

and τ is the scaling variable

$$\tau = \sqrt{-t/t_0} \ln s. \quad (3)$$

where a scale factor $s_0 = 1 \text{ GeV}^2$ is, of course, implicitly present in $\ln(s/s_0)$. In Eq. (3) t_0 is a constant less than or equal to the t -channel threshold m_π^2 ; therefore the parameter t_0 is a measure of the AKM domain in t . In this regime of maximal growth [2, 10] $F_+(s, t)$ will be a mainly imaginary function while $F_-(s, t)$ will be a mainly real function. According to the AKM theorem, the scattering amplitude must have *infinitely many zeros* in a very narrow region in t including the physical region $t \leq 0$ and it is this property which is related to the presence of oscillations. The AKM theorem does not necessarily imply the presence of visible oscillations in the differential cross section : additional constraints [11] are needed in order to get zeros on the real τ axis.

Of course, at *finite energies* (like $\sqrt{s_1} = 541 \text{ GeV}$), only a finite number of zeros would be present in the possible oscillatory part $F_{osc}(s, t)$ of the nuclear amplitude, which is not yet dominant. In the following we study if such oscillations are or are not compatible with the UA4/2 data.

If such oscillations exist in dN/dt , they will be swamped by the large contribution of the Coulomb amplitude $F_c(s, t)$ and the usual χ^2 fits will be unable to detect them. It is therefore interesting to make a zoom in the very small- t region (say $0 < |t| \lesssim 0.01 \text{ GeV}^2$) of the quantity $(1/K) dN/dt(s_1, t) - |F_c(s_1, t)|^2$, where K is the normalization factor used by the UA4/2 Collaboration [5, 6] and where $F_c(s, t)$ amplitude is taken in its standard form [12]. It is seen from Fig.1 that at least one visible oscillation - a kind of "bump" - seems to occur around $|t| \simeq 2 \cdot 10^{-3} \text{ GeV}^2$.

The existence of oscillations can be studied by defining the ratio R at $\sqrt{s_1} = 541 \text{ GeV}$:

$$R(s_1, t) = \frac{d\sigma/dt(s_1, t)}{d\sigma/dt(s_1, t)_{expon}} - 1, \quad (4)$$

where $d\sigma/dt(s_1, t)$ denotes either the UA4/2 differential cross-sections or those given by a theoretical model and $d\sigma/dt(s_1, t)_{\text{expon}}$ is, *by definition*, the theoretical model involving the exponential form of the nuclear scattering amplitude

$$F_N(s_1, t)_{\text{expon}} = (\rho + i) \sigma_T \exp(bt/2), \quad (5)$$

with $\sigma_T = 62.2 \text{ mb}$ [13], $\rho = 0.135$ [5], $b = 15.5 \text{ GeV}^{-2}$ [5] (i.e. σ_T, ρ and b are constants *fixed* by the central values given by the χ^2 method of the UA4 group in two different experiments [5, 13]).

The quantity R given by (4) is directly linked to the UA4/2 differential cross sections. The large exponential background is partly extracted from $d\sigma/dt$ and therefore R is a good indicator of phenomena occurring in the Coulomb-nuclear interference region. The large Coulomb contribution is also partly extracted from $d\sigma/dt$. At $t = 0$ its dominance imposes the value $R(s_1, 0) = 0$. Let us note that if the theoretical model tested by Eq. (4) is just the exponential one, the ratio R is *zero* at any t .

In Fig.2 we show the ratio R corresponding to the experimental UA4/2 data. R is plotted versus $\sqrt{t}$, proportional to the scaling τ variable. It is seen that R is oscillating around the value 0. It is also seen that R is *positive* for $4 \cdot 10^{-2} \lesssim \sqrt{|t|} \lesssim 5 \cdot 10^{-2} \text{ GeV}$ and *negative* for $3 \cdot 10^{-2} \lesssim \sqrt{|t|} \lesssim 4 \cdot 10^{-2} \text{ GeV}$ and for $5 \cdot 10^{-2} \lesssim \sqrt{|t|} \lesssim 6 \cdot 10^{-2} \text{ GeV}$. The experimental data are therefore compatible with the presence of approximately equally-spaced zeros at $\sqrt{|t|}$ values of $\approx 3, 4$ and $5 \cdot 10^{-2} \text{ GeV}$.

These regularities are impressive and highly suggestive of AKM structure associated with oscillatory entire functions of order $1/2$. In the present case, these functions seems to be also quasi-periodic.

For example, let us take, as a toy model satisfying the AKM theorem, the periodic function $(\sin \tau)/\tau$ of order $1/2$ and period 2π . The zeros of this function are equally-spaced by the interval π . The corresponding interval in t is obtained from the equality

$\tau = \pi$, i.e.

$$\delta(|t|) = t_0 \cdot \pi^2 / (\ln s_1)^2 \leq 1.2 \cdot 10^{-3} \text{ GeV}^2, \quad (6)$$

by taking into account that $t_0 \leq m_\pi^2$. The function $(\sin \tau)/\tau$ induces equally-spaced zeros in $R(s_1, t)$ separated by the interval $\delta(\sqrt{|t|}) \lesssim 3.5 \cdot 10^{-2} \text{ GeV}$. It is seen from Fig. 2 that this inequality is compatible with the experimental value $\delta(\sqrt{|t|}) \simeq 10^{-2} \text{ GeV}$ of the distance between two successive zeros of $R(s_1, t)$. This agreement is striking and can hardly be thought of as pure coincidence : the new structure is precisely located in the t -region of the UA4/2 experiment which overlaps the AKM domain $t \leq t_0$. The experimental value $\delta(\sqrt{|t|})$ immediately fixes the value of t_0 through the relation :

$$\sqrt{t_0} = \delta(\sqrt{|t|}) \cdot \frac{\ln s}{\pi}, \quad (7)$$

i.e. $\sqrt{t_0} \simeq 40 \text{ MeV} \simeq (1/3)m_\pi$.

The above remarks strongly suggest that further zeros are likely to be present at $\sqrt{|t|} \simeq 1 \cdot 10^{-2} \text{ GeV}$ and $\sqrt{|t|} \simeq 2 \cdot 10^{-2} \text{ GeV}$, in a region $|t| < 8.75 \cdot 10^{-4} \text{ GeV}^2$ not yet explored by experiment. At $t = 0$, due to the normalization condition (2), there is no zero induced by the oscillations present in the nuclear amplitude. However, at extremely small t -values the Coulomb amplitude is dominant and therefore $R = 0$ at $t = 0$.

Let us note that AKM functions $2 \cdot J_1(\tau)/\tau$ and $(\sin \tau)/\tau$, appearing in the dominant $\ln^2 s$ terms at $s \rightarrow \infty$ in $F_\pm(s, t)$ [2, 10], correspond to *damped* oscillation in τ . Such a damping is also compatible with the R data at the finite energy $\sqrt{s_1}$. The damping increases rapidly with increasing τ : from Eqs. (3) and (7) it is seen that τ reaches a large value $\tau \simeq \ln s_1$ already in the region of the bump in $d\sigma/dt$.

For the moment, we have shown that the oscillations seen in the experimental values of R displayed in Fig.2 (i.e. in the UA4/2 raw data dN/dt), if statistically significant, are fairly compatible with the AKM theorem. It remains to establish the statistical significance of these oscillations.

Considering the entire experimental region (99 experimental points) we get $\chi^2/dof \simeq 1.1$ for the description of the dN/dt data by the exponential form (5). From this value of χ^2/dof itself we can not yet conclude about the presence or absence of some additional effect to the exponential form of the scattering amplitude.

In order to check further for the presence of oscillations and therefore the deviation of the experimental data from the exponential form of the nuclear amplitude we adopt a procedure similar to that used in the analysis of periodic radio signals. Namely, we define

$$L(n_1, n_2) = \sum_{i=n_1}^{n_2} (R_i^{exper} - R_i^{theor})/\delta(R_i), \quad (8)$$

where n_1, n_2, i are integer numbers such that $1 \leq n_1 < n_2 \leq 99$. $i = 1, 2, \dots, 99$; R_i^{exper} is the value at $\sqrt{|t|} = \sqrt{|t_i|}$ of R (Eq. (4)) with $d\sigma/dt$ given by the experimental UA4/2 data; $\delta(R_i)$ is the corresponding experimental error; R_i^{theor} is the value at $\sqrt{|t|} = \sqrt{|t_i|}$ of R (Eq. (4)) with $d\sigma/dt$ given either by the exponential model or by a model including oscillations. We split the experimental t -domain into several intervals counting the “pulses” in terms of an index k ($k = 1, 2, \dots$), such that

$$k_{max} \cdot \Delta = \sqrt{|t_{99}|} - \sqrt{|t_1|}, \quad (9)$$

Δ being the length of the interval in $\sqrt{|t|}$ where the “pulse” is located. Of course, once t_1 and Δ are fixed, the values of n_1 and n_2 , corresponding to the experimental points in the respective “pulse” are fixed. We further define

$$L_k^{up} = \sum_{k_{even}} L(n_1, n_2) \quad and \quad L_k^{down} = \sum_{k_{odd}} L(n_1, n_2). \quad (10)$$

If the exponential model is right we have to get $\sum_k L_k^{up} \simeq \sum_k L_k^{down} \simeq 0$. However, detailed study on these lines shows that, in the region of interest ($\sqrt{|t|} \lesssim 0.1$ GeV), with $\sqrt{|t_1|} \simeq 3 \cdot 10^{-2}$ GeV and $\Delta \simeq 10^{-2}$ GeV, L_k^{up} is a *positive* increasing function, while L_k^{down} is a *negative* decreasing function. Moreover, when the sum over all possible values of k

is done, we get that both $|L|$ values are much bigger than 1. This is a strong statistical indication in favour of the presence of oscillations.

Let us now consider the modification of the nuclear amplitude (5) in the following way:

$$F_N(s_1, t) = F_N(s_1, t)_{\text{expon}} + F_N(s_1, t)_{\text{osc}}, \quad (11)$$

where $F_N(s_1, t)_{\text{osc}}$ is again the toy model, the complex oscillatory function of order 1/2

$$F_{\text{osc}} = (A + B \cdot i) \cdot (\sin \tau) / \tau, \quad (12)$$

where A, B and t_0 are free parameters.

The details of the fit are not important here. What is important is the fact that χ^2/dof decreases by 10%, in the entire experimental domain, as compared with the exponential case. The improvement is even more important in the small t region : by 30% in the zoom region and by 60 % for the first 15 experimental points. Moreover, both L_k^{up} and L_k^{down} are now compatible with zero. All these results reinforce the statistical indications in favour of the presence of oscillations at very small t . The exponential model fails to reproduce the data at very small t . This general conclusion about the non-validity of the exponential form of the nuclear amplitude at very small t is, in fact, valid for a much larger class of functions. Any *smooth* function for which $ReF(s, t \simeq 0) \simeq \rho \cdot \sigma_T$ and $ImF(s, t \simeq 0) \simeq \sigma_T$ will not describe the dN/dt UA4/2 data.

Let us now illustrate the physical effects induced by oscillations by considering their consequences for the crucial parameter $\rho(s, t = 0)$, which is sensitive to the presence of the odd-under-crossing amplitude $F_-(s, t)$. We note that, in order to keep σ_T inside the experimental range [13] 62.2 ± 1.5 mb, B has to be of the order of the experimental error of σ_T , i.e. 1.5 mb. In other words, $ImF_N(s_1, t)$ in Eq. (13) is still described, at this fixed energy $\sqrt{s_1} = 541$ GeV, by an effective exponential form (5). By using $ImF_N(s, t)$ as

given by Eqs. (5), (11) and (12) we can perform a *direct calculation of $ReF_N(s, t)$ from the data* through the obvious formula derived from $d\sigma/dt$:

$$ReF_N(s_1, t)_{exper} = -ReF_C(s_1, t) + [16\pi(d\sigma/dt)(s_1, t) - (ImF_C(s_1, t) + ImF_N(s_1, t))^2]^{1/2}. \quad (13)$$

Fig. 3 clearly shows the presence of oscillations around 0.135 in $\rho(s_1, t)$ which is now a function of t and not a constant as in the exponential case. The configurations of zeros in $\rho(s_1, t) - 0.135$ (equivalent to that in $R(s_1, t)$) immediately indicates that $\rho(s_1, t = 0)$ has to be larger than 0.135. On the other hand, the function $F_{osc}(s, t)/F_{osc}(s, t = 0)$ has to be 1 at $\tau = 0$ via the normalization condition (2), while it is much smaller than 1 in the region $\tau \simeq \ln s_1$ of the bump in $d\sigma/dt$, because of the fast $1/\tau$ damping previously discussed. Therefore a small oscillation in the region of the bump will induce a large effect at $t = 0$. Of course, this large effect will be somewhat attenuated by the presence of non-asymptotic terms at finite energies. Nevertheless from the trend of the $\rho(s_1, t)$ “data” in Fig. 3, it is seen that the bound $\rho(s_1, t = 0) \lesssim 0.23$, derived several years ago by an almost model-independent analysis [14] of the experimental data at lower energies, could easily be saturated.

A firm prediction of the value of $\rho(s_1, t = 0)$ would require a much more detailed study of $F_{osc}(s, t)$ which is beyond the scope of the present paper. The only assertion we can make now is that there are strong indications that this value is substantially higher than 0.135, a phenomenon which can be understood through the persistence of $F_-(s, t)$ at high energies. The existence of visible oscillations then suggests a general mechanism of saturation of the axiomatic bounds, of which the persistence of $F_-(s, t)$ at high energy is an example.

Of course, the above considerations are somewhat speculative in the presence of just one set of high-precision data in the *TeV* region of energy. A detailed dynamical, phe-

nomenological analysis is premature now, because it would require much more information on the experimental side.

The problem of oscillations was, in fact, recently revived in literature [15-17]. However, the oscillations considered in Refs. [16, 17] have a period of oscillation in t two orders of magnitude larger than the oscillations considered in the present paper. Moreover, they are periodic oscillations in t and not in τ . Therefore they are clearly not oscillations of the AKM type. It is also interesting to note that T.T.Wu and collaborators [18] observed that there is a certain possible disagreement between the eikonal models and the UA4/2 data, which could be in fact related to the presence of oscillations in the data.

In conclusion, in the present paper we reveal, for the first time, possible evidence for the AKM structure at high s ($\sqrt{s} = 541 \text{ GeV}$) and very small t , present in the very precise UA4/2 measurement of dN/dt . Of course, strictly speaking, the AKM structure was theoretically established in the framework of asymptotic theorems, i.e. at $s \rightarrow \infty$. However, in [19] it was shown that the transition region between low energy behaviour and high energy behaviour of the hadronic scattering amplitudes corresponds phenomenologically to the ISR energy range. Therefore our considerations should be valid in the TeV region of energy.

Future experiments in the crucially interesting TeV region of energy - at Tevatron, RHIC and LHC - are needed in order to confirm this first indication of oscillations at very small t in the high-precision UA4/2 dN/dt data and their possible relation with the AKM structure.

Acknowledgments. The authors thank Prof. Elliot Leader for very useful discussions. Two of us (P.G. and B.N.) would like to thank Dr. Maurice Haguenaier for the interesting discussions we had in 1992-1993 on what we called, at that time, "the strange structure".

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Figure Captions

Fig. 1. Zoom in the small- t region of the UA4/2 dN/dt data.

Fig. 2. The ratio R (see Eq. (4)) vs $\sqrt{|t|}$. The solid curve corresponds to the toy model (Eq. (12)).

Fig. 3. The function $\rho(s_1, t)$ vs $\sqrt{|t|}$ at $\sqrt{s_1} = 541$ GeV. The dashed line corresponds to $\rho(s_1, t) = \text{const.} = 0.135$. The solid curve corresponds to the toy model (Eq. (12)). Its possible extrapolation towards the bound $\rho(s_1, t = 0) \lesssim 0.23$ is also shown

Fig.1

Fig.2

Fig.3