BRST Quantization of the Proca Model based on the BFT and the BFV Formalism

Yong-Wan Kim†, Mu-In Park†, Young-Jai Park†, and Sean J. Yoon†,*

† Department of Physics and Basic Science Research Institute
Sogang University, C.P.O. Box 1142, Seoul 100-611, Korea

and

* LG Electronics Research Center, Seoul 137-140, Korea

ABSTRACT

The BRST quantization of the Abelian Proca model is performed using the Batalin-Fradkin-Tyutin and the Batalin-Fradkin-Vilkovisky formalism. First, the BFT Hamiltonian method is applied in order to systematically convert a second class constraint system of the model into an effectively first class one by introducing new fields. In finding the involutive Hamiltonian we adopt a new approach which is more simpler than the usual one. We also show that in our model the Dirac brackets of the phase space variables in the original second class constraint system are exactly the same as the Poisson brackets of the corresponding modified fields in the extended phase space due to the linear character of the constraints comparing the Dirac or Faddeev-Jackiw formalisms. Then, according to the BFV formalism we obtain that the desired resulting Lagrangian preserving BRST symmetry in the standard local gauge fixing procedure naturally includes the St"uckelberg scalar related to the explicit gauge symmetry breaking effect due to the presence of the mass term. We also analyze the nonstandard nonlocal gauge fixing procedure.

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1 Introduction

The Dirac method has been widely used in the Hamiltonian formalism\(^1\) to quantize the first and the second class constraint systems generally, which do and do not form a closed constraint algebra in Poisson brackets, respectively. However, since the resulting Dirac brackets are generally field-dependent and nonlocal, and have a serious ordering problem, the quantization is under unfavorable circumstances because of, essentially, the difficulty in finding the canonically conjugate pairs. On the other hand, the quantization of first class constraint systems established by Batalin, Fradkin, and Vilkovisky (BFV),\(^2,3\) which does not have the previously noted problems of the Dirac method from the start, has been well appreciated in a gauge invariant manner with preserving Becci-Rouet-Stora-Tyutin (BRST) symmetry.\(^4,5\) After their works, this procedure has been generalized to include the second class constraints by Batalin, Fradkin, and Tyutin (BFT)\(^6,7\) in the canonical formalism, and applied to various models\(^8\text{"--"}10\) obtaining the Wess-Zumino (WZ) actions.\(^11,12\)

Recently, Banerjee\(^13\) has applied the BFT Hamiltonian method\(^7\) to the second class constraint system of the Abelian Chern-Simons (CS) field theory,\(^14\text{"--"}16\) which yields the strongly involutive first class constraint algebra in an extended phase space by introducing new fields. As a result, he has obtained a new type of an Abelian WZ action, which cannot be obtained in the usual path-integral framework. Very recently, we have quantized several interesting models\(^17\) as well as the non-Abelian CS case, which yields the weakly involutive first class constraint system originating from the non-Abelian nature of the second class constraints of the system, by considering the generalized form of the BFT formalism.\(^18\) As shown in all these works, the nature of the second class constraint algebra originates from only the symplectic structure of the CS term, not due to the local gauge symmetry breaking. Banerjee’s and Ghosh\(^19\) have also considered the Abelian and non-Abelian (incompletely) Proca model,\(^20\) which have the explicit gauge-symmetry breaking term by extending the BFT approach to the case of rank-1 non-Abelian conversion compared to the Abelian (rank-0) conversion. As a result, the extra field in this approach has identified with the Stückelberg scalar.\(^21\) However, all these analysis do not carry out the covariant gauge fixing procedure.
preserving the BRST symmetry based on the BFV formalism hence completing the
BRST quantization procedure. Furthermore, up to now all above authors do not
explicitly treat the Dirac brackets in this BFT formalism although the general but
classical relation between the Dirac bracket and the Poisson bracket in the extended
phase space are formally reported for the case of Abelian conversion. 7

In the present paper, the BRST quantization of the Abelian Proca model 20 is
performed completely by using the usual BFT 7 and the BFV 2,3 formalism. In section
2, we will apply the usual BFT formalism 7 to the Abelian Proca model in order to
convert the second class constraint system into a first class one by introducing new
auxiliary fields. Here, we newly obtain the relation that the well-known Dirac brackets
between the phase space variables in our starting second class constraint system of
the Abelian Proca model are the same as the Poisson brackets of the corresponding
modified ones in the extended phase space without Φ → 0 limiting procedure of the
general formula of BFT 7 due to essentially the linear character of the constraint.
It is also compared with the Dirac 1 or Faddeev-Jackiw (FJ) symplectic formalism,
22 which is to be regarded as the improved version of the Dirac one. Furthermore,
we adopt a new approach, which is more simpler than the usual one, in finding the
involutive Hamiltonian by using these modified variables. In section 3, we will consider
the completion of the BRST quantization based on the BFV formalism. As a result we
show that by identifying a new auxiliary field with the Stückelberg scalar we naturally
derive the Stückelberg scalar term related to the explicit gauge symmetry breaking
mass term through a BRST invariant and local gauge fixing procedure according to
the BFV formalism. We also analyze the nonlocal gauge fixing procedure which has
been recently studied by several authors. 23 Our conclusions are given in section 4.

2 BFT Formalism

2.1 Proca Model and Constraints

Now, we first apply the usual BFT formalism which assumes the Abelian conversion
of the second class constraint of the original system 7 to the Abelian Proca model of
the massive photon in four dimensions, \(^{20}\) whose dynamics are given by

\[
S = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_{\mu} A^\mu \right],
\]  

(1)

where \(F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu},\) and \(g_{\mu\nu} = \text{diag}(+, -, -, -).\)

The canonical momenta of gauge fields are given by

\[
\pi_0 \equiv \frac{\delta S}{\delta \dot{A}_0} \approx 0,
\]

\[
\pi_i \equiv \frac{\delta S}{\delta \dot{A}_i} = F_{i0}
\]

(2)

with the Poisson algebra \(\{A^\mu(x), \pi_\nu(y)\} = \delta^\mu_\nu \delta(x - y).\) The weak equality ‘\(\approx\)’ means the equality is not applied before all involved calculations are finished. \(^{1}\) In contrast, the strong equality ‘\(\approx\)’ means the equality can be applied at all the steps of the calculations.

Then, \(\Omega_1 \equiv \pi_0 \approx 0\) is a primary constraint. \(^{1}\) The total Hamiltonian is

\[
H_T = H_c + \int d^3x u \Omega_1
\]

(3)

with the multiplier \(u\) and the canonical Hamiltonian

\[
H_c = \int d^3x \left[ \frac{1}{2} \pi_i^2 + \frac{1}{4} F_{ij} F^{ij} + \frac{1}{2} m^2 \left\{ (A^0)^2 + (A^i)^2 \right\} - A_0 \Omega_2 \right],
\]

(4)

where \(\Omega_2\) is the Gauss’ law constraint, which comes from the time evolution of \(\Omega_1\) with \(H_T,\) defined by

\[
\Omega_2 = \partial^i \pi_i + m^2 A^0 \approx 0.
\]

(5)

Note that the time evolution of the Gauss’ law constraint with \(H_T\) generates no more additional constraints but only determine the multiplier \(u \approx -\partial_i A^i.\) As a result, the full constraints of this model are \(\Omega_i\) \((i, j = 1, 2)\) which satisfy the second class constraint algebra as follows

\[
\Delta_{ij}(x, y) \equiv \{\Omega_i(x), \Omega_j(y)\} = -m^2 \epsilon_{ij} \delta^3(x - y),
\]

(6)

\[
\det \Delta_{ij}(x, y) \neq 0,
\]

where we denote \(x = (t, \mathbf{x})\) and three-space vector \(\mathbf{x} = (x^1, x^2, x^3)\) and \(\epsilon_{12} = -\epsilon_{21} = 1.\)
We now introduce new auxiliary fields $\Phi^i$ to convert the second class constraints $\Omega_i$ into first class ones in the extended phase space with the Poisson algebra

$$\{A^\mu (\text{or } \pi_\mu), \Phi^i\} = 0,$$

$$\{\Phi^i(x), \Phi^j(y)\} = \omega^{ij}(x, y) = -\omega^{ji}(y, x).$$  (7)

Here, the constancy, i.e., the field independence of $\omega^{ij}(x, y)$, is considered for simplicity.

According to the usual BFT method, \(^7\) the modified constraints $\tilde{\Omega}_i$ with the property

$$\{\tilde{\Omega}_i, \tilde{\Omega}_j\} = 0,$$  (8)

which is called the Abelian conversion, which is rank-0, of the second class constraint (6) are generally given by

$$\tilde{\Omega}_i(A^\mu, \pi_\mu; \Phi^j) = \Omega_i + \sum_{n=1}^{\infty} \tilde{\Omega}^{(n)}_i = 0; \quad \Omega_i^{(n)} \sim (\Phi^j)^n$$  (9)

satisfying the boundary conditions, $\tilde{\Omega}_i(A^\mu, \pi_\mu; 0) = \Omega_i$. Note that the modified constraints $\tilde{\Omega}_i$ become strongly zero by introducing the auxiliary fields $\Phi^i$, i.e., enlarging the phase space, while the original constraints $\Omega_i$ are weakly zero. As will be shown later, essentially due to this property, the result of the Dirac formalism can be easily read off from the BFT formalism. The first order correction terms in the infinite series \(^7\) are simply given by

$$\tilde{\Omega}_i^{(1)}(x) = \int d^3y X_{ij}(x, y) \Phi^j(y),$$  (10)

and the first class constraint algebra (8) of $\tilde{\Omega}_i$ requires the following relation

$$\Delta_{ij}(x, y) + \int d^3w \ d^3z \ X_{ik}(x, w) \omega^{kl}(w, z) X_{jl}(y, z) = 0.$$  (11)

However, as was emphasized in Refs. 13, 18, and 19, there is a natural arbitrariness in choosing the matrices $\omega^{ij}$ and $X_{ij}$ from Eqs. (7) and (10), which corresponds to canonical transformation in the extended phase space. \(^6,7\) Here we note that Eq. (11) can not be considered as the matrix multiplication exactly unless $X_{ji}(y, z)$ is the symmetric matrix, i.e., $X_{ji}(y, z) = X_{ij}(z, y)$ because of the form of the last two product of the matrices $\int d^3z \omega^{kl}(w, z) X_{jl}(y, z)$ in the right hand side of Eq. (11). Thus, using this
arbitrariness we can take the simple solutions without any loss of generality, which are compatible with Eqs. (7) and (11) as

\[ \omega_{ij}(x,y) = \epsilon_{ij}\delta^{3}(x-y), \]
\[ X_{ij}(x,y) = m\delta_{ij}\delta^{3}(x-y) \]

(12)
i.e., antisymmetric \( \omega_{ij}(x,y) \) and symmetric \( X_{ij}(x,y) \) such that Eq. (11) is the form of the matrix multiplication exactly \(^{10,13,17-19}\)

\[ \Delta_{ij}(x,y) + \int d^{3}\omega d^{3}z X_{ik}(x,\omega)\omega^{kl}(\omega,z)X_{lj}(z,y) = 0. \]

Note that \( X_{ij}(x,y) \) needs not be generally symmetric, while \( \omega_{ij}(x,y) \) is always antisymmetric by definition of Eq. (7). However, the symmetricity of \( X_{ij}(x,y) \) is, by experience, a powerful property for the solvability of (9) with finite iteration \(^{13,17-19}\) or with infinite regular iterations. \(^{24}\)

In our model with this proper choice, the modified constraints up to the first order iteration term

\[ \tilde{\Omega}_{i} = \Omega_{i} + \Omega_{i}^{(1)} \]
\[ = \Omega_{i} + m\Phi^{i} \]

(13)
strongly form a first class constraint algebra as follows

\[ \{\Omega_{i}(x) + \tilde{\Omega}_{i}^{(1)}(x), \Omega_{j}(y) + \tilde{\Omega}_{j}^{(1)}(y)\} = 0. \]

(14)
Then, the higher order iteration terms

\[ \tilde{\Omega}_{i}^{(n+1)} = -\frac{1}{n+2}\Phi^{l}\omega_{lk}X^{kj}B_{ji}^{(n)} \quad (n \geq 1) \]

(15)
with

\[ B_{ji}^{(n)} = \sum_{m=0}^{n} \{\tilde{\Omega}_{j}^{(n-m)},\tilde{\Omega}_{i}^{(m)}\}_{(A,\pi)} + \sum_{m=0}^{n-2} \{\tilde{\Omega}_{j}^{(n-m)},\tilde{\Omega}_{i}^{(m+2)}\}_{(\Phi)} \]

(16)
are found to be vanishing without explicit calculation. Here, \( \omega_{lk} \) and \( X^{kj} \) are the inverse of \( \omega^{lk} \) and \( X_{kj} \), and the Poisson brackets including the subscripts are defined
by
\[
\{A, B\}_{(q,p)} = \frac{\partial A}{\partial q} \frac{\partial B}{\partial p} - \frac{\partial A}{\partial p} \frac{\partial B}{\partial q},
\]
\[
\{A, B\}_{(\Phi)} = \sum_{i,j} \left[ \frac{\partial A}{\partial \Phi^i} \frac{\partial B}{\partial \Phi^j} - \frac{\partial A}{\partial \Phi^j} \frac{\partial B}{\partial \Phi^i} \right],
\]
(17)

where \((q, p)\) and \((\Phi^i, \Phi^j)\) are the conjugate pairs, respectively.

2.2 Physical Variables, First Class Hamiltonian, and Dirac Brackets

Now, corresponding to the original variables \(A^\mu\) and \(\pi_\mu\), the physical variables within the Abelian conversion in the extended phase space, \(\tilde{A}^\mu\) and \(\tilde{\pi}_\mu\), which are strongly involutive, i.e.,
\[
\{\tilde{\Omega}_i, \tilde{A}^\mu\} = 0, \quad \{\tilde{\Omega}_i, \tilde{\pi}_\mu\} = 0,
\]
(18)
can be generally found as
\[
\tilde{A}^\mu(A^\nu, \pi_\nu; \Phi^j) = A^\mu + \sum_{n=1}^{\infty} \tilde{A}^{\mu(n)}, \quad \tilde{A}^{\mu(n)} \sim (\Phi^j)^n;
\]
\[
\tilde{\pi}_\mu(A^\nu, \pi_\nu; \Phi^j) = \pi_\mu + \sum_{n=1}^{\infty} \tilde{\pi}^{\mu(n)}, \quad \tilde{\pi}^{\mu(n)} \sim (\Phi^j)^n
\]
(19)
satisfying the boundary conditions, \(\tilde{A}^\mu(A^\nu, \pi_\nu; 0) = A^\mu\) and \(\tilde{\pi}_\mu(A^\nu, \pi_\nu; 0) = \pi_\mu\). Here, the first order iteration terms are given by
\[
\tilde{A}^{\mu(1)} = -\Phi^j \omega_{jk} X^{kl} \{\Omega_l, A^\mu\}_{(A, \pi)} = -\Phi^j \omega_{jk} X^{kl} \{\Omega_l, A^\mu\}_{(A, \pi)} = \left(\frac{1}{m}\Phi^2, m\partial_1 \Phi^1\right),
\]
\[
\tilde{\pi}^{\mu(1)} = -\Phi^j \omega_{jk} X^{kl} \{\Omega_l, \pi_\mu\}_{(A, \pi)} = \left(m\Phi^1, 0\right).
\]
(20)

Furthermore, since the modified variables up to the first iterations, \(A^\mu + \tilde{A}^{\mu(1)}\) and \(\pi_\nu + \tilde{\pi}^{\mu(1)}\) are found to involutive, i.e., to satisfy Eq. (18), the higher order iteration
\[ \tilde{A}^{\mu(n+1)} = -\frac{1}{n+1} \Phi^j \omega^i_{jk} X^{kl} (A^{\mu})^{(n)}_l, \]
\[ \tilde{\pi}_{\mu(n+1)} = -\frac{1}{n+1} \Phi^j \omega^i_{jk} X^{kl} (H^{\mu})^{(n)}_l \]

with
\begin{align*}
(G^{\mu})^{(n)}_l &= \sum_{m=0}^{n} \{ \Omega^{(n-m)}_i, \tilde{A}^{\mu(m)}_l \}_{(A, \pi)} + \sum_{m=0}^{n-2} \{ \Omega^{(n-m)}_i, \tilde{A}^{\mu(m+2)}_l \}_{(\Phi)} + \{ \Omega^{(n+1)}_i, \tilde{A}^{\mu(1)}_l \}_{(\Phi)}, \\
(H^{\mu})^{(n)}_l &= \sum_{m=0}^{n} \{ \Omega^{(n-m)}_i, \tilde{\pi}^{(m)}_{\mu} \}_{(A, \pi)} + \sum_{m=0}^{n-2} \{ \Omega^{(n-m)}_i, \tilde{\pi}^{(m+2)}_{\mu} \}_{(\Phi)} + \{ \Omega^{(n+1)}_i, \tilde{\pi}^{(1)}_{\mu} \}_{(\Phi)} 
\end{align*}

are also found to be automatically vanishing. Hence, the physical variables in the extended phase space are finally found to be
\begin{align*}
\tilde{A}^{\mu} &= A^{\mu} + \tilde{A}^{\mu(1)} \\
&= (A^0 + \frac{1}{m} \Phi^2, A^i + \frac{1}{m} \partial^i \Phi^1), \\
\tilde{\pi}_{\mu} &= \pi_{\mu} + \tilde{\pi}_{\mu}^{(1)} \\
&= (\pi_0 + m \Phi^1, \pi_i) \\
&= (\tilde{\Omega}_1, \pi_i).
\end{align*}

Similar to the physical phase space variables \( \tilde{A}^{\mu} \) and \( \tilde{\pi}_{\mu} \), all other physical quantities, which correspond to the functions of \( A^{\mu} \) and \( \pi_{\mu} \), can be also found in principle by considering the solutions like as Eq. (19). However, it is expected that this procedure of finding the physical quantities may not be simple depending on the complexity of the functions.

In this paper, we consider a new approach using the property
\[ \tilde{K}(A^{\mu}, \pi_{\mu}; \Phi^i) = K(\tilde{A}^{\mu}, \tilde{\pi}_{\mu}) \]

for the arbitrary function or functional \( K \) defined on the original phase space variables unless \( K \) has the time derivatives: the following relation
\[ \{ K(\tilde{A}^{\mu}, \tilde{\pi}_{\mu}), \tilde{\Omega}_i \} = 0 \]
is satisfied for any function \( K \) not having the time derivatives because \( \tilde{A}^\mu \) and \( \tilde{\pi}_\mu \) and their spatial derivatives already commute with \( \tilde{\Omega}_i \) at equal times by definition. On the other hand, since the solution \( K \) of Eq. (24) is unique up to the power of the first class constraints \( \tilde{\Omega}_i \), \( K(\tilde{A}^\mu, \tilde{\pi}_\mu) \) can be identified with \( \tilde{K}(A^\mu, \pi_\mu; \Phi) \) modulus the power of the first class constraints \( \tilde{\Omega}_i \). However, note that this property is not satisfied when the time derivatives exist.

Using this elegant property we can directly obtain the desired first class Hamiltonian \( \tilde{H}_T \) corresponding to the total Hamiltonian \( H_T \) of Eq. (3) as follows

\[
\tilde{H}_T(A^\mu, \pi_\nu; \Phi^i) = H_T(\tilde{A}^\mu, \tilde{\pi}_\nu) + \int d^3x \left[ \frac{1}{2}(\partial_i \Phi^1)^2 + \frac{1}{2}(\Phi^2)^2 + \frac{1}{m} \partial_i \partial_1 \tilde{\Omega}_1 - \frac{1}{m} \Phi^2 \tilde{\Omega}_2 \right].
\]

(26)

On the other hand, since the first class Hamiltonian \( \tilde{H}_c \) corresponding to the canonical Hamiltonian \( H_c \) of Eq. (4) can be similarly obtained as follows

\[
\tilde{H}_c(A^\mu, \pi_\nu; \Phi^i) = H_c(\tilde{A}^\mu, \tilde{\pi}_\nu) + \int d^3x \left[ \frac{1}{2}(\partial_i \Phi^1)^2 + \frac{1}{2}(\Phi^2)^2 + m \Phi^1 \partial_i A_i - \frac{1}{m} \Phi^2 \tilde{\Omega}_2 \right],
\]

(27)

Eq. (26) can be re-expressed as follows

\[
\tilde{H}_T(A^\mu, \pi_\nu; \Phi^i) = \tilde{H}_c(A^\mu, \pi_\nu; \Phi^i) - \int d^3x (\partial_i \tilde{A}^i) \tilde{\Omega}_1
\]

(28)

as it should be according to Eq. (3).

This is the same result as the usual approach (See the Appendix A) and, by construction, both \( \tilde{H}_T \) and \( \tilde{H}_c \) are automatically strongly involutive,

\[
\{ \tilde{\Omega}_i, \tilde{H}_T \} = 0, \quad \{ \tilde{\Omega}_i, \tilde{H}_c \} = 0.
\]

(29)

Note that all our constraints have already this property, i.e., \( \tilde{\Omega}_i(A^\mu, \pi_\mu; \Phi) = \Omega_i(\tilde{A}^\mu, \tilde{\pi}_\mu) \).

In this way, the second class constraints system \( \Omega_i(A^\mu, \pi_\mu; \Phi) \approx 0 \) is converted into the first class constraints one \( \tilde{\Omega}_i(A^\mu, \pi_\mu; \Phi) = 0 \) with the boundary conditions \( \tilde{\Omega}_i|_{\Phi=0} = \Omega_i \).
On the other hand, in the Dirac formalism one can make the second class constraint system \( \Omega_i \approx 0 \) into the first class constraint one \( \Omega_i(A_\mu, \pi^\mu) = 0 \) only by deforming the phase space \((A_\mu, \pi^\mu)\) without introducing any new fields. Hence, it seems that these two formalisms are drastically different ones. However, remarkably the Dirac formalism can be easily read off from the usual BFT-formalism by noting that the Poisson bracket in the extended phase space with \( \Phi \to 0 \) limit becomes

\[
\{ \tilde{A}, \tilde{B} \}_{\Phi=0} = \{ A, B \} - \{ A, \Omega_k \} \Delta^{kk'} \{ \Omega_{k'}, B \} \\
= \{ A, B \}_D
\]

(30)

where \( \Delta^{kk'} = -X^{lk} \omega_{ll'} X^{l'k'} \) is the inverse of \( \Delta_{kk'} \) in Eq. (6). About this remarkable relation, we note that this is essentially due to the Abelian conversion method of the original second class constraint: In this case the Poisson brackets between the constraints and the other things in the extended phase space are already strongly zero

\[
\{ \tilde{\Omega}_i, \tilde{A} \} = 0, \\
\{ \tilde{\Omega}_i, \tilde{\Omega}_j \} = 0,
\]

(31)

which resembles the property of the Dirac bracket in the non-extended phase space

\[
\{ \Omega_i, A \}_D = 0, \\
\{ \Omega_i, \Omega_j \}_D = 0,
\]

(32)

such that

\[
\{ \tilde{\Omega}_i, \tilde{A} \}_{\Phi=0} \equiv \{ \Omega_i, A \}^* = 0, \\
\{ \tilde{\Omega}_i, \tilde{\Omega}_j \}_{\Phi=0} \equiv \{ \Omega_i, \Omega_j \}^* = 0
\]

(33)

are satisfied for some bracket in the non-extended phase space \( \{ , , \}^* \). However, due to the uniqueness of the Dirac bracket it is natural to expect the previous result (29) is satisfied, i.e.,

\[
\{ , , \}^* = \{ , , \}_D
\]

(34)
without explicit manipulation. Moreover we add that, due to similar reason, some non-Abelian generalization of the Abelian conversion as

\[
\{\tilde{\Omega}_i, \tilde{A}\} = \alpha_{ij} \Phi^j + \alpha_{ijk} \Phi^j \Phi^k + \cdots, \\
\{\tilde{\Omega}_i, \tilde{\Omega}_j\} = \beta_{ijk} \Phi^k + \beta_{ijkl} \Phi^k \Phi^l + \cdots
\] (35)

also gives the same result (30) with the functions \(\alpha_{ij}, \alpha_{ijk}, \beta_{ijk}, \ldots\) of the original phase variables \(A_\mu, \pi^\nu\). As an specific example, let us consider the brackets between the phase space variables in Eq. (23). The results are as follows

\[
\begin{align*}
\{\tilde{A}^0(x), \tilde{A}^j(y)\}|_{\Phi=0} &= \{\tilde{A}^0(x), \tilde{A}^j(y)\} = \frac{1}{m^2} \partial^j_x \delta(x - y), \\
\{\tilde{A}^0(x), \tilde{A}^0(y)\}|_{\Phi=0} &= \{\tilde{A}^0(x), \tilde{A}^0(y)\} = 0, \\
\{\tilde{A}^i(x), \tilde{A}^k(y)\}|_{\Phi=0} &= \{\tilde{A}^i(x), \tilde{A}^k(y)\} = 0, \\
\{\tilde{\Pi}_\mu(x), \tilde{\Pi}_\nu(y)\}|_{\Phi=0} &= \{\tilde{\Pi}_\mu(x), \tilde{\Pi}_\nu(y)\} = 0, \\
\{\tilde{A}^i(x), \tilde{\Pi}_j(y)\}|_{\Phi=0} &= \{\tilde{A}^i(x), \tilde{\Pi}_j(y)\} = \delta_{ij} \delta(x - y), \\
\{\tilde{A}^0(x), \tilde{\Pi}_\nu(y)\}|_{\Phi=0} &= \{\tilde{A}^0(x), \tilde{\Pi}_\nu(y)\} = 0, \\
\{\tilde{A}^i(x), \tilde{\Pi}_0(y)\}|_{\Phi=0} &= \{\tilde{A}^i(x), \tilde{\Pi}_0(y)\} = 0
\end{align*}
\] (36)

due to the linear correction (i.e., only the first order correction) of the modified fields \(\tilde{A}^\mu\) and \(\tilde{\pi}_\mu\), which are the same as the usual Dirac brackets. \(^{19}\) Hence, in the case of the phase space variables of the model the well-known Dirac brackets of the fields are exactly the Poisson brackets of the corresponding modified fields in the extended phase space contrast to the general formula (30). Note that the FJ symplectic formalism, \(^{22}\) which is the improved version of the Dirac method, also gives the same result (See Appendix B for this matter).

Now, since in the Hamiltonian formalism the first class constraint system without the CS term \(^{13,18}\) indicates the presence of a local symmetry, this completes the operatorial conversion of the original second class system with the Hamiltonian \(H_T\) and the constraints \(\Omega_i\) into first class one with the Hamiltonian \(\tilde{H}_T\) and the constraints \(\tilde{\Omega}_i\). From Eqs. (13) and (28), one can easily see that the original second class constraint system is converted into the effectively first class one if one introduces two fields, which are conjugated with each other in the extended phase space. Note that for the Proca
case the origin of the second class constraint is due to the explicit gauge symmetry breaking term in the action (1).

### 2.3 Corresponding First Class Lagrangian

Next, we consider the partition function of the model in order to present the Lagrangian corresponding to $\tilde{H}_T$ in the canonical Hamiltonian formalism. However, the result is the same with $\tilde{H}_c$. As a result, we will unravel the correspondence of the Hamiltonian approach with the well-known Stückelberg’s formalism. First, let us identify the new variables $\Phi_i$ as a canonically conjugate pair $(\rho, \pi_\rho)$ in the Hamiltonian formalism,

$$(\Phi^i) \rightarrow (m\rho, \frac{1}{m}\pi_\rho),$$

satisfying Eqs. (7) and (12). Then, the starting phase space partition function is given by the Faddeev formula as follows

$$Z = \int \mathcal{D}A^\mu \mathcal{D}\pi_\mu \mathcal{D}\rho \mathcal{D}\pi_\rho \prod_{i,j=1}^{2} \delta(\tilde{\Omega}_i)\delta(\Gamma_j)\det \{\tilde{\Omega}_i, \Gamma_j\} | e^{iS},$$

where

$$S = \int d^4x \left( \pi_\mu \dot{A}^\mu + \pi_\rho \dot{\rho} - \tilde{H}_T \right),$$

with the Hamiltonian density $\tilde{H}_T$ corresponding to the Hamiltonian $\tilde{H}_T$ of Eq. (25), which is now expressed in terms of $(\rho, \pi_\rho)$ instead of $\Phi^i$. Note that the gauge fixing conditions $\Gamma_i$ are chosen so that the determinant occurring in the functional measure is nonvanishing. Moreover, $\Gamma_i$ may be assumed to be independent of the momenta so that these are considered as the Faddeev-Popov type gauge conditions.

Before performing the momentum integrations to obtain the partition function in the configuration space, it seems appropriate to comment on the involutive Hamiltonian. If we directly use the above Hamiltonian following the previous analysis done by Banerjee et al., we will finally obtain the non-local action corresponding to this Hamiltonian due to the existence of $(\partial^2 \pi_i)^2$–term in the action when we carry out the functional integration over $\pi_\rho$ later. Furthermore, if we use the above Hamiltonian, we can not also naturally generate the first class Gauss' law constraint $\tilde{\Omega}_2$ from the time
evolution of the primary constraint $\tilde{\Omega}_1$, i.e., $\{\tilde{\Omega}_1, \tilde{H_T}\} = 0$. Therefore, in order to avoid these unwanted situations, we use the equivalent first class Hamiltonian without any loss of generality, which differs from the involutive Hamiltonian (26) by adding a term proportional to the first class constraint $\tilde{\Omega}_2$ as follows

$$\tilde{H}_T' = \tilde{H}_T + \int d^3x m^2 \tilde{\Omega}_2.$$  

(40)

Then, we have the desired first constraint system such that

$$\{\tilde{\Omega}_1, \tilde{H}_T'\} = \tilde{\Omega}_2,$$

$$\{\tilde{\Omega}_2, \tilde{H}_T'\} = 0.$$  

(41)

Note that when we operate this modified Hamiltonian on physical states, the difference is trivial because such states are annihilated by the first class constraints. Similarly, the equations of motion for gauge invariant variables will also be unaffected by this difference since $\tilde{\Omega}_2$ can be regarded as the generator of the gauge transformations.

Now, we consider the following effective phase space partition function

$$Z = \int \mathcal{D}\pi_i \mathcal{D}A^\mu \mathcal{D}\pi_\rho \mathcal{D}_\rho \prod_{i,j=1}^2 \delta(\tilde{\Omega}_i) \delta(\Gamma_j) \det | \tilde{\Omega}_i, \Gamma_j | e^{iS'},$$

$$S' = \int d^4x (\pi^i_\mu A^\mu_i + \pi_\mu \dot{\rho} - \tilde{\mathcal{H}}_T').$$  

(42)

The $\pi_0$ integral is performed trivially by exploiting the delta function $\delta(\tilde{\Omega}_1) = \delta(\pi_0 + m^2 \rho)$ in Eq. (42). On the other hand, the other delta function $\delta(\tilde{\Omega}_2) = \delta(\partial^i \pi_i + m^2 A^0 + \pi_\rho)$ can be expressed by its Fourier transform with Fourier variable $\xi$ as follows

$$\delta(\tilde{\Omega}_2) = \int \mathcal{D}\xi e^{-i \int d^4x \xi \tilde{\Omega}_2}.$$  

(43)

Making a change of variable $A^0 \rightarrow A^0 + \xi$, we obtain the action

$$S = \int d^4x [\pi_i \dot{A}_i - m^2 \rho(\dot{A}_0 + \xi) + \pi_\rho \dot{\rho} - \frac{1}{2} \pi_i^2 - \frac{1}{4} F_{ij} F^{ij} + \frac{1}{2} m^2 (A^0)^2 + \frac{1}{2} m^2 A_i A^i + A^0 \partial^i \pi_i - m^2 \partial_i A^i \rho - \frac{1}{2} m^2 \partial_\rho \partial^i \rho - \xi \pi_\rho - \frac{1}{2} m^2 \xi^2],$$  

(44)

where the corresponding measure is given by

$$[\mathcal{D}_\mu] = \mathcal{D}\xi \mathcal{D}\pi_i \mathcal{D}A^\mu \mathcal{D}\rho \mathcal{D}_\rho \prod_j \delta[\Gamma_j(A^0 + \xi, A^i, \pi_i, \rho)] \det | \tilde{\Omega}_i, \Gamma_j |.$$  

(45)
Performing the Gaussian integral over \( \pi_i \), this yields the intermediate action as follows

\[
S_u = \int d^4x \left[ \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu 
+ \pi_\rho (\dot{\rho} - \xi - \frac{1}{2m^2} \pi_\rho) - m^2 \rho(\dot{A}^0 + \dot{\xi}) - m^2 \partial_i A^i + \frac{1}{2} m^2 \partial_i \rho \partial^i \rho - \frac{1}{2} m^2 \xi^2 \right].
\]

(46)

To realize the Stückelberg term through the BFT analysis, we choose the Faddeev-Popov-like gauges \(^{10,13,17,27,28}\), which do not involve the momenta. After the Gaussian integration over \( \pi_\rho \), we finally obtain the well-known action up to the total divergence by identifying the extra field \( \rho \) with the Stückelberg scalar as follows

\[
S = \int d^4x \left[ \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 (A_\mu + \partial_\mu \rho)^2 \right],
\]

(47)

which is invariant under the gauge transformations as \( \delta A_\mu = \partial_\mu \Lambda \) and \( \delta \rho = -\Lambda \). As expected, the Stückelberg scalar \( \rho \) is introduced in the mass term.

It seems to appropriate to comment on the original unitary gauge fixing. If we choose the gauge as follows

\[
\Gamma_i = (\rho, \pi_\rho),
\]

(48)

we get

\[
\{ \Omega_i(x), \Gamma_j(y) \} = \epsilon_{ij}\delta^3(x - y).
\]

(49)

Then, integrating over the variables \( \rho \) and \( \pi_\rho \) we reproduce the original partition function as follows

\[
Z = \int \mathcal{D}A_\mu e^i \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 A_\mu A^\mu \right].
\]

(50)

The physical meaning of this result is that the original action can be regarded as a gauge-fixed version of the first constraint system (18) and (40). Note that this gauge fixing is consistent because when we take the gauge fixing condition \( \rho \approx 0 \), the condition \( \pi_\rho \approx 0 \) is naturally generated from the time evolution of \( \rho \), i.e., \( \dot{\rho} = \{\rho, H_u\} = -\frac{1}{m^2} \pi_\rho \approx 0 \), where the Hamiltonian \( H_u \) corresponds to the intermediate action \( S_u \).

### 3 BFV-BRST Gauge Fixing
3.1 Basic Structure

In this subsection, we briefly recapitulate the BFV formalism\(^2,3\) which is applicable for general theories with the first-class constraints. For simplicity, this formalism is restricted to a finite number of phase space variables. This makes the discussion simpler and conclusions more apparent.

First of all, consider a phase space of the bosonic canonical variables \(q^i, p_i\) \((i = 1, 2, \cdots, n)\) in terms of which canonical Hamiltonian \(H_c(q^i, p_i)\) and constraints \(\Omega_a(q^i, p_i) \approx 0\) \((a = 1, 2, \cdots, m)\), which being bosonic also, are given. We assume that the constraints satisfy the following constraint algebra \(^2,3,9\)

\[
\{\Omega_a, \Omega_b\} = U^c_{ab} \Omega_c, \\
\{H_c, \Omega_a\} = V^b_a \Omega_b, 
\]

where the structure coefficients \(U^c_{ab}\) and \(V^b_a\) are functions of the canonical variables. We also assume that the constraints are irreducible, which means that there locally exists an invertible change of the variables such that \(\Omega_a\) can be identified with the \(m\)-unphysical momenta.

In order to single out the physical variables, we can introduce the additional bosonic conditions \(\Gamma^a(q^i, p_i) \approx 0\) with \(\det |\{\Gamma^a, \Omega_b\}| \neq 0\) at least in the vicinity of the constraint surface \(\Gamma^a \approx 0\) and \(\Omega_a \approx 0\). Then, \(\Gamma^a\) play the roles of gauge-fixing functions. That is to say, from the condition of time stability of the constraints, a family of phase space trajectories is possible. By selecting one of these trajectories through the conditions of \(\Gamma^a \approx 0\), we can get the \(2(n - m)\) dimensional physical sub-phase space denoted by \(q^*, p^*\). And then, \(\Gamma^a(q^i, p_i)\) can be identified with the \(m\)-unphysical coordinates.

The described dynamical system with the partition function

\[
\mathcal{Z} = \int[dq^i dp_i] \delta(\Omega_a) \delta(\Gamma^b) \det |\{\Gamma^b, \Omega_a\}| e^{i \int dx (p \dot{q} - H_c)}
\]

(52)
is then completely equivalent to the effective quantum theory with the following partition function

\[
\mathcal{Z}_{eq.} = \int[dq^* dp^*] e^{i \int dx [p^* \dot{q}^* - H_{phys}(q^*, p^*)]}
\]

(53)

which only depending on the canonical variables \(q^*, p^*\) of the physical sub-phase space,
And the constraints $\Omega_a \approx 0$ and $\Gamma^a \approx 0$ together with the Hamilton equations may be obtained from a action

$$S = \int dt \left( p_i \dot{q}^i - H - \lambda^a \Omega_a + \pi_a \Gamma^a \right), \quad (54)$$

where $\lambda^a$ and $\pi_a$ are the bosonic Lagrange multiplier fields canonically conjugated to each other, obeying the Poisson bracket relations

$$\{\lambda^a, \pi_a\} = \delta^a_b. \quad (55)$$

Note that the gauge-fixing conditions contain $\lambda^a$ in the following general form

$$\Gamma^a = \dot{\lambda}^a + \chi^a(q^i, p_i, \lambda), \quad (56)$$

where $\chi^a$ are arbitrary functions. And we can see that the Lagrange multiplier $\lambda^a$ become dynamically active, and $\pi_a$ serve as their conjugate momenta. This consideration naturally leads to the canonical formalism in an extended phase space.

In order to make the equivalence to the initial theory with the constraints in the reduced phase space, we may introduce two sets of canonically conjugate, fermionic ghost coordinates and momenta $C^a, \overline{P}_a$ and $P^a, C_a$ such that

$$\{C^a, \overline{P}_b\} = \{P^a, C_b\} = \delta^a_b \quad (57)$$

with the super-Poisson bracket

$$\{A, B\} = \frac{\delta A}{\delta q} \frac{\delta B}{\delta p} _r - (-1)^{\eta A \eta B} \frac{\delta B}{\delta q} \frac{\delta A}{\delta p} _l, \quad (59)$$

where $\eta_A$ denotes the number of fermions called ghost number in $A$, and subscript "r" and "l" right and left derivatives.

The quantum theory is now defined by the extended phase space functional integral

$$Z_\Psi = \int [dq^i dp_i] [d\lambda^a d\pi_a] [dC^a d\overline{P}_a] [dP^a dC_a] e^{iS_\Psi}, \quad (58)$$

where the action is

$$S_\Psi = \int dt \left\{ p_i \dot{q}^i + \pi_a \dot{\lambda}^a + \overline{P}_a \dot{C}^a + C_a \dot{P}^a - H_m + \{Q, \Psi\} \right\}. \quad (59)$$
Here, the BRST-charge $Q$ and the fermionic gauge-fixing function $\Psi$ are defined by

$$\begin{align*}
Q &= C^a \Omega_a - \frac{1}{2} C^b C^c U_{cb} \bar{\mathcal{P}}_a + \mathcal{P}^a \pi_a , \\
\Psi &= \bar{\mathcal{C}}_a \chi^a + \mathcal{P}_a \lambda^a ,
\end{align*}$$

respectively. $H_m$ is the BRST invariant Hamiltonian, called the minimal Hamiltonian,

$$H_m = H_c + C^a V_b^b \mathcal{P}_a .$$

The measure in $Z_\Psi$ is the Liouville measure on the covariant phase space. Furthermore, if we choose the fermionic gauge-fixing function $\Psi$ properly, we can obtain manifestly covariant expression. And the equivalence of the dimensionality $2n + 6m$ in the extended phase space, including the canonical ghost variables, to the original dimensionality $2n - 2m$ in the reduced phase space can be seen by identifying the ghost variables with the negative-dimensional canonical degree of freedom, which is suggested by the Parisi-Sourlas’ original work related with the superrotation Osp(1,1|2) in the extended phase space.  

In order to derive the BRST gauge-fixed covariant action for the Abelian Proca theory, according to the above BFV formalism in the extended phase space, let us introduce the ghosts and anti-ghosts together with auxiliary fields as follows

$$(\mathcal{C}^i, \bar{\mathcal{P}}_i), (\mathcal{P}^i, \bar{\mathcal{C}}_i), (N^i, B_i) ,$$

where $i = 1, 2$. The nilpotent BRST-charge $Q$, the fermionic gauge-fixing function $\Psi$ and the minimal Hamiltonian $H_m$ in our case are

$$\begin{align*}
Q &= \int dx \left[ C^i \bar{\Omega}_i + \mathcal{P}^i B_i \right] , \\
\Psi &= \int dx \left[ \bar{\mathcal{C}}_i \chi^i + \mathcal{P}_i N^i \right] , \\
H_m &= \bar{H}'_T - \int dx \left[ \mathcal{P}_2 C^1 \right] ,
\end{align*}$$

where $\chi^1 = A^0$, $\chi^2 = \partial_i A^i + \frac{\alpha}{2} B_2$, and $\alpha$ is an arbitrary parameter.

The BRST-charge $Q$, the fermionic gauge-fixing function $\Psi$, and the minimal Hamiltonian $H_m$ satisfy the following relations,

$$\begin{align*}
\{ Q, H_m \} &= 0 , \\
Q^2 &= \{ Q, Q \} = 0 , \\
\{ \{ \Psi, Q \}, Q \} &= 0 ,
\end{align*}$$

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which being the conditions of physical subspace after the operator quantization

$$\left[ \hat{A}, \hat{B} \right] = i\hbar \{ A, B \}$$  \hspace{1cm} (65)$$

for the quantum operators \( \hat{A} \) and \( \hat{B} \) corresponding to the classical functions \( A \) and \( B \) when there is no operator ordering problem.

The effective action is

$$S_{\text{eff}} = \int d^4x \left[ \pi_0 \dot{A}^0 + \pi_i \dot{A}^i + \pi_\rho \dot{\rho} + B_2 \dot{N}^2 + \mathcal{P}_i \dot{C}_i + \mathcal{C}_2 \dot{\mathcal{P}}^2 \right] - H_{\text{total}}$$  \hspace{1cm} (66)$$

where \( H_{\text{total}} = H_m + \{ Q, \Psi \} \), and also \( \int d^4x (B_1 \dot{N}^1 + \mathcal{C}_1 \dot{\mathcal{P}}^1) = \{ Q, \int d^4x \mathcal{C} \dot{N}^1 \} \) terms are suppressed by replacing \( \chi^1 \) with \( \chi^1 + \dot{N}^1 \) just like the cases in Refs. 8, 9.

### 3.2 Local Effective Action

Now, in order to derive the covariant effective action we first perform the path integration over the fields \( B_1, N^1, \mathcal{C}_1, \mathcal{P}_1, \mathcal{P}_1, C^1, A^0, \) and \( \pi_0 \) by using of the Gaussian integration. Then, we obtain

$$S_{\text{eff}} \equiv \int d^4x \left[ \pi_0 \dot{A}^0 + \pi_i \dot{A}^i + \pi_\rho \dot{\rho} + B_2 \dot{N}^2 + \mathcal{P}_i \dot{C}_i + \mathcal{C}_2 \dot{\mathcal{P}}^2 \right] - H_{\text{total}}$$  \hspace{1cm} (67)$$

with \( N^2 \equiv N, B_2 \equiv B, \mathcal{C}_2 \equiv \mathcal{C}, C^2 \equiv \mathcal{C}, \mathcal{P}_2 \equiv \mathcal{P}, \) and \( \mathcal{P}^2 \equiv \mathcal{P} \). Using the variations over \( \pi^i, \pi_\rho, \mathcal{P} \) and \( \mathcal{P} \), we obtain the following relations

\[
\begin{align*}
\pi_i &= \dot{A}^i - \partial_i N, \\
\pi_\rho &= m^2 (\dot{\rho} - N), \\
\mathcal{P} &= -\mathcal{C}, \quad \mathcal{P} = \dot{\mathcal{C}},
\end{align*}
\]  \hspace{1cm} (68)$$

and identifying with \( N = -A^0 \), we get the usual local form of the covariant effective action

$$S_{\text{eff}} = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 (A_\mu + \partial_\mu \rho)^2 + A^\mu \partial_\mu B - \frac{1}{2} \alpha (B)^2 - \partial_\mu \mathcal{C} \partial^\mu \mathcal{C} \right],$$  \hspace{1cm} (69)$$

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which is invariant under the standard BRST transformation

\[ \delta_B A_\mu = -\lambda \partial_\mu C, \quad \delta_B \rho = \lambda C, \]
\[ \delta_B C = 0, \quad \delta_B \bar{C} = -\lambda B, \quad \delta_B B = 0, \quad (70) \]

which is local and covariant one. This completes the procedure of BRST invariant (here standard) gauge fixing with the local action according to the BFV formalism.

Therefore, we see that the auxiliary BF field \( \rho \) is exactly the well-known Stückelberg scalar in Eq. (69).

### 3.3 Nonlocal Effective Action

Although in the previous subsection we have performed the integration over \( P \) and \( \bar{P} \) but not over \( C \) and \( \bar{C} \), it is not impossible to consider the opposite procedure, i.e., the integration over \( \bar{C} \) and \( C \) but not over \( \mathcal{P} \) and \( \bar{\mathcal{P}} \) which is dual to the previous integration. To this end, we consider the BFV formalism in the previous subsection up to the point where the integration over the momentum \( \pi_\rho \) was performed and the following effective action was obtained:

\[
S_{\text{eff}} = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 (A_\mu + \partial_\mu \rho)^2 - A^\mu \partial_\mu B + \frac{1}{2} \alpha (B)^2 - \partial_i \bar{C} \partial^i C + \bar{\mathcal{P}} \dot{C} + \bar{C} \dot{\mathcal{P}} - \bar{\mathcal{P}} \mathcal{P} \right]. \quad (71)
\]

This action is invariant under the BRST transformation which have the form

\[
\begin{align*}
\delta_B A_0 &= -\lambda \mathcal{P}, \quad \delta_B A_i = -\lambda \partial_i C, \quad \delta_B \rho = \lambda C, \\
\delta_B C &= 0, \quad \delta_B \bar{C} = -\lambda B, \quad \delta_B B = 0, \\
\delta_B \mathcal{P} &= 0, \quad \delta_B \bar{\mathcal{P}} = -\lambda (-\partial_i F^{0i} + m^2 (\dot{\rho} + A^0)). \quad (72)
\end{align*}
\]

Now, let us perform the integration over \( C, \bar{C} \) instead of their conjugated ones \( \mathcal{P}, \bar{\mathcal{P}} \). First, performing the integration over the ghost field \( C \), we get the following delta function

\[
\delta(\partial_i \partial^i C - \dot{\mathcal{P}}) = \det(\partial_i \partial^i) \delta(C - \frac{1}{\partial_i \partial^i} \dot{\mathcal{P}}). \quad (73)
\]
Next, performing the integration over $\overline{C}$, we get the unusual non-local form of the effective action

$$S_{\text{eff}} = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 (A_\mu + \partial_\mu \rho)^2 - A^\mu \partial_\mu B + \frac{1}{2} \alpha (B)^2 + \overline{\mathcal{P}} \frac{1}{\partial_i \partial^i} \dot{\mathcal{P}} - \mathcal{P} \mathcal{P} \right]$$  \hspace{1cm} (74)

which is non-covariant. Notice that the appearance of the nonlocal term in the ghost action was a result of the unusual integration. But, we may find that this form is also obtained by the change of variables

$$C \rightarrow \frac{1}{\partial_i \partial^i} \mathcal{P}, \quad \overline{C} \rightarrow \mathcal{P}$$  \hspace{1cm} (75)

in the Eq. (69). Under these replacements, we have the nonlocal BRST charge $Q'$ given by

$$Q' = \int d^3x \left[ B\overline{C} + (-\partial_i F^{0i} + m^2 (\dot{\rho} + A^0)) \frac{1}{\partial_i \partial^i} \mathcal{P} \right].$$  \hspace{1cm} (76)

Then, the effective action is invariant under the following nonstandard BRST transformation as

$$\delta'_{B} A_\mu = -\lambda \partial_\mu \left( \frac{1}{\partial_i \partial^i} \mathcal{P} \right), \quad \delta'_{B} \rho = \lambda \frac{1}{\partial_i \partial^i} \mathcal{P},$$

$$\delta'_{B} \overline{\mathcal{P}} = 0, \quad \delta'_{B} \mathcal{P} = -\lambda B, \quad \delta'_{B} B = 0,$$  \hspace{1cm} (77)

which is non-local and non-covariant and hence can be categorized as the symmetry recently proposed in QED by Lavelle and McMullan. Moreover, this nonlocal BRST symmetry yields a conserved current through the Noether’s theorem as follows

$$J'_{B\mu} = F_{\mu\nu} \partial^\nu \frac{1}{\partial_i \partial^i} \mathcal{P} + m^2 (A_\mu + \partial_\mu \rho) \frac{1}{\partial_i \partial^i} \overline{\mathcal{P}} + B \partial_\mu \frac{1}{\partial_i \partial^i} \mathcal{P}.$$  \hspace{1cm} (78)

This completes the procedure of the BRST invariant (here nonstandard) gauge fixing with the non-local action according to BFV formalism. As a result, we have recognized that these nonlocal symmetry and the conserved current are nothing but the original local theory performing the change of variable (75).

### 4 Conclusion

In conclusion, we have applied the BFT and the BFV method to covariantly quantize the second class constraint system of the Abelian Proca model without spoiling
the unitarity. First, by applying the BFT method, we have systematically converted the second class system of the model into the effectively first class one in the extended phase space. We have shown the relation that, due to the linear character of the constraint, the Dirac brackets between the phase space variables in the original second class system are exactly the Poisson brackets of the corresponding modified ones in the extended phase space without \( \Phi \rightarrow 0 \) limiting procedure of the general formula of BFT by comparing the Dirac or Faddeev-Jackiw symplectic formalism.

Furthermore, we have noted that, like as this relation, the general relation (30) of the Dirac brackets in the non-extended phase space and the Poisson brackets in the extended phase space is essentially due to the Abelian conversion (31) of the second class constraint into the first class one and we have added also that some more general conversion method (35) may be considered without spoiling this nice relation.

Moreover, we have adopted a new approach, which is more simpler than the usual one, in finding the involutive Hamiltonian by using these modified variables according to the important property (24). Now, with this first class constraint we have applied the BFV method to covariantly quantize the Proca model without spoiling the unitarity. By identifying a new auxiliary field with the St"uckelberg scalar we have naturally derived the St"uckelberg scalar term related to the explicit gauge symmetry breaking mass term through the usual local gauge fixing procedure with the standard BRST symmetry according to the BFV formalism. We have also analyzed the nonlocal gauge fixing procedure with the nonstandard BRST symmetry which has been recently studied by several authors.

As final remarks, we first note that although we have successfully applied the rather simple Abelian case, it is not clear whether the non-Abelian generalization of our model is possible or not in a priori. However, considering the recent failure of the complete conversion of the second class constraints of this non-Abelian model into the first class ones, which does not use the symmetric \( X_{ij}(x,y) \)-matrix, and the power of the formalism with the symmetric \( X_{ij}(x,y) \) as noted in Section 2.1 we expect that whether we can find the symmetric \( X_{ij}(x,y) \) or not is crucial point for the successful application of our BFT formalism. We are in progress in this direction. In another direction, a new formalism may be considered to solve this problem, but it is not clear whether the
complete conversion of the system into the first class one is possible and furthermore that formalism is equivalent to the BFT formalism or not. On the other hand, it is interesting to note that the non-Abelian model of the Chern-Simons theory allows the symmetric \( X_{ij}(x, y) \) and the solution of the first class system is found by finite iterations. 

**Appendix A**

In this appendix, we derive the first class Hamiltonians (26) and (27) in the extended phase space corresponding to the total Hamiltonian \( H_T \) of (3) and canonical Hamiltonian \( H_c \) of (4) by using the usual straightforward approach. Let us first consider the first class Hamiltonian \( \widetilde{H}_T \) corresponding to \( H_T \). It is given by the infinite series,

\[
\widetilde{H}_T = H_T + \sum_{n=1}^{\infty} H^{(n)}_T; \quad H^{(n)}_T \sim (\Phi^i)^n,
\]

satisfying the initial condition, \( \widetilde{H}_T(\pi_\mu, A^\mu; 0) = H_T \). The general solution \(^7\) for the involution of \( \widetilde{H}_T \) is given by

\[
H^{(n)}_T = -\frac{1}{n} \int d^3 x d^3 y d^3 z \Phi^i(x)\omega_{ij}(x, y)X^k(y, z)G^{(n-1)}(z) \quad (n \geq 1),
\]

where the generating functions \( G^{(n)}_k \) are given by

\[
G^{(0)}_i = \{\Omega^{(0)}_i, H_T\},
\]

\[
G^{(n)}_i = \{\Omega^{(0)}_i, H^{(n)}_T\}_O + \{\Omega^{(1)}_i, H^{(n-1)}_T\}_O \quad (n \geq 1),
\]

where the symbol \( O \) in Eq. (80) represents that the Poisson brackets are calculated among the original variables, i.e., \( O = (A^\mu, \pi_\mu) \). Here, \( \omega_{ij} \) and \( X^{ij} \) are the inverse matrices of \( \omega^{ij} \) and \( X_{ij} \) respectively as in the text. Explicit calculations yields

\[
G^{(0)}_1 = \Omega_2,
\]

\[
G^{(0)}_2 = \partial_i \partial_i \Omega_1,
\]

which are substituted in (80) to obtain \( H^{(1)}_T \),

\[
H^{(1)}_T = \int d^3 x \left[ \frac{1}{m} (\partial_i \partial_i \Phi^1) \Omega_1 - \frac{1}{m} \Phi^2 \Omega_2 \right].
\]
This is inserted back in Eq. (81) in order to deduce $G^{(1)}_i$ as follows
\[
G^{(1)}_1 = m \Phi^2,
\]
\[
G^{(1)}_2 = m \partial_i \partial_i \Phi^1.
\] (84)

Then, we obtain $H^{(2)}_T$ by substituting $G^{(1)}_i$ in Eq. (76)
\[
H^{(2)}_T = \int d^3x \left[ -\frac{1}{2} (\partial_i \Phi^1) (\partial_i \Phi^1) - \frac{1}{2} (\Phi^2)^2 \right].
\] (85)

Finally, since
\[
G^{(n)}_i = 0 \quad (n \geq 2),
\] (86)
due to the proper choice (12) we obtain the complete form of the Hamiltonian $\tilde{H}$ as follows
\[
\tilde{H}_T = H_T + H^{(1)}_T + H^{(2)}_T,
\] (87)
which, by construction, is strongly involutive,
\[
\{ \tilde{\Omega}_i, \tilde{H}_T \} = 0.
\] (88)

Similarly, for the canonical Hamiltonian we can easily obtain it’s first class Hamiltonian $\tilde{H}_c$ as follows
\[
\tilde{H}_c = H_c + H^{(1)}_c + H^{(2)}_c,
\] (89)
where
\[
H^{(1)}_c = \int d^3x \left[ m \Phi^1 (\partial_i A^i) - \frac{1}{m} \Phi^2 \Omega_2 \right],
\] (90)
\[
H^{(2)}_c = \int d^3x \left[ \frac{1}{2} (\partial_i \Phi^1) (\partial_i \Phi^1) - \frac{1}{2} (\Phi^2)^2 \right].
\] (91)

Here, we used
\[
G^{(0)}_1 = \Omega_2,
\]
\[
G^{(0)}_2 = m^2 \partial_i A^i,
\]
\[
G^{(1)}_1 = m \Phi^2,
\]
\[
G^{(1)}_2 = -m \partial_i \partial_i \Phi^1.
\]
\[
G^{(n)}_i = 0 \quad (n \geq 2).
\] (92)
Note the differences in $G_2^{(0)}, G_2^{(1)}$ and $\Phi^1$-dependent terms in $H^{(1)}$ and $H^{(2)}$ for the total and canonical Hamiltonians. Moreover, $\tilde{H}_c$ is also, by construction, strongly involutive,

$$\{\tilde{\Omega}_i, \tilde{H}_c\} = 0. \quad (93)$$

### Appendix B

In this appendix, we obtain the FJ symplectic brackets comparing them with both the orthodox Dirac brackets and the modified Poisson brackets (36) in the extended phase space in the section 2.2.

According to the FJ formalism, which is to be regarded as the improved version of the Dirac one, we rewrite the first order Lagrangian corresponding to the Proca model (1) as

$$\mathcal{L}^{(0)} = \pi_i \dot{A}^i - \mathcal{H}^{(0)}, \quad (94)$$

where the conjugate momenta (2) of the gauge fields and the canonical Hamiltonian $\mathcal{H}_c$ in which we denote it as $\mathcal{H}^{(0)}$ showing the iterative nature of the formalism are used.

In order to find the FJ symplectic brackets we introduce the sets of the symplectic variables $\xi^{(0)k}$ and the conjugate momenta $a_k^{(0)}$ as follows

$$\begin{align*}
\xi^{(0)k} &= (A^0, A^i, \pi_i), \\
a_k^{(0)} &= (0, \pi_i, 0),
\end{align*} \quad (95)$$

which are usually read off from the form of the canonical sector of the first order Lagrangian (94), respectively.

Then, the dynamics of the model is governed by the invertible symplectic two-form matrix such that

$$f^{(0)}_{kl}(x, y) = \frac{\partial a_k^{(0)}(y)}{\partial \xi^{(0)k}(x)} - \frac{\partial a_l^{(0)}(x)}{\partial \xi^{(0)l}(y)}, \quad (96)$$

through the equations of motion

$$\dot{\xi}^k(x) = \int d^3y f^{(0)kl}(x, y) \frac{\partial \mathcal{H}^{(0)}(x)}{\partial \xi^{(0)l}(y)}, \quad (97)$$

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where \( f^{(0)kl}(x, y) \) is an inverse of \( f_{kl}(x, y) \). However, in the Proca model the symplectic two-form matrix is given by

\[
f^{(0)kl}(x, y) = \begin{pmatrix} 0 & 0 & 0 & \delta^3(x - y) \\ 0 & 0 & -\delta_{ij} & 0 \\ 0 & \delta_{ij} & 0 & -\partial^1_x \\ -\delta_{ij} & 0 & \partial^1_y & 0 \end{pmatrix}
\]

showing the matrix \( f^{(0)kl}(x, y) \) is singular. As it happens, the symplectic two-form matrix has a zero mode, i.e., \( \tilde{\nu}^{(0)}_l = (v_1, 0, 0) \), where \( v_1 \) is an arbitrary function. Furthermore, this zero mode generates constraint \( \Omega^{(1)} \) in the context of the FJ formalism 22 as follows

\[
0 = \int d^3x \, v_1(x) \frac{\delta}{\delta A^0(x)} \int d^3y \mathcal{H}^{(0)}(y) \\
= -\int d^3x \, v_1(\partial^i \pi_i + m^2 A^0) \\
\equiv -\int d^3x \, v_1 \Omega^{(1)},
\]

which will be added into the canonical sector of the Lagrangian (94) enlarging the symplectic phase space with the Lagrange multiplier \( \alpha \). Then, the iterated, first order Lagrangian is given by

\[
\mathcal{L}^{(1)} = \pi_i \dot{A}^i + (\partial^i \pi_i + m^2 A^0) \dot{\alpha} - \mathcal{H}^{(1)},
\]

where the corresponding first iterated Hamiltonian \( \mathcal{H}^{(1)} \) is given by

\[
\mathcal{H}^{(1)}(\xi) |_{\Omega^{(1)}=0} = \frac{1}{2} (\pi_i)^2 + \frac{1}{4} F_{ij} F^{ij} + \frac{1}{2} m^2 (A^0)^2 + \frac{1}{2} m^2 (A^1)^2.
\]

The situations we stand is exactly the same as before except we now have the first order Lagrangian (100) and the Hamiltonian (101). In other words, we can set again the symplectic variables and the conjugate momenta as follows

\[
\xi^{(1)k} = (A^0, A^i, \pi_i, \alpha), \\
\alpha_{k}^{(1)} = (0, \pi_i, 0, \partial^i \pi_i + m^2 A^0)
\]

reading off from the Lagrangian (100). From this set of the variables, the first iterated symplectic two-form matrix is given by

\[
f^{(1)kl}(x, y) = \begin{pmatrix} 0 & 0 & 0 & m^2 \\ 0 & 0 & -\delta_{ij} & 0 \\ 0 & \delta_{ij} & 0 & -\partial^1_x \\ -m^2 & 0 & \partial^1_y & 0 \end{pmatrix} \delta^3(x - y),
\]

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and its inverse matrix is easily obtained

\[
f^{kl(1)}(x, y) = \begin{pmatrix}
0 & \frac{1}{m^2} \partial_x^i & 0 & -\frac{1}{m^2} \\
\frac{1}{m^2} \partial_x^i & 0 & \delta_{ij} & 0 \\
0 & -\delta_{ij} & 0 & 0 \\
\frac{1}{m^2} & 0 & 0 & 0
\end{pmatrix} \delta^3(x - y).
\] (104)

Now, according to the FJ formalism, this inverse symplectic two-form matrix gives the symplectic brackets of the Proca model

\[
\{\xi^k(x), \xi^l(y)\}_{\text{symp}} = f^{kl(1)}(x, y)
\] (105)
in the case of having the invertible symplectic matrix, i.e., at the final stage of iteration, such that

\[
\begin{align*}
\{A^0(x), A^j(y)\}_{\text{symp}} &= \frac{1}{m^2} \partial_x^j \delta(x - y), \\
\{A^0(x), A^0(y)\}_{\text{symp}} &= 0, \\
\{A^j(x), A^k(y)\}_{\text{symp}} &= 0, \\
\{\Pi_{\mu}(x), \Pi_{\nu}(y)\}_{\text{symp}} &= 0, \\
\{A^i(x), \Pi_{j}(y)\}_{\text{symp}} &= \delta_{ij} \delta(x - y), \\
\{A^0(x), \Pi_{\nu}(y)\}_{\text{symp}} &= 0, \\
\{A^i(x), \Pi_{0}(y)\}_{\text{symp}} &= 0
\end{align*}
\] (106)

showing that the symplectic brackets are exactly same both as the Dirac brackets and the modified Poisson brackets in the extended phase space (36).

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References


