The Intracluster Gas Fraction in X–ray Clusters : Constraints on the Clustered Mass Density

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ABSTRACT

The mean intracluster gas fraction of X–ray clusters within their hydrostatic regions is derived from recent observational compilations of David, Jones & Forman and White & Fabian. At radii encompassing a mean density 500 times the critical value, the individual sample bi-weight means are moderately (2.4σ) discrepant; revising binding masses with a virial relation calibrated by numerical simulations removes the discrepancy and results in a combined sample mean and standard error $\bar{f}_{\text{gas}}(r_{500}) = (0.060 \pm 0.003) h^{-3/2}$. For hierarchical clustering models with an extreme physical assumption to maximize cluster gas content, this value constrains the universal ratio of total, clustered to baryonic mass $\Omega_m/\Omega_b \leq 23.1 h^{3/2}$; combining with the primordial nucleosynthesis upper limit on $\Omega_b$ results in $\Omega_m h^{1/2} < 0.60$. A less conservative, physically plausible approach based on low D/H inferences from quasar absorption spectra and accounting for baryons within cluster galaxies yields an estimate $\Omega_m h^{2/3} = 0.28 \pm 0.07$ with sources of systematic error involved in the derivation providing approximately 35% uncertainty. Additional effects which could provide consistency with the Einstein–deSitter case $\Omega_m = 1$ are presented, and their observable signatures discussed.

Key words: cosmology: observations – cosmology: theory – cosmology: dark matter – galaxies: clusters

1 INTRODUCTION

Clusters of galaxies provide a number of interesting cosmological diagnostics. In particular, the relative amount of baryons and dark matter within their hydrostatic regions provides a measure of the cosmic mix of these components, i.e., a measure of the ratio of density parameters $\Omega_m/\Omega_b$ in the Friedmann–Lemaître world model.*

Employing this measure in practice requires accurate observational data along with estimates of possible systematic biases. Likely sources of bias include systematic errors in component mass estimates and deviation of the local cluster ratio of baryonic–to–total mass arising, for example, from the different dynamical histories of the two components. Within the context of hierarchical clustering scenarios, these effects have been calibrated by numerical simulations of cluster formation. The results, discussed in detail below, indicate that the magnitude of these effects are small — a few tens of percent or less — in observationally accessible regions of clusters. If our current description of cluster formation dynamics is physically accurate, then determination of the cosmic baryon fraction from X–ray cluster data is straightforward.

The baryonic component of the richest clusters is dominated by the X–ray emitting intracluster gas rather than the mass associated with the optical light of the cluster galaxies (Forman & Jones 1982; Sarazin 1986; Mushotzky 1994). Taking Coma as an example, White et al. (1993) estimate the ratio of gas to galaxy mass to be $M_{\text{gas}}/M_{\text{gal}} = (5.5 \pm 1.5) h^{-3/2}$ (with $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$). This number is typical of the values seen in larger samples for clusters comparable in temperature to Coma (David et al. 1990; Arnaud et al. 1992). The data also indicate a dependence of the ratio $M_{\text{gas}}/M_{\text{gal}}$ on cluster temperature, with

* In this paper, $\Omega_m$ refers to the contribution of all clustered matter (including baryons) to the stress–energy density.
poorer clusters and groups possessing less gas per massive galaxy than their larger counterparts.

The gas mass thus provides not only a formal lower limit to the total baryon cluster content, but a fairly accurate estimate of the baryon mass in rich clusters if the dark matter is assumed non–baryonic. Recent compilations of X–ray cluster data by White & Fabian (1995) and David, Jones & Forman (1995) provide gas and total mass estimates for samples of moderate size. In this paper, I use these data, along with guidance from numerical simulations, to estimate the sample mean gas mass fraction \( f_{gas} \) within a characteristic radius defining the hydrostatic boundary of clusters. Motivations for the specific approach adopted here are provided in §2 and the observational data are analysed in §3. Implied constraints on the universal baryon fraction and the value of \( \Omega_m \) are provided in §4, including a thorough discussion of systematic errors.

## 2 PHYSICAL EXPECTATIONS

The value of any individual cluster’s baryon-to-total mass inferred from observations will, in general, differ from the cosmic value. The difference can be partly intrinsic — reflecting a true baryon enhancement or deficit within the cluster — and partly due to errors in estimates of the component masses.

To the extent that the physical processes responsible for a cluster’s structure are independent of its total mass, any intrinsic bias in the baryon fraction can, to first approximation, be expressed as a function of a scaled radial variable, equivalent to the local density contrast. Introducing notation used below, \( \delta_c \) denotes the mean interior density contrast with respect to the critical value \( \delta_c \equiv \rho(\langle r \rangle) / \rho_c \) with \( \rho_c \equiv 3 \ H_0^2 / 8 \pi \mathcal{G} \). Similarly, \( r_{500} \) is the radius at which a density contrast \( \delta_c \) is attained.

### 2.1 Intrinsic component bias

Because the horizon mass scale at the baryogenesis epoch is many orders of magnitude smaller than the typical cluster mass (e.g., Kolb & Turner 1990), there is no causal mechanism which can generate primordial fluctuations in the baryon-to-total mass ratio on cluster scales. This argument holds as long as inflation precedes baryogenesis. Any intrinsic bias must therefore be set by dynamical processes operating differentially on the baryonic and dark matter.

Gravity alone is not a powerful segregating mechanism. In the simplest example, consider clusters forming from spherically symmetric, scale–free initial density perturbations. Self–similar solutions to the dynamical equations (Fillmore & Goldreich 1984; Bertschinger 1985; Chièze, Teyssier & Alimi 1996) exhibit two characteristic regions — a nearly hydrostatic, inner body surrounded by an outer, infalling envelope which merges seamlessly into the Hubble flow at large radii. The flow changes discontinuously at the boundary, implying the development of a shock for the collisional baryons or a caustic surface for the collisionless dark matter (or galaxies). The position and velocity of the shock for a \( \gamma = 5 / 3 \) ideal gas is very close to that of the outermost caustic surface for the dark matter; together they define a unique radius (commonly referred to as the virial radius \( r_{vir} \)) within which the mean enclosed density is \( \sim 100 \) times the background value. Since all matter outside the virial surface is infalling with the cosmic mix of components, then, by continuity, the baryon mass fraction measured at the virial radius must be unbiased \( f_b(r_{vir}) \equiv \Omega_b / \Omega_m \).

The realistic case differs from the spherical one in several respects. Clusters forming in a fully three–dimensional, hierarchical fashion experience lumpy, asymmetric accretion directed by connecting filaments (West, Dekel & Oemler 1987; Frenk et al. 1990; Evrard 1990a,b; Kang et al. 1994; Navarro, Frenk & White 1995; Tormen 1996). In addition, the ability of the intracluster gas to lose entropy via radiative cooling or increase it via feedback from galactic winds calls into question lessons learned assuming gravitationally induced shocks are the only entropy changing mechanism.

These issues led White et al. (1993) to explore extreme models for dynamical baryon enhancement. They examined the evolution of infinitely dissipational (pressureless) gas and dark matter using both a spherical model based on Bertschinger’s (1985) solutions and three dimensional, gas dynamic simulations. Following their notation, define \( \Upsilon(\delta_c) \) as the ratio of enclosed baryon fraction within radius \( r_{500} \) to the cosmic value

\[
f_b(r_{500}) \equiv \Upsilon(\delta_c) \frac{\Omega_b}{\Omega_m}. \tag{1}
\]

For the extreme case of a zero temperature gas, the White et al. three dimensional simulations show baryon enhancements smaller than the spherical case. For example, at \( \delta_c = 500 \), the spherical model predicts \( \Upsilon(500) = 1.6 \) while the average of twenty simulations is \( \Upsilon(500) = 1.25 \) and the maximum simulation value is 1.5. Although the simulation results strictly apply to the case of a standard CDM initial fluctuation spectrum, results for other cosmologically reasonable \( \Omega_m = 1 \) power spectra should not differ substantially, as the sensitivity of cluster dynamical histories to spectral shape is fairly mild (Lacey & Cole 1993; Crone, Evrard & Richstone 1994; Navarro, Frenk & White 1996).

When a realistic gas equation of state is used, combined N-body and gas dynamic simulations generally show the gas to be slightly more extended than the dark matter (Evrard 1990a; Thomas & Couchman 1992; Cen & Ostriker 1993; Kang et al. 1994; Metzler & Evrard 1994; Navarro, Frenk & White 1995; Lubin et al. 1996), though the evidence is not universal (Anninos & Norman 1996). Energy transfer between these components during major mergers is a likely physical explanation (Navarro & White 1993; Pearce, Thomas & Couchman 1994), though the origin may be unrelated to mergers (Chièze, Teyssier & Alimi 1996). The extended gas structure implies a weakly rising baryon fraction with radius \( f_{gas}(r) \sim r^3 \) with \( \gamma \sim 0.1 - 0.2 \) near the virial radius, and a modest, overall baryon diminution \( \Upsilon(500) \sim 0.9 \) (Frenk et al. 1996). The mild rise of gas fraction with radius appears consistent with the observations discussed below.

### 2.2 Cluster mass estimation

The bias and variance in the mass estimates inferred from observations have also been calibrated by numerical experiments of cluster formation incorporating gravity and gas dynamics. A number of independent experiments over the years (Evrard 1990a,b; Tsai, Katz & Bertschinger 1994; Metzler &...
Evrard 1994; Navarro, Frenk & White 1995; Schindler 1996; Evrard, Metzler & Navarro 1996; Roettiger, Burns & Loken 1996) show that, when exercised judiciously, accurate binding mass estimates can be made with the standard, $\beta$-model approach (Cavaliere & Fusco-Fumiano 1976). "Judicious exercising" here means avoiding the cores of clusters where cooling flows and other complications occur (Tsai, Katz & Bertschinger 1994), avoiding clusters engaged in obvious major mergers (Roettiger et al. 1996), and avoiding extrapolating to very large radii where an equilibrium assumption is not justifiable.

From an analysis of gas velocity moments, Evrard et al. (1996, hereafter EMN) propose $r_{500}$ as a conservative choice for the outer hydrostatic boundary of clusters. Using a sample of 56 cluster simulations evolved in different cosmological backgrounds, and with a subset including input of mass and energy from galactic winds, they show that, within this radius, the gas is very close to hydrostatic, with a mean, mass weighted, radial Mach number of only a few percent (see Tables 3 and 4 of EMN). Binding mass estimates based on the standard, $\beta$-model approach are nearly unbiased and have an intrinsic scatter of $\sim 30\%$ at $r_{500}$. This compares well with the 15\% rms deviation quoted by Schindler (1996) and the tens of percent variations seen in the experiments of Roettiger et al. (1996).

In addition, EMN find that the variance in the mass estimates can be considerably reduced by eliminating the $\beta$ parameter entirely (i.e., ignoring the X-ray image) and deriving the binding mass directly from the global, emission weighted temperature $T$. The resulting scaling relations are consistent with virial equilibrium expectations at a fixed density contrast $\delta \sim GM/r^2$ (see Tables 3 and 4 of EMN). Binding mass estimates based on the numerical simulations of EMN, and is consistent with the rise of gas fraction with radius in both the WF and DJF samples. The mean values of $\eta$ in the WF data, derived by comparing the baryon fractions at 0.5 $h^{-1}$ Mpc and $r_X$, is 0.13. This is biased somewhat low by the few clusters with short lever arm — removing 3 clusters with $r_X < 0.6 h^{-1}$ Mpc yields a mean $\eta = 0.15$, with all but one value in the range 0.05–0.28.

The resultant gas fractions at $r_{500}(T_X)$ for the WF sample are plotted against cluster temperature as filled circles in Figure 1. Error bars on the gas fraction are 90\% confidence limits and include contributions from the binding and gas mass errors. The errors quoted in Table 2 of WF include only the gas contribution. To estimate binding mass uncertainty, I assume a fractional error equal to that in cluster temperature. Because the latter is generally asymmetric, so also is the binding mass error. Temperature values and errors are taken from the deprojected values in Table 1 of WF, except for two cases of very hot clusters — A2142 and A2163 — which have unusually large allowed lower temperature ranges from the deprojection method. For these, I use the “reference” lower bound on temperature quoted by WF rather than the deprojected values.

The temperature errors are comparable in magnitude to those of the gas; average 1$\sigma$ fractional errors are 8.5\% for the gas and 6.6/12.8\% for the upper/lower temperature uncertainties. Because the gas emissivity in the IPC band is fairly insensitive to temperature over most of the range spanned by the WF data, the derived gas masses are nearly independent of temperature. Errors in gas and total masses can then be assumed uncorrelated, implying the fractional

\[ f_{gas}(r_{500}(T_X)) = f_{gas}(r_X) \left( \frac{r_{500}(T_X)}{r_X} \right)^{\eta} \]  \hspace{1cm} (4)

with $\eta = 0.17$. This value of $\eta$ is derived from the numerical simulations of EMN, and is consistent with the rise of gas fraction with radius in both the WF and DJF samples. The mean values of $\eta$ in the WF data, derived by comparing the baryon fractions at 0.5 $h^{-1}$ Mpc and $r_X$, is 0.13. This is biased somewhat low by the few clusters with short lever arm — removing 3 clusters with $r_X < 0.6 h^{-1}$ Mpc yields a mean $\eta = 0.15$, with all but one value in the range 0.05–0.28.

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Figure 1. The intracoustic gas fraction within $r_{500}(T_X)$ from eqn(4) plotted against X-ray temperature for the observational samples of WF (solid dots) and DJF (crosses). Errors in the temperature, omitted for clarity, are fractionally comparable to those in gas fraction. Estimates of the samples’ cumulative parent probability distributions, derived from the normalized sum of the individual cluster asymmetric Gaussian contributions, are shown by the solid (WF) and dotted (DJF) lines.

The range of likely baryon fraction values, as measured by the 5 to 95% confidence intervals, is large, $4 - 10 \, h^{-3/2}\%$, and the two data sets provide consistent estimates for this range. However, they differ substantially in their median values of $5.7 \, h^{-3/2}$ (WF) and $7.7 \, h^{-3/2}$ (DJF) percent, reflecting the fact that the distributions are asymmetric and oppositely skewed. A likelihood analysis of the data yields most likely values of $5.6 \, h^{-3/2}$ and $7.3 \, h^{-3/2}$, respectively, very similar to the median values.

A more robust assessment of sample location is made by employing a bootstrap procedure to estimate the range of sample means consistent with each data set. A bi-weight estimator of location is used, because of its superior performance for small data samples (Beers, Flynn & Gebhardt 1990), but results using an ordinary mean or median are similar. The procedure creates a large number of trial samples with replacement, assuming data values distributed in an asymmetric Gaussian fashion. For each trial, the bi-weight mean is estimated and the resulting distribution over many trials constructed.

The results, shown in panel (a) of Figure 2, are very nearly symmetric and Gaussian, the DJF sample less so because of the influence of A2063, an outlier with $T_X > 1 \, keV$; namely, A539, A262, A2589, A2063, A1795, A85 and A2029. Errors in the binding masses for these systems are again taken to be proportional to the temperature errors listed in Table 1 of DJF. Gas mass errors are not quoted in their Table, for these I assume a fixed 1σ error of 8.5%, the mean of the WF sample. This is likely to be an overestimate of the actual typical error, given the improved image quality of the ROSAT PSPC over the Einstein IPC. The results are insensitive to the choice of assumed error over the range 0 − 15%. The data are plotted as crosses in Figure 1.

The cooler ROSAT sample appears to be more gas rich than its hotter IPC counterpart. Non-parametric statistical tests of location, such as the Mann–Whitney test and the Wilcoxon Signed-rank test, indicate sample inconsistency at better than 99% confidence for the raw data values. A simple way to incorporate errors is through estimates of the samples’ parent probability distributions, derived from an unweighted sum of the individual cluster asymmetric Gaussian contributions. The cumulative version of these are shown for each sample as the solid and dotted lines in Figure 1.

The direction of the discrepancy — lower temperature DJF groups having higher gas fractions than the richer clusters of the WF sample — is surprising, because adding the galaxy contribution to the baryon mass would serve to exaggerate the difference, not correct it. Adding the galaxies would imply significantly larger baryon fractions in poor groups compared to rich clusters within the virial radius, an outcome not anticipated in hierarchical clustering scenarios. In fact, the opposite is the more likely expectation. Feedback from galactic winds drives baryons preferentially out of low temperature systems (Yahil & Ostriker 1973; White 1991; Metzler & Evrard 1994). It is possible that the discrepancy arises from sample selection criteria, but the fact that
the density contrast
appropriate corrections have been made to maintain estimates at
the revised binding masses is shown in Figure 2(b). Appro-
plication (White from the different dynamical histories/states of a coeval pop-
ulation (White et al. 1993; Cen & Ostriker 1993; Kang et al. 1994). Galactic feedback can cause gas loss by subsonic winds after cluster formation or by hindering collapse itself through early preheating. Though nearly all the 26 clusters
in Figure 1 have 90% confidence limits which overlap the limits in eq’n(5), there are clusters which have significantly less gas. An example is A576, which has a gas fraction lower by about a factor 2 than the mean (Mohr et al. 1996). Compact groups are extreme in this regard (Ponman et al. 1996; Pildis, Evrard & Bregman 1996). Loewenstein & Mushotzky (1996) present two cool clusters with different baryon fractions within their virial regions. Intrinsic variations in cluster gas content are thus both expected and observed, with the caveat that large amplitude variations go in only one direction, that of reducing gas content. There is currently no empirical evidence or theoretical justification supporting large baryon enhancements near the virial radius in clusters.

4 IMPLICATIONS FOR $\Omega_M$

The arguments presented in §2 indicate that the mean baryon fraction within $\delta_c = 500$ is enhanced by at most a factor 1.25 by dynamical means during hierarchical clustering and is more likely slightly below the universal value. This leads to two avenues for using the mean intracluster gas fraction to constrain $\Omega_m$, which are considered here.

4.1 Upper limit approach

Because the contribution of baryon sources add linearly, the mean baryon content of clusters must exceed the mean gas content $\bar{f}_b \geq \bar{f}_{gas}$. The mean gas fraction can thus be used to place an upper limit on $\Omega_m/\Omega_b$ in eq’n(1)

$$\frac{\Omega_m}{\Omega_b} \leq \bar{f}_{gas} f^{-1}_b (r_{500}).$$

(6)

From the perspective of maximizing $\Omega_m$, it is appropriate to take the 5%-ile lower limit on $f_{gas}$ along with the largest possible baryon enhancement. The simulations of White et al. yield an average enhancement $\bar{T}(500) = 1.25$, and it is reasonable to assume this is appropriate for the effect on the population mean. The upper bound on the ratio of total, clustered to baryonic mass density is then

$$\frac{\Omega_m}{\Omega_b} < 23.1 h^{3/2}.$$  

(7)

Comparison between the light elemental composition of the universe and primordial nucleosynthesis expectations places an upper limit on the mean baryon density. This can be combined with the above to express an upper limit on $\Omega_m$ directly. The exact value of the baryon density derived in this way continues to be a subject of current debate (Krauss & Kernan 1994; Copi, Schramm & Turner 1995; Steigman 1995; Sasselov & Goldwirth 1995; Hata et al. 1996; Tytler, Fan & Burles 1996; Rugers & Hogan 1996). However, a consensus view is that a firm upper limit $\Omega_b \leq 0.026 h^{-2}$ exists simply from the abundance of fragile deuterium in the hostile environment of the local solar neighborhood (Linsky et al. 1995). This value produces the constraint

$$\Omega_m h^{1/2} < 0.60.$$  

(8)

The Einstein–de Sitter case $\Omega_m = 1$ requires a very low Hubble constant $h \leq 0.36$ (Bartlett et al. 1995).

4.2 Best estimate approach

This upper limit is generous in that it: (i) ignores the contribution of galaxies to the baryon fraction; (ii) assumes maximal, dynamical baryon enhancement through use of an unphysical gas equation of state and (iii) uses the largest realistic value of $\Omega_b$ combined with the smallest allowed value of
An alternate perspective is to use “realistic” parameter values to provide a “best” estimate of $\Omega_m$. For this, I will assume: (i) galaxies make a $20 \pm 5\%$ contribution relative to the gas $f_g(r_{500}) = (1 + 0.2 h^{3/2}) f_{gas}(r_{500})$ as appropriate for Coma (White et al. 1993) and (ii) a realistic equation of state for the gas leads to a modest baryon diminution $T(500) = 0.85$. Utilizing the central value of $f_{gas}(r_{500})$ from eq’n (5) results in

$$\Omega_m/\Omega_b = (11.8 \pm 0.7) \frac{1.2 h^{3/2}}{1 + 0.2 h^{3/2}} \approx (11.8 \pm 0.7) h^{4/3} \ (9)$$

which is a factor of two smaller than the generous upper limit in eq’n (7). The $1\sigma$ error quoted above is derived from the propagated statistical error of $f_{gas}$. The odd-looking slope of $4/3$ in the second expression is a power law approximation to the $1.2 h^{3/2} / (1 + 0.2 h^{3/2})$ term; it is accurate to 2 percent over the range $h \in [0, 4.5, 1]$.

The final step to estimate $\Omega_m$ requires a value for $\Omega_b$. Recent inferences of the primordial deuterium abundance from quasar absorption line spectra produce two different preferred values, a low value $\Omega_b h^2 = 6.2 \pm 0.8 \times 10^{-3}$ (Rugers & Hogan 1996) and a high value $\Omega_b h^2 = 0.024 \pm 0.006$ (Tytler, Fan & Burles 1996) derived from their respective high and low estimates for $D/H$. These values result in estimates and $1\sigma$ errors of

$$\Omega_m h^{2/3} = 0.07 \pm 0.01 \quad \text{high D/H}, \quad (10)$$

$$\Omega_m h^{2/3} = 0.28 \pm 0.07 \quad \text{low D/H}. \quad (11)$$

The latter value should be preferred over the former for several reasons. First, a recent analysis of high resolution Keck spectra of Q0014+813 — the best “high D/H” candidate — by Tytler, Burles & Kirkman (1996) provides no support for a level of high deuterium absorption, and suggests hydrogen interlopers as a likely explanation for the data. In addition, the value $\Omega_m \sim 0.3$ is currently concordant with large-scale structure observations (e.g., Ostriker & Steinhardt 1995), while the low value $\Omega_m < 0.1$ deduced from the Rugers & Hogan D/H estimate is decidedly difficult in this regard.

### 4.3 Systematic effects

The modest statistical error in these estimates is deceptive, since there are systematic uncertainties in the steps involved in the derivation. In the “minimal” approach adopted above, there is perhaps a factor two uncertainty in the contribution of galaxies relative to gas, implying about a 20% error in the baryon fraction estimate. In addition, there is approximately 15% uncertainty in the appropriate value of $T(500)$. Conservatively adding these contributions leads to an estimate of the overall systematic uncertainty of 35%. In addition, there are systematic effects in the nucleosynthesis determination of $\Omega$ which would enlarge the quoted statistical error (Audouze, Olive & Truran 1997), but values $\Omega_b h^2 > 0.024$ are unlikely because of additional constraints from $^4\text{He}$, $^9\text{Li}$ and $^7\text{Li}$ abundances (e.g., Lemoine et al. 1997).

The actual error in this analysis could be larger if other systematic effects play a significant role. Current possibilities include the following:

(i) **Multi-phase intracluster gas** — One can imagine a multi-phase structure, with cooler gas perhaps entrained along loops of magnetic field surrounded by hotter, more dilute plasma. In order to be competitive with thermal pressure and create a significant mass estimate bias through clumping, the magnetic pressure must be close to the thermal pressure outside the loops and the mass filling factor within the loops should be large. No specific model for the origin and maintenance of such a configuration has been proposed. On energetic grounds alone, it is difficult to imagine galaxies supplying enough magnetic field, and observations of Faraday rotation in background sources indicate that any strong fields must have small coherence lengths (Kim et al. 1990; Kronberg 1994). Another energetic argument against significant magnetic pressure is the fact that the specific thermal energy of the intracluster gas is very nearly equal to the specific kinetic energy of the galaxies (Jones & Forman 1984; Lubin & Bahcall 1993), as expected if the thermal and kinetic pressures respectively support each component within the same potential well (the assumption underlying the $\beta$-model of Cavaliere & Fusco-Femiano 1976). Spatially resolved X-ray spectroscopy, particularly of line emission in cooler (2 – 6 keV) clusters, will provide tests of such models, which are broadly similar to multi-phase cooling flow models (Sarazin 1996). Limited information is already available in broad-beam colors (Henriksen & White 1996). Data from ASCA, SAX, and soon XMM and AXAF, coupled with realistic, dynamical and thermodynamical modeling (Teyssier, Chièze & Alimi 1996) should place stringent constraints on such multiphase models.

In addition, since the electron pressure structure in the gas in a multi-phase model will differ from the standard, unclumped model, Sunyaev-Zel’dovich measurements can be used to provide independent constraints on clumping. Roettiger’s (1996) gas dynamic merger simulations show that errors in SZ Hubble constant determinations are small in the standard scenario, providing hopeful prospects of detecting a modest signal from clumpiness. Present data probably cannot rule out clumping factors as large as 50%, but factors $\gtrsim 2$ seem unlikely.

(ii) **Mass estimate errors** — The simulations may be giving a misleadingly simple picture of mass estimate accuracy. The good agreement of independent codes employing different gas dynamic methods on the same problem points the finger at missing physics, rather than numerical inaccuracy, if a significant effect is to be found. Neither feedback from galactic winds (EMN) nor the inclusion of non-equilibrium thermodynamics in the intracluster plasma (Teyssier, Chièze & Alimi 1996) significantly alter the virial scaling relation, eq’n (3). The fact that this binding mass estimator is able to reconcile the modest WF and DJF sample difference can be taken as a measure of empirical support for the simulation results, but that could be a misleading interpretation. Independent measures of cluster mass, particularly weak gravitational lensing (Tyson, Valdes & Wenk 1990; Kaiser & Squires 1993) are needed to provide additional measures of mass estimate accuracy. Comparison of the two methods near the virial radius in a few clusters indicates consistency within modest ($\sim 50\%$) statistical error ranges (Squires et al. 1996). Extending such studies to larger, statistical samples is imperative.

(iii) **Extrapolation errors** — The X-ray data of the WF sample do not extend to $r_{500}$, making extrapolation necessary. The extrapolation in gas fraction is modest, about
$1 \, h^{-3/2} \%$ on average. As a further check on the employed procedure, the data values at $r_X$ can be used to predict values at $0.5 \, h^{-1} \, \text{Mpc}$, and the underlying distribution function constructed at that radius. The resultant 5, 50 and 95% confidence limits using the extrapolated gas fraction of 9.7, 13.8, and 22.9%, respectively (quoted for $h=0.5$), agree well with the directly determined values of 10.0, 13.8 and 22.3 from WF (their Figure 6). Significant errors from extrapolation are thus very unlikely.

(iv) Hot dark matter — A sea of massive, light neutrinos would cluster differently from the cold dark matter typically assumed in the dynamical simulations; their high entropy would prevent them from clustering in small potential wells. Experiments using viable cold plus hot dark matter (CHDM) models show that this effect is negligible in rich clusters at radii close to $r_{500}$ (Kofman et al. 1996).

(v) Additional baryon contributions — Sources of baryons in clusters besides gas and galaxies exist, in the form of diffuse intracluster stars (Uson, Boughn, & Kuhn 1991; Theuns & Warren 1996) and MACHOS (Gates, Gyuk, & Turner 1995; Alcock et al. 1996). The latter have not been directly detected in intracluster environments, and they may be simply connected to the former. Regardless, extra baryons only drive the constraints on $\Omega_m$ to lower values. Their contribution probably does not exceed that of galactic starlight.

(vi) Initial baryon fraction inhomogeneities — An early universe model which generates pre-inflationary baryon–total mass fluctuations and manages to preserve them into the post-inflationary epoch would circumvent the causality argument mentioned in §2. Such a model would need to naturally couple high baryon overdensity to high mass overdensity in order to produce an overestimate of the cosmic baryon fraction in clusters. Such a correlation would also avoid producing baryon deficient clusters, a comforting situation since no large population of deep, “empty” potential wells exists (although Bonnet, Mellier & Fort (1994) have a candidate in the field of CL0024+1654). Consistency with $\Omega_m = 1$ would presumably be a natural feature of such models.

The sources of systematic uncertainty above (except the last) added in quadrature allow room for perhaps 70% additional upward error in $\Omega_m$. However, the magnitude of these effects — particularly of gas clumping for which there are no specific, dynamical models — are currently not well understood. Forcing all effects in the same direction could probably manage consistency with $\Omega_m = 1$ and, in this case, the observational tests cited above should start uncovering the effects responsible in the near future.

5 SUMMARY

The gas fraction in clusters of galaxies provides information on the cosmic baryon mass fraction. Dynamical biases of clusters’ baryon content can be minimized by measuring masses near the virial radius, where gas dynamic experiments show equilibrium is valid and segregation processes inefficient. At $r_{500}$, the radius where the mean interior density is a factor 500 times the critical value, the bi-weight mean gas fraction of the combined, revised WF and DJF X-ray cluster samples is $0.060 \pm 0.003 h^{-3/2}$. This value, when combined with our current understanding of cluster formation history and limits on $\Omega_m$ from primordial nucleosynthesis, strongly favors $\Omega_m h^{2/3} \approx 0.3$ and rules out the possibility of $\Omega_m = 1$ with high statistical significance, unless the Hubble constant is very low $h < 0.4$. These conclusions reinforce, at greater statistical significance, earlier work based on different methods and data than those used here (e.g., Henriksen & Mamon 1994; Steigman & Felten 1995; Lubin et al. 1996).

How serious is the case against $\Omega_m = 1$? A die-hard Einstein-deSitter advocate could espouse all the elements of the analysis leading to the upper limit in eq’n (8) and claim only a small (30% for $h=0.7$) systematic effect is missing. However, this approach accounts for neither the hot, X–ray emitting gas nor the galaxies in clusters, and one might well be suspicious of an argument which ignores these two principal, observable components. The best estimate approach leaves one short by a factor $\gtrsim 3$. It is possible that this gap is plugged not by a single large effect, but by several effects acting in concert, a situation reminiscent of the so-called “$\beta$–discrepancy” in clusters (Evrard 1990h; Lubin & Bahcall 1995). There remains the possibility that a new element of X–ray cluster physics is simply missing from the present picture. Observational constraints on known sources of systematic error should be vigorously pursued, along with more sophisticated theoretical modeling of intracluster plasma dynamics and thermodynamics.

The arguments presented here are independent of the value of the cosmological constant. If one favors a spatially flat universe, as motivated by simple models of inflation, obtained through a non-zero cosmological constant $\Lambda$, then the limits on the clustered matter component predict values of the required vacuum energy density $\Omega_\Lambda \equiv \Lambda/3 \, H_0^2 = 1 - \Omega_m$. For example, for $h = 0.7$, the upper limit on $\Omega_m$ requires $\Omega_\Lambda \geq 0.28$ while the best estimate approach gives $\Omega_\Lambda = 0.64 \pm 0.09$. The latter is consistent with the limit $\Omega_\Lambda < 0.66$ derived from gravitational lensing arguments (Kochanek 1996) while marginally inconsistent with the $\Omega_\Lambda < 0.51$ inferred recently by Perlmutter et al. (1996) from the magnitude–redshift relation for Type–Ia supernovae.

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