Non-Universal Soft SUSY Breaking and Dark Matter

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Abstract

An analysis is given of the effects of non-universal soft SUSY breaking masses in the Higgs sector and in the third generation squark sector, and it is shown that they are highly coupled. Analytic expressions are obtained for their effects on the parameters $\mu, m_A$ and on the third generation squark masses. Non-universality effects on dark matter event rates in neutralino-nucleus scattering are analysed. It is found that the effects are maximal in the range $m_{\tilde{\chi}_1} \leq 65$ GeV where the relic density is governed by the Z and Higgs poles. In this range the minimum event rates can be increased or decreased by factors of $O(10)$ depending on the sign of non-universality. Above this range Landau pole effects arising from the heavy top mass tend to suppress the non-universality effects. The effect of more precise measurements of cosmological parameters on event rates, which is expected to occur in the next round of COBE like satellite experiments, is also investigated. Implications for the analysis for dark matter searches are discussed.

1. Introduction

Although there is currently a great deal of evidence for the existence of dark matter in the universe\cite{1}, the nature of such dark matter is still uncertain. COBE and other experiments have provided further insights into the nature of dark matter which strongly suggests more than one component to its structure\cite{2}. Thus the simplest types of models to fit the power spectrum seen by COBE and other experiments require two components, a hot component and a cold component. Several other possibilities have also been discussed such as those using a cosmological constant.

In our analysis here we shall adhere to the simple two component picture of dark matter, with a hot component (which could be a light neutrino with mass in the range of a few electron volts), and a cold component which we
assume is the lightest supersymmetric neutral particle, the neutralino. (For a review of supersymmetric dark matter see ref.3). For the purpose of our analysis we assume a mix of hot and cold dark matter in the ratio of \( \Omega_{HDM} : \Omega_{CDM} = 1 : 2 \) (consistent with the COBE data), where \( \Omega_i = \rho_i / \rho_c \) where \( \rho_i \) is the matter density in the universe due to matter of type \( i \) and \( \rho_c \) is the critical matter density needed to close the universe. Assuming an inflationary scenario with \( \Omega = 1 \), the condition that \( \Omega_B \leq 0.1 \) as implied by the Big Bang Nucleosynthesis, and \( h \) in the experimental range \( 0.4 \leq h \leq 0.8 \) where \( h \) is the Hubble parameter in units of 100 km/s Mpc, one finds \( \Omega_{\tilde{\chi}_1} h^2 \) in the range

\[
0.1 \leq \Omega_{\tilde{\chi}_1} h^2 \leq 0.4
\]  

(1)

\( \Omega_{\tilde{\chi}_1} h^2 \) is the quantity computed theoretically and thus the above condition acts as the primary constraint on the neutralino dark matter analysis. We shall discuss in the latter part of this paper the effect of constraining the \( \Omega_{\tilde{\chi}_1} h^2 \) range more narrowly.

The purpose of this paper is to analyse the effects of non-universal boundary conditions on soft SUSY breaking masses at the GUT scale on event rates in neutralino-nucleus scattering, as almost all of the previous analyses, with the exception of the work of Ref. (4), are in the framework of supergravity unification using only the universal boundary conditions.

The outline of the paper is as follows: in Sec. 2 we give the salient points of supergravity unification which provides the framework of our analysis. In Sec. 3 we discuss non-universal soft breaking in the Higgs sector and in the third generation squark sector and show that their effects at low energy are highly coupled. We then obtain solutions in a closed form at the electro-weak scale of the relevant low energy parameters that contain the non-universalities. In Sec. 4 we discuss the constraints that are imposed on the event rate analysis. In Sec. 5 we give the basic formulae that enter in the analysis of scattering of the neutralino by nuclear targets. In Sec. 6 results of the event rate analysis are given and a comparison made between the cases with universal and non-universal soft SUSY breaking boundary conditions at the GUT scale. In Sec. 7 we discuss the effects of grand unification on non-universal parameters. In Sec. 8 we discuss the effects that a more accurate determination of the Hubble parameter, which is expected to occur in the next round of COBE like satellite experiments, will have on the event rate analysis. Conclusions are given in Sec. 9. New results of this paper are contained in Secs. 3,6,7,8, in Figs. 1-7, in Tables 1-5 and in Appendix A.
2. Neutralino Dark Matter in Supergravity Grand Unification

In the analysis of this paper we use the framework of supergravity grand unification (SUGRA) [5]. In this picture one assumes the existence of a supergravity grand unified theory which in its symmetric phase operates in the region between the string scale and the grand unification scale. Supersymmetry is broken in the theory via a hidden sector. In the minimal supergravity unification one finds that after breaking of supersymmetry and of the grand unified symmetry (where one assumes that the grand unified theory breaks to the gauge group $SU(3) \times SU(2) \times U(1)$ ) one has a low energy theory (obtained by integrating out the heavy modes and the modes of the hidden sector) which can be given in terms of just five arbitrary parameters [5]–[8].

These consist of the universal soft SUSY scalar mass at the GUT scale $m_0$, the universal gaugino mass at the GUT scale $m_{1/2}$, the universal trilinear scalar coupling at the GUT scale $A_0$, $B_0$ which is the coefficient of the bilinear term $\mu_0 H_1 H_2$ at the GUT scale, and the Higgs mixing parameter $\mu_0$ at the GUT scale. In addition there are the GUT parameters $M_G$ and $\alpha_G$ which are determined by the grand unification condition.

A remarkable aspect of supergravity grand unification is the breaking of the electro-weak symmetry by radiative effects. Radiative breaking is governed by the equations [9]

$$\mu^2 = \mu_1^2 - \mu_2^2 \tan^2 \beta - \frac{1}{2} M_Z^2, \quad \sin 2\beta = \frac{-2B\mu}{2\mu^2 + \mu_1^2 + \mu_2^2}$$

where $B$ is the value of the parameter $B_0$ at the electro-weak scale, $\mu_i^2 = m_{H_i}^2 + \Sigma_i$ where $m_{H_i}^2$ is the running $H_i$ mass at the scale $Q \approx M_Z$ and $\Sigma_i$ are loop corrections [10, 11]. $m_{H_i}^2$ are given by

$$m_{H_i}^2 = m_o^2 + m_{1/2}^2 g(t)$$

$$m_{H_2}^2 = m_{1/2}^2 e(t) + A_o m_{1/2} f(t) + m_o^2 h(t) - k(t) A_0^2$$

Here $t = \ln(M_G^2/Q^2)$, and e,f,g,h,k are form factors defined in Ref. (12). On using the radiative breaking equations one can determine the parameter $\mu$ from the Z boson mass and one can also eliminate the parameter $B$ in favor of $\tan \beta \equiv <H_2> / <H_1>$. Thus in the low energy domain, the theory can be described by the parameters

$$m_0, m_{\tilde{g}}, A_t, \tan \beta, \text{sign}(\mu)$$

where $A_t$ is the value of $A_0$ at the electro-weak scale. Further, at the electro-weak scale the $SU(3) \times SU(2) \times SU(1)$ gaugino masses are given by $m_i =$
(\alpha_i/\alpha_G)m_{1/2}$. (For radiative corrections to this formula see Refs. (13-14)). We note that the situation in minimal supergravity is in sharp contrast to that for MSSM. In MSSM one has 110 parameters in the low energy domain. In contrast, in minimal supergravity unification (MSGM) one has only four parameters (and one sign) as discussed above. The minimal model has 32 supersymmetric particles, and all their masses and coupling constants are determined by just the parameters of Eq. (5).

One of the interesting features that emerges from supergravity unification is that the model predicts as a consequence of its dynamics which particle is the lowest lying supersymmetric particle (LSP). One finds that over almost all of the parameter space that the neutralino is the LSP. Thus with the assumption of R parity invariance, one finds that supergravity unified theory provides a candidate for cold dark matter. The LSP neutralino is an admixture of four neutral states, i.e., of gauginos $\tilde{W}_3, \tilde{B}$ and of Higgsinos $\tilde{H}_1, \tilde{H}_2$. Denoting the lightest neutralino by $\tilde{\chi}_1$ we may write

$$\tilde{\chi}_1 = n_1 \tilde{W}_3 + n_2 \tilde{B} + n_3 \tilde{H}_1 + n_4 \tilde{H}_2$$

where the co-efficients $n_i$ are to be determined by diagonalizing the neutralino mass matrix given in Appendix A. An interesting property of radiative breaking is that it produces the phenomenon of scaling in a significant part of the parameter space[15]. In the region of scaling one has $\mu^2 >> M_Z^2$. In this region the eigenvalues and the eigenvectors of the neutralino mass matrix can be computed in a perturbative fashion by expanding in powers of $M_Z/\mu$[16]. The analysis shows that in the scaling limit the neutralino is mostly a Bino with $n_2 > 0.95$. Further one finds that $n_3, n_4$ and $n_1$ are typically of first order, i.e., $O(M_Z/\mu)$. These expansions are useful in understanding the relic density and the event rates. We add here a note of caution. Inspite of the fact that the Higgsino components are generally small their effects on event rates are generally very significant. Thus large inaccuracies can result from a neglect of the Higgsino components.

3. Effects of Non-universal Soft SUSY Breaking

In this section we want to study the effects of non-universalities in soft SUSY breaking on the low energy parameters. While most of the previous analyses in supergravity grand unification have been carried out with universal soft SUSY breaking, the general framework of the theory allows for non-universal soft SUSY breaking terms. Thus, for example, under the assumption of a general Kahler potential, deviations from universalities can be generated[17]. Of course the non-universalities are severely constrained, most significantly from flavor changing neutral current (FCNC) constraints. In the analysis here we
shall limit the nature of non-universalities to the Higgs sector and to the third generation sector. Higgs sector non-universalities are not strongly constrained by FCNC and have been discussed in the literature\cite{18, 19, 20, 4} often in the context of certain SO(10) models to achieve radiative breaking of the electro-weak symmetry. In our analysis here we show that the non-universalities in the Higgs sector and the non-universalities in the third generation up squark sector are highly coupled due to renormalization group effects below the GUT scale. It is convenient to parametrise the non-universalities in the Higgs sector by $\delta_1, \delta_2$ and the non-universalities in the third generation squark sector by $\delta_3, \delta_4$ so that at the GUT scale $M_G$ one has

$$m_{H_1}^2 = m_0^2 (1 + \delta_1), \quad m_{H_2}^2 = m_0^2 (1 + \delta_2)$$

(7)

$$m_{\tilde{Q}_L}^2 = m_0^2 (1 + \delta_3), \quad m_{\tilde{U}_R}^2 = m_0^2 (1 + \delta_4)$$

(8)

where $m_0$ is the universal scalar mass of the first two generation masses. We shall assume that the remaining sectors of the theory are universal. We shall give an analytic solution to the non-universal effects on the mass spectra at low energy. We begin by a discussion of the case of the $H_1$ mass and the masses of the first two generations of sparticles. For $m_{H_1}^2$ we obtain

$$m_{H_1}^2 = m_0^2 (1 + \delta_1) + m_1^2 g(t) + \frac{3}{5} S_0 p$$

(9)

Here the last term arises from the $Tr(Y m^2)$ term in the renormalization group evolution equations\cite{13}. This term vanishes in the universal case because of the anomaly cancellation constraint $Tr(Y) = 0$. However, for the non-universal case it is non-vanishing and one has

$$S_0 = Tr(Y m^2) = m_{H_2}^2 - m_{H_1}^2 + \sum_{i=1}^{n_g} (m_{\tilde{d}_L}^2 - 2 m_{\tilde{u}_R}^2 + m_{\tilde{d}_R}^2 - m_{\tilde{d}_L}^2 + m_{\tilde{e}_R}^2)$$

(10)

where all the masses are at the GUT scale and $n_g$ is the number of generations. In Eq. (11) $p$ is defined by

$$p = \frac{5}{66} [1 - \left( \frac{\tilde{\alpha}_1(t)}{\tilde{\alpha}_1(0)} \right)]$$

(11)

where $\tilde{\alpha}_1 = g_1^2(0)/(4\pi) \quad \text{and} \quad g_1(0)$ is the U(1) gauge coupling constant at the GUT scale. The corrections to the sparticle masses for the first two generations are listed in Appendix A. We note that although the non-universalities at the GUT scale in the first two generations vanish there are effects at the electro-weak scale in the sparticle masses of the first two generations because of non-universalities in the Higgs sector and in the third generation via the trace
anomaly term. To get an idea of the size of the effects from this term, for
$M_G = 10^{16.2}$ GeV and $\alpha_G = 1/24$ one has $p=0.0446$. Thus the effect of the
trace term is order a few percent for $|\delta_i| < 1$.

The analysis of non-universalities on the squark masses in the third gen-
eration is more complicated. Here $m_{H_2}^2$, $m_{U}^2$ and $m_{Q}^2$ obey the coupled renor-
malization group equations (61)-(63) of Appendix A. Solution to these gives
for $m_{H_2}^2$

$$m_{H_2}^2 = m_0^2 \Delta_{H_2} + m_{1/2}^2 e(t) + A_0 m_{1/2} f(t) + m_0^2 h(t) - k(t) A_o^2 - \frac{3}{5} S_0 p$$  \hspace{1cm} (12)

where $\Delta_{H_2}$ is given by

$$\Delta_{H_2} = \left( \frac{D_0 - 1}{2} \right) (\delta_2 + \delta_3 + \delta_4) + \delta_2$$  \hspace{1cm} (13)

and $D_0$ defines the top Landau pole, i.e.,

$$y_0 = \frac{y_t}{E(t) D_0}; D_0 = 1 - 6 y_t \frac{F(t)}{E(t)}$$  \hspace{1cm} (14)

Here $y_0 = h_t^2/(4\pi)^2$ where $h_t$ is the top Yukawa coupling, and

$$E(t) = (1 + \beta_3(t))^{\frac{16}{21}} (1 + \beta_2(t))^{\frac{2}{15}} (1 + \beta_1(t))^{\frac{12}{5}}$$  \hspace{1cm} (15)

In Eq. (15) $\beta_i = \alpha_i(0) b_i/4\pi$, where $\alpha_i(0) = \alpha_G$ are the gauge coupling constant
co-efficients at the GUT scale ($\alpha_1 = (5/3)\alpha_Y$), $b_i$ are the one loop beta function
coefficients defined by $(b_1, b_2, b_3) = (33/5, 1, -3)$, and $F(t) = \int_0^t E(t) dt$. The non-universality effects in the stop matrix enter via $m_{\tilde{Q}}$ and $m_{\tilde{U}}$ where the stop mass matrix is

$$\begin{pmatrix}
m_{\tilde{t}_L}^2 & -m_t(A_t + \mu c t \beta) \\
-m_t(A_t + \mu c t \beta) & m_{\tilde{t}_R}^2
\end{pmatrix}$$  \hspace{1cm} (16)

and $m_{\tilde{t}_L}^2$, $m_{\tilde{t}_R}^2$ are given by

$$m_{\tilde{t}_L}^2 = m_{\tilde{Q}}^2 + m_t^2 + \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) M_Z^2 \cos 2\beta$$  \hspace{1cm} (17)

and

$$m_{\tilde{t}_R}^2 = m_{\tilde{U}}^2 + m_t^2 + \left( \frac{2}{3} \sin^2 \theta_w M_Z^2 \cos 2\beta \right)$$  \hspace{1cm} (18)
Here $m_\tilde{Q}^2$ and $m_\tilde{U}^2$ obey the set of coupled equations given in Appendix A. With the assumption of non-universalities of Eqs. (7) and (8) one has for $m_\tilde{Q}^2$

$$m_\tilde{Q}^2 = m_0^2 \Delta_{\tilde{Q}} + \frac{2}{3} m_0^2 + \frac{1}{3} (m_0^2 h - k A_0^2 + m_1 A_0 f + m_1^2 e)$$

$$+ \frac{\alpha G}{4\pi} (\frac{8}{3} f_3 + f_2 - \frac{1}{15} f_1) m_2^2 - \frac{1}{5} S_0 p$$

where

$$\Delta_{\tilde{Q}} = \frac{(D_0 - 1)}{6} (\delta_2 + \delta_3 + \delta_4) + \delta_3$$

Similarly for $m_\tilde{U}^2$ one has

$$m_\tilde{U}^2 = m_0^2 \Delta_{\tilde{U}} + \frac{1}{3} m_0^2 + \frac{2}{3} (m_0^2 h - k A_0^2 + m_1 A_0 f + m_1^2 e)$$

$$+ \frac{\alpha G}{4\pi} (\frac{8}{3} f_3 - f_2 + \frac{1}{3} f_1) m_2^2 + \frac{4}{5} S_0 p$$

where

$$\Delta_{\tilde{U}} = \frac{(D_0 - 1)}{3} (\delta_2 + \delta_3 + \delta_4) + \delta_4$$

In the above, the non-universalities are all contained in the quantities $\Delta_{H_2}$, $\Delta_{\tilde{Q}}$, $\Delta_{\tilde{U}}$ and in the corrections involving the term $S_0 p$. We note that because of a peculiar accident that the sum of the corrections proportional to $S_0 p$ in the subsectors involving $H_2, \tilde{Q}$ and $\tilde{U}$ vanishes, the trace anomaly receives no top quark Landau pole enhancement. A proof of this result is given in Appendix A. An analysis similar to the above holds in the bottom squark sector. Details are given in Appendix A.

In addition to above, the quantities $\mu$ and $m_A$ are also effected. Using the radiative breaking relation Eq. (2) we find the following closed form solution for $\mu^2$ to one loop order:

$$\mu^2 = m_0^2 C_1 + A_0^2 C_2 + m_1^2 C_3 + m_1 A_0 C_4 - \frac{1}{2} M_Z^2 + \frac{3 t^2 + 1}{5 t^2 - 1} S_0 p$$

Here

$$C_1 = \frac{1}{t^2 - 1} (1 - \frac{3 D_0 - 1}{2} t^2) + \frac{1}{t^2 - 1} (\delta_1 - \delta_2 t^2 - \frac{D_0 - 1}{2} (\delta_2 + \delta_3 + \delta_4) t^2)$$

$$C_2 = -\frac{t^2 - 1}{5 t^2 - 1} k, \ C_3 = -\frac{1}{t^2 - 1} (g - t^2 e), \ C_4 = -\frac{t^2}{t^2 - 1} f$$
where $t \equiv \tan \beta$ and where the functions $e,f,g,k$ are as defined in Eqs. (3) and (4). Similarly, for $m_A^2$ one has

$$m_A^2 = m_0^2 D_1 + A_0^2 D_2 + m_{1/2}^2 D_3 + m_{1/2} A_0 D_4 - \frac{1}{2} M_Z^2 + \frac{6 t^2 + 1}{5 t^2 - 1} S_0 p$$

(26)

where

$$D_1 = \frac{t^2 + 1}{t^2 - 1} (1 - \frac{3 D_0 - 1}{2}) + \frac{t^2 + 1}{t^2 - 1} (\delta_1 - \delta_2 - \frac{D_0 - 1}{2} (\delta_2 + \delta_3 + \delta_4))$$

(27)

$$D_2 = -\frac{t^2 + 1}{t^2 - 1} k, \quad D_3 = -\frac{t^2 + 1}{t^2 - 1} (g - e), \quad D_4 = -\frac{t^2 + 1}{t^2 - 1} f$$

(28)

In the above we have assumed the existence of non-universalities on phenomenological grounds. Theoretically there are several possible sources from where such non-universalities can arise. One source of non-universalities is the general Kahler potential. In general the Kahler potential can have generational dependent couplings which lead to non-universalities in the visible sector after the breaking of supersymmetry in the hidden sector. However, even if the couplings in the Kahler potential were generation blind and the soft SUSY breaking masses for the scalars were universal at a scale higher than the GUT scale one will have non-universalities at the GUT scale arising from renormalization group running of the soft SUSY breaking parameters. Thus, for example, if one has universality of the soft parameters at the string scale, the running of these parameters from the string scale down to the GUT scale will generate non-universalities at the GUT scale. Model calculations indicate that such running typically generates $|\delta_{1,2}| \leq 0.5$, and a similar level of non-universality in the relevant mass spectra at the electro-weak scale is expected. However, enhancement of non-universalities can occur under special circumstances. To show this enhancement we consider the expression for $\mu^2$. Here the non-universalities are all contained in $C_1$. If the dominant universal terms cancel, then the non-universal effects become large. This situation can be easily seen to arise for large $\tan \beta$. Here for the case when $m_t \approx 167$ GeV, the universal part cancels since $D_0 \approx 1/3$ and thus the non-universality effects get enhanced. A similar enhancement of non-universalities also occurs from the $A_0$ dependence, although in a somewhat different manner. Here the enhancement occurs when the residue of the Landau pole in $A_0$ vanishes. All three quantities, $m_{H^2}^2$, $m_{\tilde{Q}}^2$ and $m_{\tilde{U}}^2$, depend on $A_0$ in the combination $k A_0^2 - f A_0 m_{1/2}$. Using the renormalization group equation for $A_0$

$$A_0 = \frac{A_R}{D_0} - \frac{H_3}{F} m_{1/2}; \quad H_3 = t E - F$$

(29)
where \( A_R = A t - m_{1/2} (H_2 - H_3/F) \) with \( H_2 \) defined in Ref. (12), one finds that

\[
k A_0^2 - f A_0 m_{1/2} = \frac{1}{2} (1 - D_0) \frac{A_R^2}{D_0} + \frac{1}{2} D_0 (1 - D_0) \left( \frac{H_3}{F} \right)^2 m_{1/2}^2
\]

(30)

The Landau pole contribution vanishes when \( A_R = 0 \) which occurs when \( A_t \approx 0.61 m_\tilde{g} \)[21]. We give a graphical description of this phenomenon in Fig. 1 where \( \mu \) is plotted as a function of \( A_t \). For the specific set of parameters chosen in Fig. 1 the residue of the Landau pole vanishes when \( A_t/m_0 \approx 0.7 \). One finds that Landau pole effects are suppressed close to this value of \( A_t \), and there is considerable dispersion among the three curves corresponding to the sets of values \( \delta_1 = 0 = \delta_2 \), \( \delta_1 = 1 = -\delta_2 \) and \( \delta_1 = -1 = \delta_2 \). However, away from the region of \( A_t/m_0 \approx 0.7 \) one finds that the Landau pole effects begin to dominate and diminish the effect of non-universalities.

4. Constraints on Dark Matter

We explore the parameter space of supergravity unified models under the naturalness constraints which we assume to imply

\[
m_0 \leq 1 \text{ TeV}, \ m_\tilde{g} \leq 1 \text{ TeV}, \ \tan \beta \leq 25
\]

(31)

The constraints of electro-weak breaking and the condition that there be no tachyons then imply the following range for \( A_t \)[21, 16]:

\[
A_t/m_0 \geq -0.5
\]

(32)

Constraints also arise from imposition of the condition that there be no nearby minima with lower energies that break color and charge conservation[22](CCB constraints), and from experimental lower limits on the SUSY mass spectra given by CDF, D0 and LEP experiments.

In addition to the above there is an important constraint that arises from the decay \( b \to s + \gamma \). This decay proceeds via loop corrections and is thus sensitive to physics beyond the Standard Model(SM). For the case of the SM, the loop contribution involves a W-t exchange, while for the supersymmetric case one has additional contributions arising from \( W - H^- \), \( \tilde{W} - \tilde{t} \), and \( \tilde{Z} - \tilde{b} \) exchanges. Thus SUSY contributions are a priori as important as the SM contributions, and so one expects \( b \to s + \gamma \) decay to act as an important constraint on the parameter space of supergravity models. The CLEO value for the branching ratio for \( B \to X_s + \gamma \) is [23]

\[
BR(B \to X_s \gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}
\]

(33)
If one combines the errors in quadrature then \( \text{BR}(B \to X_s \gamma) \cong (2.32 \pm 0.67) \times 10^{-4} \). The SM prediction gives \( \text{BR}[B \to X_s \gamma] \cong (3.28 \pm 0.33) \times 10^{-4} \) for \( m_t = 174 \text{ GeV} \). In supersymmetry, theoretical analyses when subject to the current experimental limits, impose serious constraints on the parameter space of the theory. The parameter space after the above constraints are imposed must still be subject to the constraint of relic density of Eq. (1). The quantity \( \Omega_{\tilde{\chi}_1^1} h^2 \) is computed theoretically in each point in the supergravity parameter space using the relation\( [28, 29] \)

\[
\Omega_{\tilde{\chi}_1^1} h^2 \cong 2.48 \times 10^{-11} \left(\frac{T_{\tilde{\chi}_0^0}}{T_\gamma}\right)^3 \left(\frac{T_\gamma}{2.73}\right)^3 \frac{N_f^{1/2}}{\mathcal{J}(x_f)}
\]  

(34)

where \( T_\gamma \) is the current background temperature, \( T_f \) is the freezeout temperature, \( (T_{\tilde{\chi}_0^0}/T_\gamma)^3 \) is the reheating factor, \( N_f \) is number of massless degrees of freedom at freezeout, \( x_f = kT_f/m_{\tilde{\chi}_1^1} \), and

\[
J(x_f) = \int_0^{x_f} dx \langle \sigma v \rangle (x) \text{GeV}^{-2}
\]  

(35)

Here \( < \sigma v > \) stands for the thermal average, where \( \sigma \) is the the neutralino annihilation cross-section and \( v \) is the neutralino relative velocity. In the analysis of the relic density we have used the accurate method for its computation[30, 31, 32, 33] which carries out a correct thermal averaging over the Z and the Higgs poles which appear in the s channel in the neutralino annihilation. The accurate method for the computation of relic density is important as it has significant effects in the event rate analysis. The relic density computed in the above fashion is then subject to the constraints of Eq. (1) which further limits the parameter space of the theory.

5. Detection of Neutralino Dark Matter

We discuss now the possibilities for the detection of the neutralino dark matter. Several techniques, both direct and indirect, have been discussed over the years for its detection. The direct method includes (i) scattering by nuclei[34]-[43] in terestial detectors, (ii) scattering by bound electrons[44], (iii) use of overheated microbubbles[45], while the indirect methods include, (iv) annihilation in the center of sun and earth[46, 47], and (v) annihilation in the halo of galaxies. For our analysis here we limit ourselves to consideration of case (i).

The total interaction in the scattering of neutralinos by quarks is given by

\[
L_{\text{eff}} = (\bar{\tilde{\chi}}_1 \gamma^\mu \gamma_5 \tilde{\chi}_1) [\bar{q} \gamma_\mu (A_L P_L + A_R P_R) q] + (\bar{\tilde{\chi}}_1 \tilde{\chi}_1)(\bar{q} C m_q q)
\]  

(36)
where \( P_{R,L} = (1 \pm \gamma_5)/2 \). The part with terms \( A_L, A_R \) is the spin dependent (SD) part while the remainder is the spin independent (SI) part. The total event rate can thus be written as follows:

\[
R = [R_{SI} + R_{SD}] \left[ \frac{\rho_{\tilde{\chi}_1}}{0.3 \text{GeV cm}^{-3}} \right] \left[ \frac{v_{\tilde{\chi}_1}}{320 \text{km/s}} \right] \text{ events kg da} \tag{37}
\]

where \( \rho_{\tilde{\chi}_1} \) is the local mass density of \( \tilde{\chi}_1 \) incident on the detector, and \( v_{\tilde{\chi}_1} \) is the incident \( \tilde{\chi}_1 \) velocity. \( R_{SI} \), is given by

\[
R_{SI} = \frac{16m_{\tilde{\chi}_1} M_N^3 M_Z^4}{[M_N + m_{\tilde{\chi}_1}]^2} |A_{SI}|^2 \tag{38}
\]

and the spin dependent rate is given by

\[
R_{SD} = \frac{16m_{\tilde{\chi}_1} M_N}{[M_N + m_{\tilde{\chi}_1}]^2} \lambda^2 J(J+1) |A_{SD}|^2 \tag{39}
\]

where \( J \) is the nuclear spin and \( \lambda \) is defined by \( \langle N | \sum \vec{S}_i | N \rangle = \lambda \langle N | \vec{J} | N \rangle \). \( A_{SI} \) and \( A_{SD} \) are the corresponding amplitudes. We note that for large \( M_N \), \( R_{SI} \sim M_N \) while \( R_{SD} \sim 1/M_N \). These results imply that the spin independent scattering becomes more dominant as the nucleus becomes heavier. Both \( R_{SI} \) and \( R_{SD} \) contain theoretical and experimental uncertainties. A major source of uncertainty in \( R_{SI} \) arises from the uncertainty in the matrix elements of the operator \( m_q \bar{q} q \) between nucleon states. There can be as much as 50% uncertainty in the strange quark contributions\([48]\) leading to a 30% uncertainty in \( R_{SI} \). For the case of \( R_{SD} \) the major source of ambiguity arises from the matrix elements of the axial current between nucleon states, i.e., of \( \Delta q \) defined by \( \langle n | \vec{q} \gamma^\mu \gamma_5 q | n \rangle = 2s^\mu \Delta q \), where \( s^\mu \) is the nucleon spin 4-vector. There exist two sets of determinations of \( \Delta q \), an old determination using the EMC data\([49]\) and a new determination using the SMC data\([50]\). For most nuclei the difference is not major and in our analysis here we use the values of \( \Delta q \) from the newer determination\([50]\). In addition both rates have uncertainties due to the nature of the nuclear form factors. Thus theoretical predictions are accurate to within perhaps a factor of two.

6. Event Rates with Universal and Non-universal Soft Breaking

We give now an analysis of the event rates and draw a comparison between the case where one uses universal boundary conditions at the GUT scale for the soft SUSY breaking parameters and the case when one has non-universal boundary conditions at the GUT scale. We begin with a discussion of event rates when one includes non-universalities in the Higgs sector, i.e., when \( \delta_3 = 0 = \delta_4 \).
From Eqs. (23) and (24) one can see that a positive $\delta_1$ and a negative $\delta_2$ make a positive contribution to $\mu^2$ while a negative $\delta_1$ and a positive $\delta_2$ make a negative contribution to $\mu^2$. If one limits $\delta_1$ and $\delta_2$ so that $|\delta_i| \leq 1 (i=1,2)$, then the case $\delta_1 = 1 = -\delta_2$ gives the largest positive contribution to $\mu^2$ while the case $\delta_1 = -1 = -\delta_2$ gives the largest negative contribution to $\mu^2$. These cases represent the extreme limits of how large the non-universality effects can be within our prescribed limits on $\delta_i$. With the above in mind we focus on three cases to get an idea of the differences in the event rates between the universal and the non-universal cases:

(i) $\delta_1 = 0 = \delta_2$,
(ii) $\delta_1 = -1 = -\delta_2$, and
(iii) $\delta_1 = 1 = -\delta_2$

In Fig. 2 we exhibit the maximum and minimum of event rates for xenon for the three cases listed above for $\mu > 0$. The analysis is done by mapping the full parameter space limited only by the naturalness constraints of Eq. (29), the constraints on the sparticle masses from CDF, DO and LEP experiments, the relic density constraint of Eq. (1) and the experimental constraint from $b \to s + \gamma$ of Eq. (31). The dips in the region below $m_{\tilde{\chi}_1} \leq 65$ GeV arise due to the rapid annihilation in the vicinity of the $Z$ pole and the Higgs pole. We note that the accurate method mentioned earlier for the computation of the relic density is important for the correct analysis of the event rates in this region. Comparison of cases (i) and (ii) shows that the event rates of (i) below the region $m_{\tilde{\chi}_1} \leq 65$ GeV can be enhanced by a factor of $O(10)$ or more in the minimum curves for case (ii). This effect can be understood as follows: for case (ii) one has $\delta_1 < 0$ and $\delta_2 > 0$, which according to Eq. (23) drives $\mu^2$ towards the limit $\mu^2 < 0$ eliminating such points from the parameter space. Now this effect is more important for small $\tan \beta$. Since small $\tan \beta$ governs the minimum event rates one finds that in this case since small $\tan \beta$ tend to get eliminated one has the effect that the lower limit of the event rates are raised. In contrast, for case (iii) one has $\delta_1 > 0$ and $\delta_2 < 0$ which gives the opposite effect as expected, i.e., on finds that the minimum event rates are reduced by a factor of $O(10)$. However, above $m_{\tilde{\chi}_1} \geq 65$ the non-universalities do not dramatically affect the event rates and the event rates have a uniform fall off as a function of the neutralino mass for all the three cases. A similar effect occurs for larger values of the top quark mass. However, here one finds that a larger value of the top mass gives a larger Landau pole contribution which diminishes the relative contribution of the non-universal terms.

We note that the spin independent interaction contributes significantly more than the spin dependent interaction. This result holds not only for a heavy target such as xenon but also for relatively light targets such as CaF$_2$. 

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In Tables 1, 2 and 3 we give a comparison of event rates for six additional
target materials, i.e., He, CaF$_2$, GaAs, Ge, NaI, and Pb, for the universal and
non-universal soft SUSY breaking in the Higgs sector. (Recall, however, that
there are significant uncertainties discussed above in the predictions.) Phenomena
similar to the ones discussed for xenon also appear for these cases. Many
of these materials listed in Tables 1, 2 and 3 are already being used as targets
in the dark matter detectors. We note that for all the targets considered the
event rates span several decades in magnitude within the allowed domain of
the parameter space. Thus a full exploration of the parameter space certainly
poses a formidable challenge to experimentalists.

The analysis we have given so far is for the case $\mu > 0$. As discussed earlier
for the case $\mu < 0$ a large part of the parameter space is eliminated because
of the $b \to s + \gamma$ constraint. Consequently the maximum event rates for the
$\mu < 0$ cases are much smaller than for the $\mu > 0$ case. A comparison of the
allowed range of event rates for xenon for the cases $\mu > 0$ and $\mu < 0$ is given
in Table 4 ($\mu > 0$) and Table 5 ($\mu < 0$). One finds that the event rates for
the $\mu < 0$ case are typically $O(10^{-2} - 10^{-3})$ smaller than for the $\mu > 0$ case.
Another important phenomenon is that for both signs of $\mu$ the parameter
space corresponding to $A_t/m_0 < -0.5$ is eliminated. The reason that positive
values of $A_t$ are eliminated arises from the fact that the lightest stop mass
turns tachyonic due to Landau pole effects, since the residue of the Landau
pole $A_R^2 = (A_t - 0.61m_\tilde{g})^2$ becomes large.

In the analysis of this section thus far we assumed universality in the third
generation sector and examined only the effects of non-universality in the Higgs
sector. However, as discussed in Sec. 3 the non-universalities in the Higgs
sector and in the third generation sector are highly coupled. Therefore, we
turn now to a discussion of what we might expect, from the non-universality
in the third generation sector. To get an idea of the nature of corrections
one might expect, we display below the numerical sizes of the non-universal
corrections $\Delta_{H_2}, \Delta_{\tilde{t}_L}, \Delta_{\tilde{t}_R}$. For values $M_G = 10^{16.2}$ GeV, $\alpha_G = 1/24$
and $m_t = 175$ GeV one has $D_0 \simeq 0.27$ and from Eqs. (13), (20) and (22) one finds

$$
\Delta_{H_2} \simeq 0.64\delta_2 - 0.36(\delta_3 + \delta_4) \quad (40)
$$
$$
\Delta_{\tilde{t}_L} \simeq 0.88\delta_3 - 0.12(\delta_2 + \delta_4) \quad (41)
$$
$$
\Delta_{\tilde{t}_R} \simeq 0.76\delta_4 - 0.24(\delta_2 + \delta_3) \quad (42)
$$

From Eq. (40) we find that a positive $\delta_2$ in $\Delta_{H_2}$ can be simulated by a negative
value of $\delta_3$ or a negative value of $\delta_4$, and a converse situation occurs for a
negative value of $\delta_2$. The effects of $\delta_3$ and $\delta_4$ on $\Delta \tilde{t}_L, \Delta \tilde{t}_R$ are more complicated since $\delta_3, \delta_4$ enter $\Delta \tilde{t}_L, \Delta \tilde{t}_R$ with opposite signs. The stop masses do not enter directly in any significant manner in the amplitudes in the neutralino mass range we investigate, although they do affect the event rates by affecting the parameter space. In spite of this complexity we can glean the pattern in which $\delta_3, \delta_4$ affect the event rates by examining the way they enter $\Delta H_2$. In Fig. 3 we exhibit the event rates for xenon when $\delta_1 = \delta_2 = \delta_4 = 0$ and $\delta_3$ takes on a range of values. The solid curve in Fig. 3 is for the case $\delta_3 = 0$, the dotted curve for the case $\delta_3 = 1$, and the dashed for the case $\delta_3 = -1$. Fig. 3 shows that the minimum event rates in the region below $m_{\tilde{\chi} < 65}$ GeV are lower relative to universal case for the case $\delta_3 > 0$. This is as expected from our general analysis above, i.e., it is similar to the case $\delta_2 < 0$ in Fig. 2. Similarly the minimum event rates in the region $m_{\tilde{\chi} < 65}$ GeV are enhanced relative to the universal case for $\delta_3 < 0$ which parallels the case $\delta_2 > 0$ for Fig. 2. This is again as expected. An analysis similar to the above for the case when $\delta_1 = \delta_2 = \delta_3 = 0$ and $\delta_4$ takes on a range of values is given in Fig. 4. The result of non-universalities is qualitatively similar to that of Fig. 3 as expected. We note, however, that the parameter space allowed by the non-universalities in the third generation would be somewhat different and so one does not have a complete correspondence between the non-universalities in the Higgs sector (i.e., $\delta_2$) and those in the third generation sector (i.e., $\delta_3$ and $\delta_4$).

7. Effects of Grand Unification On Non-Universal Parameters

In the previous section, the parameters describing non-universal soft SUSY breaking masses, i.e., $\delta_1, \ldots, \delta_4$, were viewed as independent quantities. This would be the case if the scale of SUSY breaking occurred below $M_G$, since there only the SM gauge group is assumed to hold. However, if the SUSY breaking scale occurs above $M_G$ (e.g., at the string scale $M_{str}$), then the soft breaking masses at $M_G$ must obey constraints imposed by the grand unification group $G$. We consider here this situation for the case where $G$ contains $SU(5)$ as a subgroup. This includes $G=SU(5)$, $SO(N)$ for $N \geq 10$ or $E(6)$ which essentially include all the grand unification groups that have been examined in significant detail.

For this class of groups, we may characterize the light matter at $M_G$ in terms of the $SU(5)$ quantum numbers, i.e., matter occurs in $10_a$ and $\bar{5}_a$ representations ($a=1,2,3$ is a generation index) and the light Higgs doublet in a 5 and 5 representation. One has then for the sfermions[51]

\begin{align}
10_a &= (q_a \equiv (\tilde{u}_{La}, \tilde{d}_{La}); u_a \equiv \tilde{u}_{Ra}; e_a \equiv \tilde{e}_{Ra}) \\
\bar{5}_a &= (l_a \equiv (\tilde{\nu}_{La}, \tilde{e}_{La}); d_a \equiv \tilde{d}_{Ra})
\end{align}  \hspace{1cm} (43)

\begin{align}
10_a &= (q_a \equiv (\tilde{u}_{La}, \tilde{d}_{La}); u_a \equiv \tilde{u}_{Ra}; e_a \equiv \tilde{e}_{Ra}) \\
\bar{5}_a &= (l_a \equiv (\tilde{\nu}_{La}, \tilde{e}_{La}); d_a \equiv \tilde{d}_{Ra})
\end{align}  \hspace{1cm} (44)
The soft breaking masses have the form

\[ m_{10a}^2 = m_0^2(1 + \delta_{10}^a); \quad m_{5a}^2 = m_0^2(1 + \delta_5^a) \]  

(45)

with the Higgs masses obeying Eq. (7). There are therefore 8 independent soft breaking parameters for this general case. (Eqs. (7, 43) contain 9 parameters \( \delta_{10}, \delta_5^a, \delta_{1,2}, m_0, \) but actually one is redundant.). Thus we have the relations

\[ \delta_q = \delta_u = \delta_e = \delta_{10}^a; \quad \delta_l = \delta_d = \delta_{5}^a \]  

(46)

For higher gauge groups, there can be additional relations. (For example, for SO(10) with non-universalities in the third generation, some models of symmetry breaking give[53] \( \delta_5 = \delta_2 - \delta_1. \))

One may impose further group constraints on the above parameters. Thus the suppression of FCNC suggests a near degeneracy in the first two generations, which occurs naturally in a SU(2)\(_H\) model where these generations are put into a doublet (d) representation of SU(2)\(_H\) and the third generation into a singlet (s) representation[54]. Then Eqs. (45) reduce to

\[ m_{10}^2 = m_0^2(1 + \delta_{10}^{(s)}); \quad m_{10a}^2 = m_0^2(1 + \delta_{10}^{(d)}), a = 1, 2 \]  

(47)

\[ m_5^2 = m_0^2(1 + \delta_5^{(s)}); \quad m_{5a}^2 = m_0^2(1 + \delta_5^{(d)}), a = 1, 2 \]  

(48)

where quantities without generation indices are SU(2)\(_H\) singlets. This model would have 6 independent parameters.

While the FCNC constraints do not imply that the first two generation soft mass of the 10 representation is degenerate with that of the 5 representation, a further simplification occurs if one assumes that this is the case, i.e., one lets

\[ \delta_5^{(d)} = \delta_{10}^{(d)} \equiv \delta_{(d)} \]  

(49)

One can then absorb the \( (1 + \delta_{(d)} \) factor into the \( m_0^2 \) as the common mass for \( a=1,2 \), and use that as the reference mass for the third generation and Higgs masses, reducing the number of parameters to 5: \( \delta_{10}^{(s)}, \delta_5^{(s)}, \delta_{1,2} \) and \( m_0 \). In the notation of Sec. 7, this is equivalent to non-universalities specified by

\[ \delta_3 = \delta_4; \quad \delta_1; \quad \delta_2; \quad \delta_5 \]  

(50)

where \( \delta_5 \equiv \delta_5^{(s)} = \delta_{\tilde{b}_R} = \delta_3 \). The parameter \( \delta_5 \) does not enter directly into the event rate R calculation or into the Landau pole or \( b \to s + \gamma \) analyses. It does, however, affect R indirectly in that it will enter into the relic density calculation and affect the determination of the parameter space that obeys
Eq. (1). However, there it enters only into the t-channel poles which generally make small contributions to the $\chi^0_0$ annihilation cross section. We discuss now the effect of the model of Eq. (50) on event rates. For illustration we consider two cases with $\delta_i$ (i=3,4) chosen at the extreme values under the reasonable constraint $\delta_i \leq 1$. These are case(a): $\delta_3=\delta_4=1$, and case(b): $\delta_3=\delta_4=-1$ (and $\delta_5$ set to zero). For case(a) we find from Eqs. (13), (20) and (22) using $D_0 \simeq 0.27$

$$\Delta H_2 \simeq -0.73 + 0.64\delta_2 \quad (51)$$

$$\Delta Q \simeq 0.76 - 0.12\delta_2 \quad (52)$$

$$\Delta U \simeq 0.51 - 0.24\delta_2 \quad (53)$$

For $\delta_2$ in the range (1,-1) one finds that $\Delta H_2$ lies in the range (-0.1,-1.37) while $\Delta Q$ lies in the range (0.64, 0.88) and $\Delta U$ lies in the range (0.27,0.76). Because of the non-universalities arising from $\delta_3 = \delta_4 > 0$ this case is again different from the cases we have discussed before. However, we can still draw some rough comparisons with the previous cases. Thus, for example, we note that since the effective value of $\delta_2$ in this case is negative the minimum of the event rates should be similar to the minimum dotted curve of Fig. 2 where $\delta_2$ was also negative. The result of the analysis is presented in Fig. 5. We see that indeed the minimum of the event rates here is similar to the minimum of the event rates in Fig. 2 as expected. In general, the effect of non-universalities is not very large here. A similar analysis holds for case(b). Here again using Eqs. (13), (20), (22) and $D_0 \simeq 0.27$ we find that

$$\Delta H_2 \simeq 0.73 + 0.64\delta_2 \quad (54)$$

$$\Delta Q \simeq -0.76 - 0.12\delta_2 \quad (55)$$

$$\Delta U \simeq -0.51 - 0.24\delta_2 \quad (56)$$

For $\delta_2$ in the range (1,-1) one finds $\Delta H_2$ in the range (0.1,1.37), $\Delta Q$ in the range (-0.64,-0.88), and $\Delta U$ in the range (-0.27,-0.76). We note that these ranges are exact mirrors of the ranges for case(a) but with a negative sign. Of course, the full dynamics of this system is very complicated since the allowed parameter space is affected differently from previous cases because of the specific nature of non-universalities here. However, as in case(a) we can glean some understanding by a comparison with the previous analyses. We note that the effective value of $\delta_2$ is positive here. Thus one expects that the minimum event rates might show the same pattern as the positive $\delta_2$ case of Fig. 2. The result of the analysis is exhibited in Fig. 6. We find that indeed the minima curves of Fig. 6 have a behavior similar to the dashed curve of
The remarkable feature here is the increase by a factor of 10 in the maximum event rates when $\delta_2$ is positive. This is due to the fact that $\delta_2, \delta_3, \delta_4$ now act together to reduce $\mu^2$ (see Eqs. (23) and (24)) which increases the maximum event rates\(^3\).

### 8. Effects of More Accurate $\Omega h^2$ Determination

As mentioned in the Introduction the next round of satellite experiments are expected to determine the Hubble parameter to within O(10%) or better accuracy. Such an accurate determination of the Hubble parameter will put more stringent bounds on the neutralino relic density $\Omega_{\tilde{\chi}} h^2$ and hence more stringently constrain the SUGRA parameter space. We investigate here the effects of these more stringent constraints on the neutralino dark matter event rates. For specificity we consider a narrow band of $\Omega_{\tilde{\chi}} h^2$ in the range

$$0.225 < \Omega_{\tilde{\chi}} h^2 < 0.275$$

(57)

Fig. 7 gives the maximum and the minimum event rates as a function of the $\tilde{\chi}_1$ mass. A comparison of Figs. 2 and 7 shows that a narrower range $\Omega_{\tilde{\chi}} h^2$ more sharply constrains the SUGRA parameter space and the allowed band of event rates shrinks. The allowed range of the neutralino mass is also more sharply constrained since the $m_{\tilde{\chi}_1}$ mass is constrained to lie below $\approx 90$ GeV, as above this mass $\Omega h^2$ exceeds the upper limit of Eq. (57). Since the neutralino mass and the gluino mass scale one also deduces an upper limit on the gluino mass of about 650 GeV. Thus a precise measurement of $\Omega_{\tilde{\chi}} h^2$ will put an interesting upper bound on the gluino mass which would be testable at accelerators such as the upgraded Tevatron and the LHC. In general the overall effect of the more stringent constraints on $\Omega_{\tilde{\chi}} h^2$ is to eliminate a part of the parameter space allowed by Eq. (1) and thus limit more severely the permissible domain of the event rates.

### 9. Conclusions

In this paper we have investigated the effects of non-universalities in the Higgs sector and in the third generation squark sector on the low energy parameters. We have shown that the Higgs sector non-universalities and the non-universalities in the third generation squark sector are highly coupled and we have exhibited analytic forms of their effects on the parameters $\mu, m_A$ and the third generation squark masses at low energy. Next we investigated the effects of these non-universalities on the event rates in the scattering of neutralinos by nuclei. Here we found that the minimum event rates in the neutralino mass region $30 \text{ GeV} \leq m_{\tilde{\chi}_1} \leq 65 \text{ GeV}$ can be increased or decreased by a factor of O(10) depending on the sign of the non-universality. In contrast, in the neutralino mass region $m_{\tilde{\chi}_1} \geq 65 \text{ GeV}$ the minimum and the maximum event rates are not appreciably affected. These results have important implications.
for dark matter detectors. Supergravity unified models with universal boundary conditions for soft SUSY breaking parameters give event rates which lie in a wide range \( R = O(1 - 10^{-5}) \) events/kg da. Thus while the current generation of dark matter detectors can sample a part of the parameter space, one needs far more sensitive detectors (more sensitive by a factor of \( 10^3 \)) to sample the entire parameter space of the model. However, inclusion of non-universalities shows that for certain signs of the non-universality, the minimum event rates can increase by a factor of 10-100 in the region \( m_{\tilde{\chi}} \leq 65 \text{GeV} \). These models can be completely tested in this mass region if the detector sensitivity could be enhanced by a factor of about \( 10^2 \). Such an enhancement does not seem out of reach with new techniques currently being discussed\(^5\). Another interesting result seen was the appearance of large event rates, up to \( 10 \) event/kg da for the case when \( \delta_3 = \delta_4 = -1 \). This case looks very interesting from the viewpoint of experimental detection of dark matter. We have also analysed the implications of more stringent constraints on \( \Omega h^2 \) which is expected from the next round of satellite experiments, and found that while the event rates typically become smaller a significant part of the parameter space yields event rates which are within reach of dark matter experiments currently being planned.

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**Appendix A**

All the scalar sparticle masses receive contributions from the \( \text{Tr}(Y m^2) \) term. Corrections to the first two generations of sparticle masses are given by

\[
\begin{align*}
\Delta m^2_{\tilde{u}_i L} &= -\frac{1}{5}S_0 p = \Delta m^2_{\tilde{d}_i L}, \\
\Delta m^2_{\tilde{e}_i L} &= \frac{3}{5}S_0 p = \Delta m^2_{\tilde{\nu}_i} \\
\Delta m^2_{\tilde{u}_i R} &= \frac{4}{5}S_0 p = -2\Delta m^2_{\tilde{d}_i R}, \\
\Delta m^2_{\tilde{e}_i R} &= -\frac{6}{5}S_0 p
\end{align*}
\]

where \( S_0 \) and \( p \) are defined in the text. Under the assumption that the squarks and slepton masses are universal at the GUT scale for the first two generations, but with non-universalities in the Higgs sector and in the third generation we can deduce the following sum rule for the \( \text{Tr}(Y m^2) \) term:

\[
p\text{Tr}(Y m^2)_0 = -\frac{5}{6} \sin^2 \theta_W \cos 2\beta M^2_Z + \frac{1}{4}(m^2_{\tilde{e}_{i L}} + 2m^2_{\tilde{u}_{i R}}) - \frac{1}{4}(m^2_{\tilde{e}_{i R}} + m^2_{\tilde{d}_{i R}} + m^2_{\tilde{d}_{i L}})
\]

Here \( \text{Tr}(Y m^2)_0 \) is the value of \( \text{Tr}(Y m^2) \) at the GUT scale while the right hand side is evaluated at the electro-weak scale. We see from above that if
the masses of the first two generations of squarks and sleptons are known with enough precision, one can determine the value of the trace anomaly term at the GUT scale. For the case when \( Tr(Y m^2) \) vanishes the above sum rule reduces to the one given in Ref. (56).

\( m^2_{H_2}, m^2_{U} \) and \( m^2_{Q} \) satisfy the following set of coupled equations

\[
\frac{dm^2_{H_2}}{dt} = -3Y_t \Sigma_t + (3\tilde{\alpha}_2 M_2^2 + \frac{3}{5}\tilde{\alpha}_1 M_1^2) + \gamma_1 \tilde{\alpha}_1 S
\]
\[
\frac{dm^2_{U}}{dt} = -2Y_t \Sigma_t + (\frac{16}{3}\tilde{\alpha}_3 M_3^2 + \frac{16}{15}\tilde{\alpha}_1 M_1^2) + \gamma_2 \tilde{\alpha}_1 S
\]
\[
\frac{dm^2_{Q}}{dt} = -Y_t \Sigma_t + (\frac{16}{3}\tilde{\alpha}_3 M_3^2 + 3\tilde{\alpha}_2 M_2^2 + \frac{1}{15}\tilde{\alpha}_1 M_1^2) + \gamma_3 \tilde{\alpha}_1 S
\]

where \( \Sigma_t = (m^2_{H_2} + m^2_{Q} + m^2_{U} + A_t^2) \), and \( \gamma_1 = -\frac{3}{10}, \gamma_2 = \frac{2}{5} \) and \( \gamma_3 = -\frac{1}{10} \) and \( S \) is the running parameter with \( S(0) = S_0 \). We show now that the evolution of the trace anomaly term is not affected by the top Yukawa coupling. To see this it is best to go to a decoupled set of equations using the functions \( e_i(t) \) given by

\[
m^2_{H_2} = 3e_1 + e_2 + e_3
\]
\[
m^2_{U} = 2e_1 - 2e_2
\]
\[
m^2_{Q} = e_1 + e_2 - e_3
\]

It is easily seen that the evolution of \( e_2, \) and \( e_3 \) does not involve the Yukawa coupling and only the evolution of \( e_1 \) does. One has

\[
\frac{de_1}{dt} = -6Y_t e_1 - Y_t A_t^2 + (\frac{16}{9}\alpha_3 M_3^2 + \alpha_2 M_2^2 + \frac{13}{45}\alpha_1 M_1^2) + \frac{1}{6}(\gamma_1 + \gamma_2 + \gamma_3)\tilde{\alpha}_1 S
\]

However, since

\[
(\gamma_1 + \gamma_2 + \gamma_3) = 0
\]

one finds that the evolution of the anomaly terms in Eqs. (61)-(63) are not affected by the top Yukawa coupling and are thus not sensitive to the Landau pole effects. The bottom squark mass matrix is given by

\[
\begin{pmatrix}
  m^2_{b_L} & -m_b (A_b + \mu \tan \beta) \\
  -m_b (A_b + \mu \tan \beta) & m^2_{b_R}
\end{pmatrix}
\]

where

\[
m^2_{b_L} = m^2_{Q} + m^2_b + (-\frac{1}{2} + \frac{1}{3}\sin^2 \theta_W)M_Z^2 \cos 2\beta
\]
and where $m_Q^2$ is defined in the text.

**Figure Captions**

Fig. 1: $\mu$ vs $A_t$ for the case when $\delta_3 = 0 = \delta_4$, $m_0 = 300$ GeV, $m_{\tilde{g}} = 350$ GeV, and (i) $\delta_1 = 0 = \delta_2$(solid), (ii) $\delta_1 = -1 = -\delta_2$(dashed), and (iii) $\delta_1 = 1 = -\delta_2$(dotted). Here $A_R$ vanishes for $A_t/m_0 \approx 0.7$.

Fig. 2: Maximum and minimum of event rates/kg da for xenon for $\mu > 0$ for the case when $\delta_3 = 0 = \delta_4$ and (a)$\delta_1 = 0 = \delta_2$(solid), (b)$\delta_1 = 1 = -\delta_2$(dotted), and (c)$\delta_1 = -1 = -\delta_2$ (dashed) when $0.1 < \Omega_{\tilde{\chi}_1}h^2 < 0.4$, and $m_t=175$ GeV.

Fig. 3: Maximum and minimum of event rates/kg da for xenon for the case when $\delta_1 = \delta_2 = \delta_4=0$ and (a)$\delta_3 = 0$(solid), (b)$\delta_3=1$(dotted), and (c)$\delta_3=-1$ (dashed) when $0.1 < \Omega_{\tilde{\chi}_1}h^2 < 0.4$, and $m_t=175$ GeV.

Fig. 4: Maximum and minimum of event rates/kg da for xenon for the case when $\delta_1 = \delta_2 = \delta_3=0$ and (a)$\delta_4 = 0$(solid), (b)$\delta_4=1$(dotted), and (c)$\delta_4=-1$ (dashed) when $0.1 < \Omega_{\tilde{\chi}_1}h^2 < 0.4$, and $m_t=175$ GeV.

Fig. 5: Maximum and minimum event rates when $\delta_3 = \delta_4 = 1$, $\delta_5 = 0$ and (a) $\delta_1 = 0 = \delta_2$(solid), (b) $\delta_1=1= -\delta_2$ (dotted), and $\delta_1=-1=-\delta_2$ (dashed) when $0.1 < \Omega_{\tilde{\chi}_1}h^2 < 0.4$, and $m_t=175$ GeV.

Fig. 6: Same as Fig. 5 when $\delta_3 = \delta_4 = -1$.

Fig. 7: Maximum and minimum of event rates/kg da for xenon for the case when $\delta_3 = \delta_4=0$ and (i) $\delta_1 = 0 = \delta_2$(solid), (ii)$\delta_1 = -1 = -\delta_2$(dashed), and (iii)$\delta_1 = 1 = -\delta_2$(dotted) when $0.225 < \Omega_{\tilde{\chi}_1}h^2 < 0.275$, and $m_t=175$ GeV.
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Table 1: Minimum and maximum of event rates/kg da are given for the case $\delta_1 = 0 = \delta_2$ and $\mu > 0$ for several neutralino masses for the target materials He, CaF2, GaAs, Ge, NaI, and Pb.
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<td>1.50E-3</td>
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<table>
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<th>80.9</th>
<th>117.0</th>
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<td>$R_{\text{Max}}$</td>
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Table 2: Same as Table 1 except that $\delta_1 = 1 = -\delta_2$
\begin{center}
\begin{tabular}{|c|c|c|c|c|}
\hline
\textbf{$m_{\tilde{\chi}}$(GeV)} & 50.3 & 50.3 & 54.6 & 54.6 \\
\hline
 & $R_{\text{Min}}$ & $R_{\text{Max}}$ & $R_{\text{Min}}$ & $R_{\text{Max}}$ \\
\hline
He & 6.61E-3 & 1.50E-2 & 1.14E-4 & 4.36E-2 \\
CaF2 & 8.90E-3 & 0.10 & 7.33E-4 & 0.28 \\
GaAs & 3.11E-3 & 0.36 & 2.47E-3 & 0.95 \\
Ge & 2.72E-3 & 0.37 & 2.52E-3 & 0.97 \\
NaI & 3.64E-3 & 0.57 & 3.82E-3 & 1.47 \\
Pb & 5.99E-3 & 0.97 & 6.47E-3 & 2.49 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|}
\hline
$m_{\tilde{\chi}}$(GeV) & 61.2 & 61.2 & 70.9 & 70.9 \\
\hline
 & $R_{\text{Min}}$ & $R_{\text{Max}}$ & $R_{\text{Min}}$ & $R_{\text{Max}}$ \\
\hline
He & 3.71E-5 & 2.80E-2 & 3.74E-6 & 7.30E-2 \\
CaF2 & 4.09E-4 & 0.14 & 1.81E-4 & 0.16 \\
GaAs & 1.52E-3 & 0.45 & 7.46E-4 & 0.29 \\
Ge & 1.55E-3 & 0.46 & 7.65E-4 & 0.29 \\
NaI & 2.32E-3 & 0.68 & 1.12E-3 & 0.43 \\
Pb & 3.88E-3 & 1.14 & 1.84E-3 & 0.71 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|}
\hline
$m_{\tilde{\chi}}$(GeV) & 80.5 & 80.5 & 117.5 & 117.5 \\
\hline
 & $R_{\text{Min}}$ & $R_{\text{Max}}$ & $R_{\text{Min}}$ & $R_{\text{Max}}$ \\
\hline
He & 1.65E-6 & 8.20E-2 & 1.04E-4 & 8.02E-2 \\
CaF2 & 9.40E-5 & 0.18 & 2.37E-4 & 1.10E-2 \\
GaAs & 3.88E-4 & 0.33 & 3.12E-4 & 1.89E-2 \\
Ge & 3.98E-4 & 0.33 & 3.07E-4 & 1.23E-2 \\
NaI & 5.76E-4 & 0.49 & 4.18E-4 & 1.14E-2 \\
Pb & 9.33E-4 & 0.81 & 6.50E-4 & 1.57E-2 \\
\hline
\end{tabular}
\end{center}

Table 3: Same as Table 1 except that $\delta_1 = -1 = -\delta_2$
<table>
<thead>
<tr>
<th>$A_t$</th>
<th>$\delta_1 = 0 = \delta_2$</th>
<th>$\delta_1 = 1 = -\delta_2$</th>
<th>$\delta_1 = -1 = -\delta_2$</th>
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<tr>
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Table 4: A comparison of the minimum and maximum event rates/kg da for xenon for the cases $\delta_1 = 0 = \delta_2$, $\delta_1 = 1 = -\delta_2$, and $\delta_1 = -1 = -\delta_2$, and $\mu > 0$ as a function of $A_t$.

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Table 5: Same as Table 4 except that $\mu < 0$. 

24
References


[51] For the flipped SU(5) model[52] one makes the changes $\tilde{u} \leftrightarrow \tilde{d}, \tilde{e} \leftrightarrow \tilde{\nu}$ with $\tilde{e}_R$ appearing in an SU(5) singlet.


