Fermion-condensation and Boson-saturation in Cuprates

Zhe Chang¹

CCAST (World Laboratory)

P. O. Box 8730, 100080 Beijing, China

Institute of High Energy Physics, Academia Sinica

P. O. Box 918(4), 100039 Beijing, China²

and

Institute of Theoretical Physics, Academia Sinica

P. O. Box 2735, 100080 Beijing, China

SW 9707

Abstract

At the slave-boson approach, we show that the boundary terms of the t - J model turn the symmetry of the system from ordinary Lie group into quantum group. The novel property of fermion-condensation and boson-saturation is demonstrated below the critical concentration point of doping within the framework of quantum symmetry. After taking into account the novel phenomena, we get antiferromagnetic correlation and small Fermi pockets centered around the $\left(\pm\frac{\pi}{2}, \pm\frac{\pi}{2}\right)$ points at low doping which evolve into the large Fermi surface at high doping concentrations.

PACS numbers: 74.25.Jb, 71.27.+a

Key words: *high-temperature superconductor, quantum group symmetry, t - J model*

to appear in Phys. Lett. A

¹Electronic address: changz@bepc3.ihep.ac.cn

²Mailing address.

The normal state of high- T_c cuprate superconductors has shown peculiar properties in both charge and spin channels[1]. The resistivity exhibits a linear-temperature dependence up to 1000 K and down to a temperature $\simeq T_c$. This temperature dependence has been related to a scattering rate that behaves like $2\frac{k_B}{\hbar}T$ for all the optimally doped cuprates with T_c ranging from 10 K up to over 100 K. Such a linear-temperature resistivity is also accompanied by a Hall coefficient which implies hole-like charge carriers and shows a $1/T$ dependence in contrast to the T -independent Fermi liquid case. The NMR spin-lattice relaxation rate T_1^{-1} shows a strong enhancement at low temperature for planar Cu nuclei, in contrast to the usual Korringa behavior in a Fermi liquid.

Anderson[2] suggested that the physics of the cuprates is contained in a two-dimensional, large- U , single band Hubbard model, or equivalently the t - J model. A satisfactory treatment of the t - J model, which requires single electron occupation on the sites, is a long standing theoretical challenge in solid state physics. A convenient way to deal with the constraint is to formally write the electron operator $c_{j\sigma}$ ($c_{j\sigma}^\dagger$) as a product of fermion and boson operators. The overcompleteness which this decoupling unavoidably introduces is removed by a local constraint imposed at every lattice site in order to ensure single electron occupancy. Two alternative methods have been used, depending on which species carry the spin label. The slave-boson approach has been widely studied[3]. Within this approach the ground state of a doped system is the uniform Resonating Valence Bond (RVB) state[4] and shows Fermi-liquid (FL) behavior below the Bose condensation temperature. A serious difficulty of the approach is the absence of antiferromagnetic (AF) correlations. In fact, the ground state in the half-filled case does not satisfy the Marshall sign rule[5]. On the other hand, the experiments do show the strong interplay between magnetism and superconductivity[6]. The AF correlations survive even in the superconducting region. Alternatively, one can use the slave-fermion (Schwinger boson) approach[7]. At half-filling the AF long-range order is recovered if the Bose condensation is taken into account. The obtained ground-state energy is much better than in the slave-boson case and the state obeys the Marshall sign rule. However, in this approach the Fermi surface is small even for

finite doping, which is not consistent with experiments. It is well established that at optimal doping, a Fermi surface exists with area $1 - x$ where x is the concentration of doped holes[8]. On the other hand, for $x \ll 1$, the system remains AF with a doubled unit cell, and the Fermi surface is expected to be small pockets centered around the $(\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$ points[9]. Therefore a unified theoretical picture is necessary to cope correctly with both the AF correlation and the size of the Fermi surface.

In this paper we present a mechanism of fermion-condensation and boson-saturation activated by boundary terms and AF correlation. Because the slave-boson approach describes well the transport properties and the slave-fermion approach gives the correct spin-magnetic picture, we hope that a mechanism interrelating the two approaches can capture the anomalous properties of cuprate superconductors.

Recently there has been great interest in the study of quantum group and quantum symmetry[10]. Quantum group is now a popular topic in different fields of modern physics due to its richer structure than that of Lie group[11]. In particular, some integrable lattice models are constructed and solved by quantum group theory. A noticeable example of quantum group invariant models is the XXZ spin chain with proper boundary terms[12],

$$H_{XXZ} = \sum_{\langle jk \rangle} \left(S_j^x S_k^x + S_j^y S_k^y + \frac{q + q^{-1}}{2} S_j^z S_k^z \right) + \frac{q - q^{-1}}{2} \sum_{\langle jk \rangle} (S_j^z - S_k^z) - \frac{q + q^{-1}}{2}, \quad (1)$$

where q is the deformation parameter. It is easy to verify that the XXZ Hamiltonian is $SU_q(2)$ -invariant, i.e.,

$$[H_{XXZ}, SU_q(2)] = 0. \quad (2)$$

It is the boundary term which turns the symmetry of the system from ordinary $SU(2)$ into quantum $SU_q(2)$. Generalization to the two dimensional case is straightforward[13]. The boundary terms also determine the symmetry property of the two-dimensional XXZ system.

In fact for the spin system with large size and no AF correlation (long or even

short range AF order), the effects of these boundary terms can be ignored at all. Consequently one can not detect the implications of the quantum symmetry. However, when the AF correlation is present, and in particular when long-range AF order exists, these boundary terms can not be ignored and the system has to be investigated in the framework of quantum group and quantum symmetry. In the following, we generalize the argument to the case of non-zero doping.

In the slave-boson approach, one writes $c_{j\sigma} = f_{j\sigma} b_j^\dagger$ where the fermion $f_{j\sigma}$ carries the spin label. Within this formalism the ground state of a doped system is the uniform RVB state and shows FL-like behavior below the Bose condensation temperature. Various symmetry-broken phases, including the dimerized phase, flux phase, staggered flux phase, chiral phase with broken time-reversal symmetry, etc. were considered. Because of the fermionic character of the spin carriers, $f_{j\sigma}$, a large Fermi surface with area $1 - x$ is formed, which satisfies the Luttinger theorem. This is in good agreement with photoemission experiments for cuprates at optimal doping[8]. Taking into account the interaction among bosons and fermions, the gauge theory gives correct description of the linear-temperature dependence of resistivity and thermal conductivity[14].

By decreasing the concentration of holes in the cuprates, AF correlation appears, and we must take into account the boundary terms. The boundary terms turn the symmetry of the system from ordinary group into quantum group. Recently, the boson-fermion representation of quantum group was studied extensively recently[15]. The elementary excitations in quantum group invariant system are the so-called q -bosons and q -fermions instead of ordinary bosons and fermions in group invariant system. The q -boson and q -fermion excitations are connected with the ordinary bosons and fermions in the way

$$\begin{aligned} a_q &= a \frac{[a^\dagger a]}{q^\dagger a} = \frac{[aa^\dagger]}{aa^\dagger} a, & a \in \{b, f\}, \\ a_q^\dagger &= \frac{[a^\dagger a]}{a^\dagger a} a^\dagger = a^\dagger \frac{[aa^\dagger]}{aa^\dagger}, \end{aligned} \quad (3)$$

where the q number $[x]$ is defined as

$$[x] = \frac{q^x - q^{-x}}{q - q^{-1}} = \frac{\sin \eta x}{\sin \eta}, \quad q = e^{i\eta}.$$

The q -boson and q -fermion excitations show different properties compared with ordinary boson and fermion. The q -fermion operators f_q , $f_q^\dagger$ and M ($= f^\dagger f$) satisfy the quantum Clifford algebra

$$\begin{aligned} [M, f_q^\dagger] &= f_q^\dagger, & [M, f_q] &= -f_q, \\ \{f_q, f_q^\dagger\} &= \frac{[1-M]^2}{1-M} + \frac{[M]^2}{M}. \end{aligned} \quad (4)$$

From the quantum Clifford algebra, we get relations

$$\begin{aligned} \frac{[1-M]^2}{1-M} f_q^\dagger &= 0 = f_q^\dagger \frac{[M]^2}{M}, \\ \frac{[M]^2}{M} f_q &= 0 = f_q \frac{[1-M]^2}{1-M}. \end{aligned} \quad (5)$$

The consistency condition for the eigenvalues of the particle number operator M is of the form

$$\frac{[M]^2}{M} \cdot \frac{[1-M]^2}{1-M} = 0 = \frac{[1-M]^2}{1-M} \cdot \frac{[M]^2}{M}. \quad (6)$$

So that the general form of the eigenvalues of M is:

$$M = M_0 + \frac{\pi}{\eta} n, \quad M_0 \in \{0, 1\}, \quad n \in \mathbb{Z}. \quad (7)$$

For q being not a root of unity, the degeneracy of the eigenvalues of the particle number operator M is forbidden by simple physical principle: the particle number operator must be Hermitian. So that in a real physics system, $M \equiv M_0$ and the fermionic character of the spin carriers remains. This prediction is in good agreement with the fact that in a vast region of the doping concentrations the cuprates have a large Fermi surface.

For q being a root of unity, the explicit expression for the q -fermion in M -diagonal representation is

$$\begin{aligned} f_q &= \sum_{n \in \mathbb{Z}} |0, n\rangle \langle 1, -n|, \\ f_q^\dagger &= \sum_{n \in \mathbb{Z}} (-1)^n |1, -n\rangle \langle 0, n|, \end{aligned} \quad (8)$$

where we have used the notion $|M_0, n\rangle$ to denote the eigenstate of M with $M = M_0 + \pi n/\eta$ as eigenvalue.

The explicit expression shows that for fixed n the pair of states $|0, n\rangle$ and $|1, -n\rangle$ carries two-dimensional irreducible representation of the quantum Clifford algebra and the Hermitian conjugate of the operator f_q is $[M]f_q^\dagger$ rather than $f_q^\dagger$. A Hermitian set of q -fermion can be introduced to denote the real elementary excitations of the quantum group invariant system,

$$f_{q-H} = q^{-M/2} f_q, \quad f_{q-H}^\dagger = q^{(M-1)/2} f_q^\dagger \quad (9)$$

The special interested is for the case of $q = \exp(i\pi/2)$, we assume that this is occurred below the point of critical concentration. At the case,

$$M = M_0 + 2n, \quad M_0 \in \{0, 1\}, \quad n \in \mathbb{Z}.$$

i.e., M can take values of any integer. States corresponding to all integers M fall into irreducible doublets and each of them comprises $|2n\rangle$ and $|2n+1\rangle$. The physical meaning of the any integer of eigenvalues of M for elementary excitations of quantum group invariant system is that ordinary fermions condensed in it. Then the q -fermions (spin carriers of the cuprates) appear to be bosonic. The usual Fermi-Dirac distribution is more valid. The contribution of the spin sector to the Fermi surface is decreased sharply. One expects novel physical implications of the fermion-condensation.

The quantum Heisenberg algebra for $b_q, b_q^\dagger$ and $M (= b_q^\dagger b_q)$ is of the form

$$\begin{aligned} [b_q^\dagger, M] &= b_q^\dagger, & [b_q, M] &= -b_q, \\ [b_q, b_q^\dagger] &= \frac{[M+1]^2}{M+1} - \frac{[M]^2}{M}. \end{aligned} \quad (10)$$

For q being root of unity, $q = e^{i2\pi/k}$, ($k \in \mathbb{Z}$), the magnitude of the particle number operator for q -boson has maximum and minimum values

$$0 \leq M_q = |[M]| = \frac{\sin \frac{2\pi M}{k}}{\sin \frac{2\pi}{k}} \leq \left(\sin \frac{2\pi}{k} \right)^{-1}, \quad (11)$$

and the rightmost equality holds at $M = k(2l + 1)/4$. In other words, the magnitude M_q is finite regardless of the magnitude of M . The leftmost equality holds for $M = kl/2$. This property of M_q may be called the boson-saturation. In the particular case, $q = \exp(i\pi/2)$, $M_q \leq 1$ and the equality applies to $M = 2l + 1$. When M is in spectrum interval $[2l, 2l + 1)$, the magnitude M_q climbs to a maximum values 1 and then drops down. The Fock space of the q -boson is then two-dimensional and consists of two states in the ordinary Fock space for boson,

$$V_2^l = \left\{ |2l\rangle = \frac{(b_q^\dagger)^{2l}}{[2l]!} |0\rangle, |2l + 1\rangle = b_q^\dagger |2l\rangle \right\}. \quad (12)$$

And the infinite set of the ordinary boson eigenstates $|n\rangle$ is divided into an infinite set of these two-dimensional subspaces,

$$V = \bigcup_{l=0}^{\infty} V_2^l. \quad (13)$$

In fact, it can be checked easily that in space V_2^l at the critical concentration point, the q -boson turns into ordinary fermion,

$$\begin{aligned} [M, b_q^\dagger] &= b_q^\dagger, & [M, b_q] &= -b_q, \\ \{b_q, b_q^\dagger\} &= \frac{1}{2l + 1}, & b_q^2 &= 0 = (b_q^\dagger)^2. \end{aligned} \quad (14)$$

After rescaling, we obtain the standard Clifford algebra,

$$\begin{aligned} [M, \tilde{b}_q^\dagger] &= \tilde{b}_q^\dagger, & [M, \tilde{b}_q] &= -\tilde{b}_q, \\ \{\tilde{b}_q, \tilde{b}_q^\dagger\} &= 1, & \tilde{b}_q &\equiv \sqrt{2l + 1} b_q. \end{aligned} \quad (15)$$

And the newly defined number operator $\tilde{M}_q (= \tilde{b}_q^\dagger \tilde{b}_q)$ has spectrum $\{0, 1\}$.

The boson-saturation turns the charge carriers of the cuprates to be fermionic when the boundary terms and AF long-range order are present. They form small Fermi pockets at low doping concentrations. In conclusion, we see that, below the critical concentration point, the boundary terms along with the AF long-range order reciprocates the statistical property of the spin and charge carriers in cuprates.

Under the critical concentration of holes, a self-consistent perturbation theory can be developed to describe spin and hole excitations. The hole excitations can be described by a narrow quasiparticle band located at an energy of order $-t$ with a quasiparticle residue of order J/t and a bandwidth of order J . The large mass enhancement is consistent with the observed weight of the Drude peak in the ac conductivity[16]. The quasiparticle energy has its minimum value at the points $(\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$ in the Brillouin zone. According, we assume that hole quasiparticles form a weakly interacting Fermi gas. In the vicinity of the four minima $(\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$ the quasiparticle energy ϵ_k is of the form[18],

$$\epsilon_k = \epsilon_{k_i} + \frac{1}{2m_{\parallel}}k_{\parallel}^2 + \frac{1}{2m_{\perp}}k_{\perp}^2, \quad (16)$$

where $k_{\parallel}$ and $k_{\perp}$ are the components of $\mathbf{k} - \mathbf{k}_i$ in $(1, -1)$ and $(1, 1)$ directions when $\mathbf{k}_i = (\frac{\pi}{2}, \frac{\pi}{2})$. Then we have a natural Fermi surface $\tilde{F}$ and chemical potential μ ,

$$\begin{aligned} \tilde{F} : \frac{1}{\sqrt{\alpha}}k_{\parallel}^2 + \sqrt{\alpha}k_{\perp}^2 &\leq k_F^2, \\ \mu &= E_{\mathbf{k}_i} + \frac{1}{2m}k_F^2, \end{aligned} \quad (17)$$

where $k_F^2 = \pi\delta$, $\alpha = \frac{m_{\parallel}}{m_{\perp}} > 1$, and $m = \sqrt{m_{\parallel}m_{\perp}}$. In these regions the band is strongly anisotropic, $m_{\parallel}/m_{\perp} \approx 5 - 7$. Spin wave approximation can be used to investigate the magnetic property of the cuprates. The condition for finite magnon damping coincides with the condition for the appearance of low-frequency magnons. Because the magnon damping is much larger than the real part of the magnon frequency, the low-frequency magnons are overdamped when they exist. The overdamping of the low-frequency magnons means that the occupation numbers of the respective magnon modes become indefinite. This contradicts the initial assumption of the spin wave approximation about small values of these numbers. Thus the condition for appearance of low-frequency magnons gives the critical concentration of holes, x_c to be about 2%, for $x > x_c$ long-range antiferromagnetic order is destroyed by doping[19].

Near the $\Gamma(0,0)$ point, the imaginary part of the self-energy for spin excitations is of the form[20],

$$\begin{aligned}\text{Im}\Pi_R(q, \omega < T) &= -\frac{Z_k^2 t^2 m k_F}{16\sqrt{2}\pi} \frac{\omega}{T}, \\ \text{Im}\Pi_R(q, \omega > T) &= -\frac{Z_k^2 t^2 m k_F}{8\sqrt{2}\pi},\end{aligned}\tag{18}$$

where $Z_k^2 = \frac{2(J/t)^2}{\pi}$. The polarizability has the same form as in the phenomenological marginal-Fermi-liquid theory proposed by Varma *et al*[21].

In the slave-boson formalism, the ground state of doped cuprates is uniform RVB state and shows FL-like behavior below the Bose condensation temperature. After taking into account the interaction among bosons and fermions, the gauge theory gives correct description of the linear-temperature dependence of resistivity and thermal conductivity. The mechanism of fermion-condensation and boson-saturation activated by the boundary terms of the t - J model and AF long-range order reciprocates the statistical property of the spin and charge carriers of the cuprates. Thus at low doping concentrations, the slave-boson formalism turned out to be slave-fermion-like and one may obtain AF correlation and small Fermi pockets centered around the $\left(\pm\frac{\pi}{2}, \pm\frac{\pi}{2}\right)$ points which evolve into the large Fermi surface at high doping concentrations.

ACKNOWLEDGMENTS

We would like to thank Y. Zhao for useful discussions.

References

- [1] See for example, D. M. Ginsberg (ed.), *Physical Properties of High Temperature Superconductors*, Vol. 3, (World Scientific, Singapore, 1992).
- [2] P. W. Anderson, Science **235** (1987) 1196; in *Frontiers and Borderlines in Many-Particle Physics*, edited by R. A. Brogli and J. R. Schrieffer (North-Holland, New York, 1988).

- [3] For the review, see for example,
P. A. Lee, in *High Temperature Superconductivity*, edited by K. S. Bedell *et al.*
(Addison-Wesley, New York, 1990);
Yu Lu, in *Recent Progress in Many-Body Theories*, edited by T. L. Ainsworth *et al.*
(Plenum, New York, 1992), Vol.3.
- [4] G. Baskaran, Z. Zou, and P. W. Anderson, Solid State Commun. **63** (1987) 973;
P. W. Anderson and Z. Zou, Phys. Rev. Lett. **60** (1988) 132;
A. Ruckenstein, P. Hirschfeld, and J. Appell, Phys. Rev. B **36** (1987) 57.
- [5] W. Marshall, Proc. R. Soc. London, Ser. A **232** (1955) 48;
S. D. Liang, B. Doucot, and P. W. Anderson, Phys. Rev. Lett. **61** (1988) 365.
- [6] B. Batlogg, in *High Temperature Superconductivity*, edited by Bedell *et al.*
(Addison-Wesley, New York, 1990).
- [7] D. Arovas and A. Auerbach, Phys. Rev. B **38** (1988) 316;
A. Auerbach and D. Arovas, Phys. Rev. Lett. **61** (1988) 617.
- [8] See, Z. X. Shen and D. S. Dessau, Phys. Reports **253** (1995) 1;
Z. X. Shen *et al.*, Science **267** (1995) 343.
- [9] B. Shraiman and E. Siggia, Phys. Rev. Lett. **62** (1989) 1564.
- [10] V. G. Drinfel'd, in *Proceedings of International Congress of Mathematicians*,
Berkeley 1986, edited by A. M. Gleason (American Mathematical Society, Providence, RI, 1987);
M. Jimbo, Lett. Math. Phys. **10** (1985) 63.
- [11] See, Z. Chang, Physics Reports **262** (1995) 137.
- [12] F. C. Alcaraz, M. N. Barber, M. T. Batchelor, R. J. Baxter, and G. R. Quispel,
J. Phys. A **20** (1987) 6397;
F. V. Pasquier and H. Saleur, Nucl. Phys. B **330** (1990) 523.

- [13] S. M. Fei and H. Yan, Commun. Theor. Phys. **19** (1993) 49.
- [14] For the review, see for example, P. A. Lee and N. Nagaosa, Phys. Rev. B **46** (1992) 5621.
- [15] A. J. Macfarlane, J. Phys. A **22** (1989) 4581;
 L. C. Biedenharn, J. Phys. A **22** (1989) L873;
 Z. Chang, H. Y. Guo, and H. Yan, Commun. Theor. Phys. **14** (1990) 475;
 X. C. Song, Commun. Theor. Phys. **16** (1991) 241.
- [16] C. L. Kane, P. A. Lee, and N. Read, Phys. Rev. B **39** (1989) 6880;
 S. Schmitt-Rink, C. M. Varma, and A. Ruckenstein, Phys. Rev. Lett. **60** (1988) 2793;
 Y. M. Li, D. N. Sheng, Z. B. Su, and L. Yu, Phys. Rev. B **45** (1992) 5428.
- [17] J. Igarashi and P. Fulde, Phys. Rev. B **45**, 12 357 (1992); *ibid.* **48** (1993) 998.
- [18] G. A. Thomas, *et al.*, Phys. Rev. Lett. **61** (1988) 1313.
- [19] A. Sherman and M. Schreiber, Phys. Rev. B **48** (1993) 7492;
 Z. Chang, Phys. Rev. B **53** (1996) 1171.
- [20] J. X. Li and C. D. Gong, Phys. Rev. B **51** (1995) 6343.
- [21] C. M. Varma, P. B. Littlewood, S. Schmitt-Rink, E. Abrahams, and A. E. Ruckenstein, Phys. Rev. Lett. **63** (1989) 1996.