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The three Vertex in the Closed Half-String Field Theory and the General Gluing and Resmoothing Theorem

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ABSTRACT

In this letter we prove that the Half-String 3-Vertex, in Closed String Field Theory satisfies the General Gluing and Resmoothing Theorem. We also demonstrate how one calculates amplitudes in the Half-String approach to Closed String Field Theory, by working out explicitly a few simple three amplitudes.

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Motivated by the similarities between Yang-Mills theory and Witten's open string field theory [1], it was first suggested in [2,3] and proved rigorously in [4,5,6], that physical open strings can be viewed as infinite dimensional matrices. In particular; the open-string 3-vertex can be represented as a trace [4,5,6]. This trace can be generalized to represent any N-string ($N \geq 3$) tree level scattering amplitude [8]:

$$A_N = \int_{-\infty}^{\infty} \frac{d\lambda_1 \cdots d\lambda_N}{SL(2, \mathbb{R})} Tr(\exp(\lambda_1 M) A_1 \cdots \exp(\lambda_N M) A_N). \quad (.1)$$

where M is the generator of infinitesimal shifts of the mid-point of the string, so in fact one is shifting this point to every possible position.

An analogous construction for closed strings was formulated in [8,9]. Using a functional approach [8], the analogous of eq. (.1) for closed strings was shown to give the correct dual amplitudes. The HS (Half-String) operator formalism of closed string [9] (here after referred as I) indicates that the restricted polyhedra of the classical non-polynomial string field theory, can be represented as traces of infinite dimensional matrices, with operator insertions that reparametrise the half-strings. Also, in (I), factorization of closed-string was established.

In this letter, we wish to discuss one further crucial property (which was not addressed in I) that one should expect of a correct formalism of CSFT (Closed String Field Theory). Namely, the vertices of the theory must satisfy the GGR (General Gluing and Resmoothing) Theorem of ref. [10]. We will also give a few simple examples of how to calculate closed string amplitudes. Although some computational details are presented in this letter, this letter is not meant to be self-contained in the sense we rely on (I) for notation and indeed for many other details only alluded to below.

We have seen that the closed HS vertices calculated in (I), *i.e.*, $N = 1, 2$ and 3 , do agree with the standard results for the closed bosonic string. However we still have to compare HS higher order vertices with other vertices appearing in the literature. In [10], LPP (LeClair, Peskin and Preitschopf) define the 3-vertex through the relation

$$\langle V_{123} | A \rangle_1 | B \rangle_2 | C \rangle_3 = \langle T^2 h[\Theta_A] T h[\Theta_B] h[\Theta_C] \rangle,$$

here Θ_A is the normal ordered operator that creates the state $|A\rangle$

$$|A\rangle = \Theta_A|0\rangle,$$

and T is an $SL(2;C)$ transformation such that $T^3 = 1$. Higher order vertices are defined analogously.

It is precisely these type of vertices that are used, as a starting point by Kugo and Suehiro, to construct the restricted polyhedra and to show that the resulting theory is gauge invariant [11].

Lets start by comparing the two-point vertex in both formalisms (i.e., the HS one and the standard one). By definition the state $|V_{12}\rangle$ (or simply $|V^{(2)}\rangle$ in our language) imposes the condition $X_1(\sigma) = X_2(\pi - \sigma)$, which is equivalent to imposing that $\alpha_n^{(1)} = \alpha_{-n}^{(2)}$ at the level of operators. If we write the string coordinate in terms of the complex coordinate $z = \exp(\tau + i\sigma)$ (take $\tau = 0$), then this condition is equivalent to imposing $X_1(z) = X_2(-\frac{1}{z})$. This can be used to define the BPZ³ (Belavin, Polyakov and Zamolodchikov) conjugate to $|A\rangle$ as

$$\langle A| = \langle 0|I[\Theta_A(0)],$$

where $I(z) = \frac{1}{z}$ is a conformal map. i.e.

$$I[A(z, z^*)] = A'(\frac{1}{z}, \frac{1}{z^*}).$$

The (BPZ) inner product is defined as

$$\langle A|B\rangle = \langle 0|I[\Theta_A(0)]\Theta_B(0)|0\rangle = \langle V_{AB}||A\rangle|B\rangle.$$

It is easy to see that the 2-vertex constructed with LPP's prescription using the I defined above, corresponds exactly to V_2 in the HS formalism.

A very important property that any vertex must satisfy is that the new vertex produced by the contraction of two other vertices by the conformal field theory inner product BPZ, is precisely the one which results from first sewing the corresponding two Riemann surfaces via the map I , and then constructing the vertex on that surface. In other words

$$\langle V_{ABEF}^{(4)}| = \langle V_{ABC}^{(3)}| \langle V_{DEF}^{(3)}|V_{CD}^{(2)}\rangle, \quad (.2)$$

³See ref. [12]

with

$$< V_{1234} | A >_1 | B >_2 | C >_3 | D >_4 = < T^2 h[\Theta_A] T h[\Theta_B] I T^2[\Theta_C] I T[\Theta_D] > .$$

This is the *Generalized Gluing and Resmoothing Theorem* for $N = 3$ [10].

It is not hard to show that the HS vertices constructed in (I) satisfy this theorem. The proof steams out from the fact that they are written as traces, and that the transformation from half-strings to full-strings is nonsingular. To see this, denote HS string field matrices as $A_{nm} = < nm | \Lambda \rangle^4$ where the indices n and m refer to the left and right parts of the string. Completeness of the transformations means that Parseval's identity, $I = \sum_{nm} |n; m \rangle \langle n; m|$, works both ways, and $I = \int D\Lambda | \Lambda \rangle \langle \Lambda |$. The right hand side of equation (.2) can be written in the HS language (summing over repeated indices) as

$$V_{125}^{(3)} V_{56}^{(2)\dagger} V_{634}^{(3)} = \int D\Lambda^5 D\Lambda^6 < nm | \Lambda^1 \rangle < mk | \Lambda^2 \rangle < kn | \Lambda^5 \rangle \\ (\Lambda^5 | pq \rangle (\Lambda^6 | qp \rangle < rs | \Lambda^6 \rangle < sv | \Lambda^3 \rangle < vr | \Lambda^4 \rangle).$$

Using (twice) that $\int D\Lambda < kn | \Lambda \rangle (\Lambda | pq \rangle = \delta_{kp} \delta_{nq}$ (orthogonality), we can write the above identity as

$$V_{1234}^{(4)} = < nm | \Lambda^1 \rangle < mk | \Lambda^2 \rangle < kq | \Lambda^3 \rangle < qn | \Lambda^4 \rangle,$$

which is just the l.h.s. of (.2). This can be generalized to higher point vertices [13]. Notice that this result is valid for both open and closed strings.

In what follows we show how one calculates some simple three string amplitudes. At tree level the ghosts decouple and the amplitude is given by $A_3 = < V_{123} | \Psi_1 \rangle | \Psi_2 \rangle | \Psi_3 \rangle$, with $|\Psi_i \rangle$ the three external states satisfying the *physical conditions*

$$L_n | \Psi \rangle = \tilde{L}_n | \Psi \rangle = 0; \quad (L_0 - 1) | \Psi \rangle = (\tilde{L}_0 - 1) | \Psi \rangle = 0; \\ (L_0 - \tilde{L}_0) | \Psi \rangle = 0.$$

⁴Here $|\Lambda \rangle$ represent a complete basis of string states.

This can be expressed in terms of the closed-string vertex as

$$A_3 = \langle 0_{123} | N_0 \exp \frac{1}{2} (\alpha_n^r N_{nm}^{rs} \alpha_m^s + \tilde{\alpha}_n^r N_{nm}^{rs} \tilde{\alpha}_m^s) | \Psi_1 \rangle | \Psi_2 \rangle | \Psi_3 \rangle .$$

The Neumann coefficients⁵, N_{nm}^{rs} , are related to the change of representation matrices between HS and FS (Full-String) coordinates [9]. N_0 is the normalization; it is chosen such that the 3-tachyon amplitude is one. In general for closed strings $P^2 = \frac{4}{\alpha'}(1 - N) = 8(1 - N)$ where $\alpha' = 1/2$ is our Regge slope convention and N is the number operator ($N = \tilde{N}$). From the last equation one has

$$\begin{aligned} \langle V_3 | N_0 &= \langle 0_{123} | N_0 \exp \left(- \ln \frac{3^3}{2^4} (3 - \sum_{r=1}^3 N_r) + \text{higher oscillators} \right) \\ &= \langle 0_{123} | \exp \left(\ln \frac{3^3}{2^4} (\sum_{r=1}^3 N_r) + \text{higher oscillators} \right) . \end{aligned}$$

For three tachyons $N_r = 0$; $r = 1, 2, 3$, and the above amplitude is one.

Now let us look at the (Tachyon, Tachyon, Graviton) amplitude. The graviton corresponds to the first excited state ($N_G = 1$) of the closed string; it has zero mass

$$|\Psi_G \rangle = G_{\mu,\nu}(P) \alpha_{-1}^{3\mu} \tilde{\alpha}_{-1}^{3\nu} |0 \rangle ,$$

where $G_{\mu,\nu}(P)$ is a symmetric traceless tensor. The physical conditions imply

$$P^\mu G_{\mu,\nu} = P^\nu G_{\mu,\nu} = 0 .$$

If we label the two tachyon states by $i = 1, 2$ and the graviton by $i = 3$, then the amplitude is given by

$$\begin{aligned} A_3(T, T, G) &= \langle 0_{123} | \exp \frac{1}{2} \left(\sum_{n,m \geq 1} \alpha_n^r N_{nm}^{rs} \alpha_m^s + \sum_{n,m \geq 1} \tilde{\alpha}_n^r N_{nm}^{rs} \tilde{\alpha}_m^s \right. \\ &\quad \left. + \ln \frac{3^3}{2^4} \sum_{r=1}^3 N_r \right) G_{\mu,\nu}(p_3) \alpha_{-1}^{3\mu} \tilde{\alpha}_{-1}^{3\nu} |0_1 \rangle |0_2 \rangle |0_3 \rangle , \end{aligned}$$

here $\sum_{r=1}^3 N_r = 0 + 0 + 1 = 1$. Expanding the exponential in a Taylor series, the order zero terms vanish because the operators α_{-1} or $\tilde{\alpha}_{-1}$

⁵See also references 8,9,10,11,12 given in [14].

annihilate the left vacuum. For the same reason the only terms that contribute are the ones that contain one α_{-1} operator or one $\tilde{\alpha}_{-1}$, since for example commuting α_{-1} with one α_n of any of the terms quadratic in α_m 's say, gets rid of the creation operator leaving the annihilation operators free to kill the right vacuum. Using the results of (I) one obtains

$$\begin{aligned}
A_3(T, T, G) &= \langle 0_{123} | \left(\frac{3^3}{2^4} \right) \exp \left(\frac{-i\alpha_1^i}{\sqrt{2}} \vec{G}_1^{(2)'}{}^{i,j} P^j + \frac{i\tilde{\alpha}_1^i}{\sqrt{2}} \vec{G}_1^{(2)'}{}^{i,j} P^j \right) \\
&\quad \alpha_{-1}^{3\mu} \tilde{\alpha}_{-1}^{3\mu} |0_1 \rangle |0_2 \rangle |0_3 \rangle G_{\mu,\nu}(p_3) \\
&= \left(\frac{3^3}{2^4} \langle 0_{123} | \right) \left(\frac{1}{2} \vec{G}_1^{(2)'}{}^{3,j} P^{j\mu} \vec{G}_1^{(2)'}{}^{3,k} P^{k\nu} \right) \\
&\quad |0_1 \rangle |0_2 \rangle |0_3 \rangle G_{\mu,\nu}(p_3)
\end{aligned}$$

Using the value of $\vec{G}_1^{(2)'}{}^{i,j}$ given in (I), (with $U_1 = 2/3$) we arrive at the final expression for the amplitude, namely

$$\begin{aligned}
A_3(T, T, G) &= \frac{1}{16} (P_1 - P_2)^\mu (P_1 - P_2)^\nu G_{\mu,\nu}(P_3) \\
&= \frac{1}{4} \left(\frac{P_1}{2} - \frac{P_2}{2} \right)^\mu \left(\frac{P_1}{2} - \frac{P_2}{2} \right)^\nu G_{\mu,\nu}(P_3),
\end{aligned}$$

where we have made use of the fact that $G_\mu^\mu = 0$. This should be compared with the standard result

$$A^{closed}(P_3) = A^{open}(P_3/2) A^{open}(P_3/2)^*,$$

where $A^{open}(P_3)$ is the open string (Tachyon, Tachyon, Vector) amplitude which is given by

$$A^{open}(P_3) = \frac{1}{2} (P_1 - P_2)^\mu A_\mu(P_3),$$

here P_1 and P_2 are the momenta of the tachyons and $A_\mu(P_3)$ represents the vector particle (one has to make the identification $G_{\mu,\nu} \equiv A_\mu A_\nu$ at the end).

It is satisfying to see that the HS vertices calculated using the HS language do indeed satisfy the GGR Theorem and that the 3-*Amplitudes* calculated here agree with the standard results obtained in the literature. This indicates that the HS approach to string field theory is on strong footing.

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