SPONTANEOUS CP VIOLATION AND THE $B^0$ SYSTEM

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Abstract

We investigate effects of spontaneous breakdown of CP in $B^0_{d,s} - B^0_{d,s}$ systems in left-right symmetric models. Assuming that the left-right contribution to the $B^0 - B^0$ matrix element $M_{12}$ can be at most equal to the standard model one we obtain a new lower bound, $M_H \gtrsim 12$ TeV, on the flavour changing Higgs boson mass. Most importantly, the convention independent parameter $\text{Re}(\tau_B)$, which measures the amount of $\Delta B = 2$ CP violation, can be enhanced by a factor of four or more for $B^0_d$ and almost by two orders of magnitude for $B^0_s$ systems when compared with the Standard Model predictions. Therefore, interesting possibilities to observe indirect CP violation in the $B$ system are open in the planned facilities.

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1 Introduction

The success of the Standard Model (SM) teaches us very little about what to expect at energies higher than the left-handed gauge boson mass. An interesting, viable extension of the SM is the left-right symmetric $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$ model \cite{1}. Besides giving rise to a new energy scale the most interesting character of the model is the spontaneous breaking of parity invariance. In such a model of spontaneous parity violation, it seems natural to consider the possibility that CP is also broken spontaneously \cite{2, 3, 4, 5, 6, 7}. More interestingly, it has been shown \cite{3} that, to implement this symmetry breaking, it is not necessary to extend the Higgs boson sector while keeping flavour changing neutral currents under control.

Although the SM has passed all the tests in the kaon system \cite{8, 9}, intriguing hints of other plausible explanations emerge from cosmological considerations of the baryon to photon ratio in the Universe \cite{10}. The left-right symmetric model can equally well explain the existing measurements of CP violation in the neutral kaon system \cite{7}. However, by studying kaons alone it is very difficult to distinguish between different mechanisms of CP violation. Complementary tests of the origin of CP violation will be provided by the studies of $B$-systems in planned facilities \cite{11} which will start to operate in the near future. Unfortunately, the convention independent parameter $\tau$, which measures the amount of $\Delta B = 2$ CP violation, is predicted to be very small in the SM setting extremely strong requirements on the performance of these machines. In fact, it is possible that even the SM value of $\text{Re}(\tau)$ for $B^0_s$, which is expected to be an order of magnitude larger than the one for $B^0$, will not be achievable in these experiments. Therefore, measurements of large $\tau$ would inevitably mean the discovery of physics beyond the SM. It is for this reason why the exploration of CP violation in the $B$ system is so crucial.

The spontaneous violation of CP in $B^0_{s,d}$ systems have been studied earlier \cite{12}. In the present work we shall re-consider mixings and CP violation in the B systems in view of the progress in our understanding of the hadronic matrix elements and implementation of the spontaneous breakdown of CP in left-right symmetric models, not to mention the improvements in the precision of experimental data such as the strong coupling constant or top quark mass. As will be seen, our results indicate that an enhancement of about an order of magnitude compared with the SM prediction is possible leading to observable effects in the future experiments.

The outline of the paper is the following. In Section 2 we discuss the spontaneous breakdown of CP in left-right symmetric models. In Section 3 we present the general formalism for the mixing and CP violating parameters in $B^0 - \overline{B}^0$ system and in Section 4 we derive the parameters in our model. Numerical results are presented in the same Section. Our conclusions are drawn in Section 5.

2 Spontaneous CP violation in the left-right symmetric model

Here we present a brief review of the minimal $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$ model with a discrete left-right symmetry. The purpose is to set the stage for the model we consider
and establish some notation. A more detailed description of the model exists in the literature [1]. The fermionic sector of the model contains three generations of quarks which we denote as \(\Psi_{iL} \equiv (u_i, d_i)_L\) in the representation \((\frac{1}{2}, 0, 0)\) and \(\Psi_{iR} \equiv (u_i, d_i)_R\) in the representation \((0, \frac{1}{2}, \frac{1}{3})\), where \(i = 1, 2, 3\) denotes the corresponding generation. The leptonic sector does not concern us here. The Higgs boson sector contains a bidoublet \(\Phi\) in the representation \((\frac{1}{2}, \frac{1}{2}, 0)\) and two triplets, \(\Delta_L\) and \(\Delta_R\), in the representations \((1, 0, 0)\) and \((0, 1, 2)\) and \((0, 1, 2)\), respectively. They can be written as

\[
\Phi \equiv \frac{1}{2} \left( \begin{array}{cc} \phi^0_1 & \phi_2^+ \\ \phi_2^- & \phi^0_2 \end{array} \right), \quad \Delta_L \equiv \left( \begin{array}{cc} \delta^+_L/(\sqrt{2}) & \delta^+_{L}\ \\ \delta^0_L & -\delta^+_{L}/\sqrt{2} \end{array} \right). \tag{1}
\]

In order to have parity as a spontaneously broken symmetry, a discrete left-right symmetry is imposed:

\[
\Psi_{iL} \leftrightarrow \Psi_{iR}, \quad \Delta_L \leftrightarrow \Delta_R, \quad \Phi \leftrightarrow \Phi^\dagger, \tag{2}
\]

The most general Yukawa interaction invariant under (2) can be written as

\[
\mathcal{L}_Y = \sum_{i,j=1}^{3} \left( f_{ij} \bar{\Psi}^i_L \Phi \Psi^j_R + g_{ij} \bar{\Psi}^i_L \tilde{\Phi} \Psi^j_R \right) + h.c., \tag{3}
\]

where \(\tilde{\Phi} \equiv \tau_2 \Phi^* \tau_2\). After symmetry breaking, the vacuum expectation value (vev) of \(\Phi\) can be written as

\[
\langle \Phi \rangle = \begin{pmatrix} k_1 e^{i\alpha'} & 0 \\ 0 & k_2 e^{i\alpha} \end{pmatrix}. \tag{4}
\]

The quark mass matrices generated by Eqs. (3), (4) are

\[
\hat{M}^u = f k_1 e^{i\alpha'} + \hat{g} k_2 e^{-i\alpha}, \quad \hat{M}^d = \hat{g} k_1 e^{-i\alpha'} + f k_2 e^{i\alpha}, \tag{5}
\]

where \(\hat{M}^u (\hat{M}^d)\) is the up (down) quark mass matrix and the hats denote \(3 \times 3\) matrices. As a result of the left-right discrete symmetry, \(f\) and \(\hat{g}\) must be Hermitian. However, \(\hat{M}^u\) and \(\hat{M}^d\) are not Hermitian. In order to obtain the left- and right-handed Cabbibo-Kobayashi-Maskawa (CKM) matrices \(V_L\) and \(V_R\), respectively, they must be diagonalized by the usual bi-unitary transformation. In general, there is no simple relationship between \(V_L\) and \(V_R\).

The vevs of the Higgs bosons, \(\langle \Phi \rangle\) and \(\langle \Delta_{L,R} \rangle \equiv v_{L,R}\), generate the following mass matrix for the charged gauge bosons

\[
\begin{pmatrix}
\frac{1}{2} g^2 (k_1^2 + k_2^2) + g^2 v_L^2 & -g^2 k_1 k_2 e^{-i(\alpha - \alpha')} \\
-g^2 k_1 k_2 e^{i(\alpha - \alpha')} & \frac{1}{2} g^2 (k_1^2 + k_2^2) + g^2 v_R^2
\end{pmatrix}. \tag{6}
\]

Experimentally we know that \(v_R^2 \gg k_1^2, k_2^2 \gg v_L^2\), which imply that the \(W^+_1\) mass is given to a good approximation by \(M_1 \simeq \frac{1}{2} g^2 (k_1^2 + k_2^2)\), and similarly \(M_2 \simeq \frac{1}{2} g^2 v_R^2\). The \(W^+_1 - W^+_2\) mixing angle \(\xi\) is small,

\[
\xi \approx \frac{2 k_1 k_2}{k_1^2 + k_2^2} \frac{M_1^2}{M_2^2},
\]

where
as required by the low energy phenomenology [13, 14]. The most stringent lower bound on the new gauge boson mass \( W_2 \gtrsim 1.6 \) TeV derives from the analysis of the \( K_L - K_S \) mass difference [15]. However, this bound depends quite strongly on low energy QCD and different assumptions used in the literature [14]. Within the present errors, it can be as low as \( M_2 \sim 900 \) GeV, not too far from the Tevatron bound \( M_2 \sim 652 \) GeV [16]. There are two neutral flavour changing Higgs bosons in the model. To suppress their interactions, the lower bound \( M_R \gtrsim 10 \) TeV has been derived [5].

The model we want to analyse is the one we have described, except that we impose CP as a spontaneously broken symmetry. In this model it is not necessary to introduce any extra Higgs multiplet in order to break CP spontaneously [3]. In this sense, it is more natural to discuss spontaneous CP violation in models with the gauge group \( SU(2)_R \times SU(2)_L \times U(1)_{B-L} \) rather than in the standard \( SU(2)_L \times U(1) \) models in which extra Higgs multiplets are needed to generate the spontaneous CP violation. A direct consequence of imposing CP as a spontaneously broken symmetry, together with the discrete left-right symmetry (2), is that the Yukawa coupling matrices \( \hat{f} \) and \( \hat{g} \) in Eq. (2) must be real symmetric. The only complex parameter in the mass matrices \( M^{u,d} \) is a complex phase in \( h^i_8 \) that we will discuss now.

In order to break CP spontaneously, we have to look for a complex vev of the Higgs boson. The vev \( \langle \Phi \rangle \) of Eq. (4) breaks the \( U(1)_{B-L} \) symmetry with the generator \( I_{3L} - I_{3R} \). We can use this \( U(1) \) symmetry to shift the phases between the \( k_1 \) and \( k_2 \) component of the \( \langle \Phi \rangle \). As a result we can choose \( \alpha' \) to be zero without loss of generality. Similarly, also \( v_R \) can be made real by using the \( U(1)_{B-L} \) symmetry that it breaks. Working in the limit of \( k_2 \sim v_L \sim 0 \), as is usually assumed in the literature, we would find that there is no CP violation at all. A small, complex nonzero \( v_L \) will generate CP violation in the leptonic sector, but not in the pure hadronic sector.

### 3 The \( B^0 - \bar{B}^0 \) system

The flavour quantum numbers are not conserved by weak interactions. Thus a \( B^0 \) state can be transformed into its antiparticle \( \bar{B}^0 \). As a consequence, the flavour eigenstates \( B^0 \) and \( \bar{B}^0 \) are not mass eigenstates and do not follow an exponential decay law.

Let us consider an arbitrary mixture of two flavour states

\[
| \psi(t) \rangle = a(t) \ | B^0 \rangle + b(t) \ | \bar{B}^0 \rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}.
\]

(7)

Its time evolution is governed by the equation

\[
i \frac{d}{dt} | \psi(t) \rangle = \mathcal{M} | \psi(t) \rangle,
\]

where \( \mathcal{M} \) is called the \( B^0 - \bar{B}^0 \) mixing matrix. Assuming CPT symmetry to hold, this can be written as

\[
\mathcal{M} = \begin{pmatrix}
M - \frac{i}{2} \Gamma & M_{12} - \frac{i}{2} \Gamma_{12} \\
M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M - \frac{i}{2} \Gamma
\end{pmatrix}.
\]

(9)
The diagonal elements $M$ and $\Gamma$ are real parameters, which would correspond to the mass and width of the neutral mesons in the absence of mixing. The off-diagonal entries contain the dispersive and absorptive parts. If CP were an exact symmetry, $M_{12}$ and $\Gamma_{12}$ would also be real. The physical eigenstates of $\mathcal{M}$ are,

$$| B_{\pm} \rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \left( p | B^0 \rangle \pm q | B^0 \rangle \right),$$

with

$$\frac{q}{p} = \frac{1 - \bar{\tau}}{1 + \bar{\tau}} = \left( \frac{M_{12} - i\frac{1}{2} \Gamma_{12}}{M_{12} - i\frac{1}{2} \Gamma_{12}} \right)^{\frac{1}{2}}.$$

If $M_{12}$ and $\Gamma_{12}$ were real, then $\frac{q}{p} = 1$ and $| B_{\pm} \rangle$ would correspond to the CP even and CP odd eigenstates.

Note that if the $B^0 - \bar{B}^0$ violates CP, the two mass eigenstates are no longer orthogonal, and we can define

$$\langle B_- | B_+ \rangle = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} = \frac{2\text{Re}(\bar{\tau})}{1 + |\bar{\tau}|^2} \approx 2\text{Re}(\bar{\tau}),$$

which is a convention independent measure of CP violation. Notice that we have used the shorthand notation $\bar{\tau} \equiv \bar{\tau}_B$.

The time evolution of a state which was originally produced as a $B^0$ or a $\bar{B}^0$ is given by

$$\begin{pmatrix} | B^0(t) \rangle \\
| \bar{B}^0(t) \rangle 
\end{pmatrix} = \begin{pmatrix} g_1(t) & \frac{q}{p} g_2(t) \\
g_2(t) & g_1(t) \end{pmatrix} \begin{pmatrix} | B^0 \rangle \\
| \bar{B}^0 \rangle \end{pmatrix},$$

where

$$\begin{pmatrix} g_1(t) \\
g_2(t) \end{pmatrix} = e^{i\Delta M t} e^{-i\Delta \Gamma t/2} \begin{pmatrix} \cos\left(\Delta M \frac{t}{2}\Delta \Gamma \frac{t}{2}\right) \\
-i\sin\left(\Delta M \frac{t}{2}\Delta \Gamma \frac{t}{2}\right) \end{pmatrix},$$

with

$$\Delta M = M_{B_+} - M_{B_-} \quad \text{and} \quad \Delta \Gamma = \Gamma_{B_+} - \Gamma_{B_-}.$$

The mass difference $\Delta M$, and the difference of the decay widths can be calculated from the general expressions [17]

$$\Delta M = \sqrt{2} \left( \left( d_1^2 + d_2^2 \right)^{\frac{1}{2}} + d_1 \right)^{\frac{1}{2}},$$

$$\Delta \Gamma = -2\sqrt{2} \text{sgn}(d_2) \left( \left( d_1^2 + d_2^2 \right)^{\frac{1}{2}} - d_1 \right)^{\frac{1}{2}},$$

with

$$d_1 = \left| M_{12} \right|^2 - \frac{1}{4} \left| \Gamma_{12} \right|^2,$$

$$d_2 = \text{Re}(M_{12} \Gamma_{12}^*),$$

(17)
The main difference between the $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ system stems from the different kinematics involved. The light kaon mass only allows the hadronic decay modes $K^0 \rightarrow 2\pi$ and $K^0 \rightarrow 3\pi$. Since $\text{CP} \mid \pi\pi \rangle = + \mid \pi\pi \rangle$, the CP even kaon state decays into $2\pi$ whereas the CP odd one decays into the phase space suppressed $3\pi$ mode. Therefore, there is a large lifetime difference and we have a short-lived $\mid K_S \rangle \equiv \mid K^- \rangle \approx \mid K_1 \rangle + \bar{\tau}_K \mid K_2 \rangle$ and a long lived $\mid K_L \rangle \equiv \mid K_+ \rangle \approx \mid K_2 \rangle + \bar{\tau}_K \mid K_1 \rangle$ kaon, with $\Gamma_{K_L} \ll \Gamma_{K_S}$. One finds experimentally that
\[
\Delta \Gamma_{K^0} \approx -\Gamma_{K_S} \approx -2\Delta M_{K^0}
\] (18)

In the $B$ system, there are many open decay channels and a large part of them are common to both mass eigenstates. Therefore, the $\mid B_\pm \rangle$ states have a similar lifetime, i.e., $\Delta \Gamma_{B^0} \ll \Gamma_{B^0}$. Moreover, whereas the $B^0 - \bar{B}^0$ mixing is dominated by the top in the box diagram, the decay amplitude gets obviously its main contribution from the $b \rightarrow c$ transition. Thus,
\[
\frac{\Delta \Gamma_{B}}{\Delta M_{B}} \sim \frac{m_b^2}{m_t^2} \ll 1.
\] (19)

4 Left-right contributions to the $B$ system

Let us examine now the left-right contribution to the $B$ system. The charged current Lagrangian is given by
\[
\mathcal{L}_{\parallel} = \frac{g}{\sqrt{2}} \bar{t} \left( \cos \xi \gamma^\mu P_L - \sin \xi \gamma^\mu P_R \right) d W_{1\mu} + \frac{g}{\sqrt{2}} \bar{t} \left( \cos \xi \gamma^\mu P_L + \sin \xi \gamma^\mu P_R \right) d W_{2\mu} + \text{h.c.},
\] (20)

with
\[
P_{L,R} \equiv \frac{1 \mp \gamma_5}{2},
\]

where $W_1$ ($W_2$) is the charged vector boson field with the mass $M_1$ ($M_2$).

The left-right model contributions to $\langle B^0 \mid H \mid \bar{B}^0 \rangle$ are shown in Fig. 1. As in the kaon system, multiplicatively renormalizable operators of the type
\[
O_{LL} = \bar{t} \gamma^\mu P_L (\bar{t} \gamma^\mu P_L l, \quad O_S = \bar{t} P_L \bar{t} P_R l, \quad O_{V} = \bar{t} \gamma^\mu P_L (\bar{t} \gamma^\mu P_R l + \frac{2}{3} O_S),
\] (21)

with $l=$d- or s-quark, appear in the effective Hamiltonian. Their matrix elements, evaluated at an energy scale $m_b (m_b) = 4.3$ GeV, are
\[
\langle B^0 \mid O_{LL} \mid \bar{B}^0 \rangle = \frac{8}{3} B_{B} f_B^2 m_B^2,
\]
\[
\langle B^0 \mid O_{S} \mid \bar{B}^0 \rangle = \left( \frac{m_B^2}{(m_b + m_t)^2} + \frac{1}{6} \right) B'_{B} f_B^2 m_B^2,
\]
\[
\langle B^0 \mid O_{V} \mid \bar{B}^0 \rangle = -\frac{8}{3} B''_{B} f_B^2 m_B^2,
\] (22)
where \( m_B = 5.28 \text{ GeV} \) is the \( B^0 \) mass, \( f_B \) denote the \( B \) meson decay constant and the factors \( B_B, \ B'_B, \) and \( B''_B \) take into account deviations of the actual values of the hadronic matrix elements from the vacuum insertion approximation. At present, only the combination \( \sqrt{B_B f_B} = 200 \pm 50 \text{ MeV} \) is known [9]. In the following we assume that \( B_B = B'_B = B''_B \) which should be a good approximation for our numerical estimates.

With these hadronic matrix elements the SM contribution (Fig. 1a) is given by [9]

\[
M_{12}^{LL} \simeq \frac{G_F^2}{12\pi^2} B_B f_B^2 m_B M_{11}^2 \eta_2^{(B)} (\lambda_t^{LL})^2 S(x_t),
\]

where \( \eta_2^{(B)} \approx 0.55 \) is a QCD correction factor,

\[
\lambda_t^{AB} = V_{A,t} V_{B,t}^* \quad (A, B = L, R),
\]

and [18]

\[
S(x) = -\frac{3}{2} \left( \frac{x}{1-x} \right)^3 \ln(x) + x \left( \frac{1}{4} + \frac{9}{4} \frac{1}{(1-x)} - \frac{3}{2} \frac{1}{(1-x)^2} \right).
\]

The \( t\bar{t} \) exchange dominates not only the SM contribution but also the left-right contribution. For the transverse \( W_1 \) and \( W_2 \) one gets [12]

\[
M_{12}^{W_1W_2} \simeq \frac{G_F^2}{\pi^2} M_1^2 \beta B_B f_B^2 m_B \left( \frac{m_B}{m_b + m_t} \right)^2 + \frac{1}{6} \right) \lambda_t^{RL} \lambda_t^{LR} \eta_1^{LR} F_1(x_t, x_b, \beta),
\]

and for the \( S_1W_2 \) contribution in Fig. 1c (\( S_i \) denoting the Goldstone boson which becomes the longitudinal component of \( W_i \))

\[
M_{12}^{S_1W_2} \simeq -\frac{G_F^2}{4\pi^2} M_1^2 \beta B_B f_B^2 m_B \left( \frac{m_B}{m_b + m_t} \right)^2 + \frac{1}{6} \right) \lambda_t^{RL} \lambda_t^{LR} \eta_2^{LR} F_2(x_t, x_b, \beta).
\]

Here \( \beta = M_1^2/M_2^2, \ x_i = m_i^2/M_2^2 \) and \( \eta_1^{LR} \) and \( \eta_2^{LR} \) are the QCD coefficients. Following Ref. [5] and assuming \( \alpha_s(M_Z) = 0.118 \) and quark masses as given in Ref. [20] we obtain \( \eta_1^{LR} = 1.83 \) and \( \eta_2^{LR} = 1.66 \). The function \( F_1 \) is given by

\[
F_1(x_t, x_b, \beta) = \frac{x_t}{(1 - x_t \beta)(1 - x_t)} \int_0^1 da \sum_{k} \left| \sum_{1,2} \Lambda_k(\alpha, \beta) \right|,
\]

where

\[
\sum' = \sum_{1,2} - \sum_{3,4},
\]

and the functions \( \Lambda_k(\alpha, \beta) \) are defined by

\[
\begin{align*}
\Lambda_1(\alpha, \beta) & = x_t - x_b (1 - \alpha), \\
\Lambda_2(\alpha, \beta) & = 1 - \alpha + \frac{\alpha}{\beta} - x_b \alpha (1 - \alpha), \\
\Lambda_3(\alpha, \beta) & = x_t (1 - \alpha) + \frac{\alpha}{\beta} - x_b \alpha (1 - \alpha), \\
\Lambda_4(\alpha, \beta) & = 1 - \alpha + x_t \alpha - x_b \alpha (1 - \alpha).
\end{align*}
\]
Analogously,
\[
F_2(x_t, x_b, \beta) = \frac{2x_t}{(1-x_t \beta)(1-x_t)} \int_0^1 d\alpha \sum_k \Lambda_k(\alpha, \beta) \ln |\Lambda_k(\alpha, \beta)|. \tag{31}
\]

Among the three diagrams containing the unphysical scalars $S_i$, the ones which contain the $W_1S_2$ (Fig. 1d) and $S_1S_2$ (Fig. 1e) boxes are of order $\beta^2$ and can be neglected even for the intermediate top quark. Also the box diagram which contains two $W_2$ (Fig. 1f) can be neglected due to the same reason.

The tree level contributions are mediated by the flavour changing neutral Higgs bosons $\phi_1$ and $\phi_2$ and their contribution is [19]
\[
M_{12}^H \simeq -\frac{\sqrt{2}G_F m_t^2(m_b) B_B f_B^2 m_b}{M_H^2} \left( \frac{m_B}{m_b + m_t} \right)^2 \frac{1}{6} \lambda_t^{RL} \lambda_t^{LR}, \tag{32}
\]
where we have assumed a common Higgs mass $M_H$. Note that these flavour changing neutral contributions are suppressed by the same factors $\lambda_t$ as the gauge mediated amplitudes. One loop Higgs contributions to $M_{12}$ including also charged Higgs bosons are suppressed as far as $M_H \gg M_2$. Considering the kaon system, this is a quite natural assumption. In order to make the left-right contribution to the kaon mass difference smaller than the experimental value, the bounds $M_2 \gtrsim 1.6 \text{ TeV}$ and $M_H \gtrsim 10 \text{ TeV}$ have emerged.

Now we can demonstrate the relevance of the left-right contribution to the mixing of $B^0 - \bar{B}^0$. Using the formulae above we have performed a numerical computation resulting in
\[
\left| \frac{M_{12}^{W_1W_2} + M_{12}^{S_1W_2} + M_{12}^H}{M_{12}^{LL}} \right| = F(M_2) \left( \frac{1.6 \text{ TeV}}{M_2} \right)^2 + \left( \frac{12 \text{ TeV}}{M_H} \right)^2, \tag{33}
\]
where we have assumed $|\lambda_t^{RL}| = |\lambda_t^{LR}| = |\lambda_t^{LL}|$, which is guaranteed in the left-right model with the manifest left-right symmetry (2). The function $F(M_2)$ presents the non-trivial dependence of the left-right gauge boson box contribution $M_{12}^{LL}$ on $M_2$, and is plotted in Fig. 2. Comparing the numerical values one can see that for Higgs masses close to its lower bound the Higgs exchange dominates over the box contribution. Since $F(M_2)$ is an increasing function of $M_2$ it follows from Eq. (33) that scaling the masses $M_2$ and $M_H$ up by the same factor the gauge boson contribution becomes relatively more important. It is important to notice that the left-right box and Higgs contributions add up constructively. With the $W_R$ and Higgs masses close to their present lower bounds the modulus of the left-right contribution to $M_{12}$ is comparable in magnitude with the SM one. If we assume as in Ref. [5] that $|M_{12}^{LR}| \lesssim |M_{12}^{LL}|$ is allowed experimentally we can obtain from Eq. (33) a new lower bound,
\[
M_H \gtrsim 12 \text{ TeV}, \tag{34}
\]
on the Higgs boson mass.

However, due to the unknown relative phase between the SM and left-right contributions, substantial cancellation between them is possible. There are all together six phases present in the left- and right-handed CKM matrices which may cause destructive interference. If the CMK phases are small, as is expected in our model (we will see in the next
section that they are proportional to \( r \sin \alpha \) which is bounded to be a small quantity),
this cancellation may appear naturally. In conclusion, by studying the \( B \) meson mixing alone it is difficult, if not impossible, to distinguish the left-right model from the SM. We move now to investigate also the CP violating observables.

The result above does not imply that in the left-right symmetric model there is the same enhancement factor also for the CP violating observables. For the purpose of studying CP violation we need to discuss \( \Gamma_{12} \). Clearly, tree level Higgs exchange does not contribute and all terms coming from the left-right symmetric model have a suppressing \( \beta \) factor when compared to the SM contribution. Therefore, the left-right contribution is completely negligible and to a good approximation \( \Gamma_{12} \approx \Gamma_{12}^{LL} \). To calculate the transition width we use the quark box diagram approximation which is expected to give a good estimate for the \( B \) system.\(^1\) In this approximation \( \Gamma_{12} \) can be written in the form [21]

\[
\Gamma_{12} = \frac{G_F f_B^2 B_{m_B}^3}{8\pi} \left( (\lambda_u^{LL})^2 X_{uu} + (\lambda_c^{LL})^2 X_{cc} + 2\lambda_c^{LL} \lambda_u^{LL} X_{uc} \right),
\]

where the functions \( X \) will be defined later. In the three generation standard model, as well as in the minimal left-right symmetric model, the unitarity of the CKM matrix \( V_L \) entails

\[
\lambda_u^{LL} + \lambda_c^{LL} + \lambda_t^{LL} = 0,
\]

and, consequently,

\[
\Gamma_{12} = \frac{G_F f_B^2 B_{m_B}^3}{8\pi} \left[ (\lambda_u^{LL})^2 (X_{uu} - X_{uc}) + (\lambda_c^{LL})^2 (X_{cc} - X_{uc}) + (\lambda_t^{LL})^2 X_{uc} \right].
\]

Neglecting the mass of the up quark, one has \((x = m_c^2/m_B^2, y = m_b^2/m_B^2)\) [23]

\[
X_{uu} = \eta^{(s)} y, \quad X_{cc} = \eta^{(s)} y \sqrt{1 - \frac{4x}{y}} - \eta^{(a)} \frac{2}{N} x \sqrt{1 - 4x}, \quad X_{uc} = \eta^{(s)} y \left( 1 + \frac{x}{y} \right) \left( 1 - \frac{x}{y} \right) - \eta^{(a)} \frac{1}{N} x (1 - x)^2,
\]

where

\[
\eta^{(s)} = \frac{1}{2} \left( 1 + \frac{1}{N} \right) c_+^2 + \frac{1}{2} \left( 1 - \frac{1}{N} \right) c_-^2, \quad \eta^{(a)} = \left( \frac{N + 1}{2} \right)^2 c_+^2 + \left( \frac{N - 1}{2} \right)^2 c_-^2 - \frac{N^2 - 1}{2} c_+ c_-,
\]

with [24]

\[
c_+ = (c_-)^{-\frac{1}{2}} = \left( \frac{\alpha_s(m_b)}{\alpha_s(m_W)} \right)^{-\frac{6}{23}}.
\]

\(^1\)We thank A. Pich for clarifying discussions on this point.
Here $N = 3$ denotes the number of colours. Numerically the third term in Eq. (37) dominates over the first two and we obtain

$$\Gamma_{12} \approx \frac{G_F f_B^2 B m_B^3}{8\pi} (\lambda t L)^2 X_{uc}.$$  \hfill (41)

In agreement with the discussion in Section 3, the ratio

$$\left| \frac{\Gamma_{12}}{M_{12}^{LL}} \right| \frac{3 m_B^2}{2 \eta_2 (B) M_1^2} \left| \frac{X_{uc}}{S(x)} \right|$$  \hfill (42)

is, indeed, numerically small.

Now we are ready to study the CP violation convention independent measure $\tau$ defined in Eq. (12). Taking into account that in the SM the phase between $\Gamma_{12}$ and $M_{12}^{LL}$ is quite small, $O(m_c^2/m_0^2)$, we can neglect terms proportional to this phase and obtain

$$2 \text{Re}(\tau) \approx \frac{\left| \Gamma_{12} \right|}{2 \left| M_{12}^{LL} \right|^2 \left| 1 + \frac{M_{12}^{LR}}{M_{12}^{LL}} \right|^2} \sin \sigma,$$  \hfill (43)

where

$$\sigma = \text{Arg} \left( \frac{M_{12}^{LR}}{M_{12}^{LL}} \right).$$  \hfill (44)

Note that this estimate holds for both $B^0_d$ and $B^0_s$ systems as long as the result is larger than in the SM. As can be seen, in the left-right model the indirect CP violation is not related to the small relative phase of $\Gamma_{12}$ and $M_{12}^{LL}$ but to the phase between the left and right contributions to $M_{12}$ which can be appreciable. Therefore the CP violating observables can be significantly enhanced in our model when compared with the SM prediction [22]

$$\left| 2 \text{Re}(\tau) \right| \lesssim \begin{cases} 0.5 \cdot 10^{-3} & B^0_d, \\ 0.5 \cdot 10^{-4} & B^0_s. \end{cases}$$  \hfill (45)

To see this enhancement let us assume for a moment that $\sin \sigma = 1$ and turn later to the analysis of its possible values. In this case we plot in Fig. 3 the value of $2 \text{Re}(\tau)$ as a function of $M_2$ fixing the Higgs boson mass to 12 TeV and in Fig. 4 as a function of $M_H$ fixing the right-handed gauge boson mass to 1.6 TeV. With the chosen masses left-right symmetry can enhance the CP violation in $B^0_d - \bar{B}^0_d$ system more than a factor of four and in $B^0_s - \bar{B}^0_s$ system almost by two orders of magnitude. As seen in the figures, in the left-right symmetric model $2 \text{Re}(\bar{\tau})$ remains larger than the SM prediction for quite large values of the right-handed particle masses. This is a promising result for the future searches of indirect CP violation in $B$ physics. Since the parameter $\tau$ will be measured from $B$ decay asymmetries, an order of magnitude increase in the value of $\bar{\tau}$ would mean that hundred times less events are needed in order to measure it. With the designed machine parameters [11] it is not excluded that the left-right model $\bar{\tau}$ can be measured in the planned $B$ factories or hadronic machines, while the SM $\bar{\tau}$, taking into account predictions of Eq. (45), is most probably out of reach in these experiments. Therefore, stringent tests of the left-right symmetric model could be performed in the near future.
The actual value of the left-right contribution (43) will be modified by the phase $\sigma$. Therefore, we have to discuss how this phase can affect our previous conclusions. It follows from Eq. (44) that

$$e^{i\sigma} \simeq \frac{\lambda_{tL}^{RL} \lambda_{tR}^{LR}}{(\lambda_{tL}^{L})^2} = \frac{V_{Rtt} V_{Rtb}^*}{V_{Ltt} V_{Ltb}^*}. \tag{46}$$

In the left-right model with spontaneous breakdown of CP, all the CP violating phases in both $V_L$ and $V_R$ depend on a single quantity $r \sin \alpha$ ($r = k_2/k_1$). This particular feature makes the model very predictive. In practice, the phases are calculated analytically only to first order in $r \sin \alpha$. However, as shown in Ref. [6] by numerical calculations, the conventional small $r \sin \alpha$ approximation gives, indeed, very good results. An important feature that showed up in these analysis is the appearance of certain sign factors in the masses. According to the studies of the kaon system [7], some combinations of the signs can be ruled out in a phenomenological basis.

By using the derived formulae for the phases in $V_L$ and $V_R$ to first order in $r \sin \alpha$ one obtains the following approximate equations [5]

$$\sin \sigma^{(d)} \simeq \eta_d \eta_b r \sin \alpha \left[ \frac{2\mu_c}{\mu_s} + \frac{\mu_t}{\mu_b} \right],$$

$$\sin \sigma^{(s)} \simeq \eta_s \eta_b r \sin \alpha \left[ \frac{\mu_c}{\mu_s} + \frac{\mu_t}{\mu_b} \right], \tag{47}$$

where

$$\mu_i = \eta_i m_i, \quad \eta_i^2 = 1. \tag{48}$$

It is obvious from Eq. (47) that $\sin \sigma^{(s),(d)}$ depend strongly on the sign factors in Eq. (48). However, it is important to notice that both of them are enhanced by the common dominant factor $m_t/m_b$. Therefore, the values of $\sin \sigma^{(s),(d)}$ can differ maximally by a factor of 2-3 but their order of magnitude is the same. In general, we can write

$$\sin \sigma^{(d)} \sim \frac{\eta_d}{\eta_t} \sin \sigma^{(s)}. \tag{49}$$

At this stage, it is in order to remind some results from the kaon system analysis. While the value of $\eta_d$ is not restricted by the other quark signs, $\mu_s$ can flip the sign only together with $\mu_c$ or $\mu_d$. In order to find numerical estimates for $\sin \sigma^{(s),(d)}$ we have to specify the value of $r \sin \alpha$. It has been shown that, without fine tuning, the following relation holds [6]

$$| r \sin \alpha | \lesssim \frac{m_b}{m_t}. \tag{50}$$

Taking into account the enhancement factor $m_t/m_b$ in Eq. (47) one can conclude that $\sin \sigma^{(s),(d)}$ can be naturally of order one as assumed before. On the other hand, analyses of the kaon system have set lower bounds on the value of $r \sin \alpha$ which depend quite strongly on the signs of the quark masses. While for $\eta_s \eta_d = 1$ the lower bound on $r \sin \alpha$ is as low
as $5 \cdot 10^{-4}$, it is about an order of magnitude higher for $\eta_s \eta_d = -1$ [7]. In the latter case we obtain

$$| \sin \sigma^{(s,d)} | \approx 0.2,$$

which suggests that the left-right contribution to $2\text{Re}(\bar{\epsilon})$ is larger than the SM one. In order to obtain the correct value for the measured $\epsilon_K$, the values of $r \sin \alpha$ and $M_2$ are correlated. For small, close to the minimally allowed, values of $r \sin \alpha$ one obtains a very narrow region of $M_2$. For most of the parameter space the values of $\sin \sigma$ in Eq.(47) can be large and no significant suppression of the left-right CP violating effects due to the phase $\sigma$ is implied by existing phenomenology. Unlike stated in previous works, enhancement of indirect CP violation in the $B_{s,d}^0$ systems in the left-right symmetric models can occur in both cases $\eta_s \eta_d = \pm 1$, and particularly for $\eta_s \eta_d = -1$.

5 Conclusions

Left-right symmetry provides a promising scenario for explaining the origin of CP violation. Quite independently of phenomenological considerations, the left-right symmetric gauge model possesses the attractive feature that CP can be violated spontaneously already with the minimal content of the Higgs sector. In this framework, we have considered the model where all the CP violating quantities depend on a single phase $\alpha$ of the scalar bidoublet vev $\langle \Phi \rangle$. This model has already been successfully tested in the kaon system from which several constraints on the model parameters have been derived.

In the present work we have studied the $B^0 - \bar{B}^0$ system in the left-right model with spontaneous breakdown of CP. Our results can be summarized as follows. When the right-handed gauge boson and the flavour changing Higgs boson masses are close to their present lower bounds, the left-right contribution to the $B^0 - \bar{B}^0$ mass matrix element $M_{12}$ is comparable in magnitude with the SM one. Assuming that the left-right contribution does not exceed the SM one, we obtain a new lower bound on the Higgs mass $M_H \approx 12$ TeV. However, large cancellations between the two contributions are possible due to the different phases in the left and right CKM matrices.

While the amount of mixing is comparable with the SM one, CP violating observables of the model are much more promising candidates to discover deviations from the SM. The reason is that the almost common phase of the SM quantities $M_{12}^{LL}$ and $\Gamma_{12}$ (the relative phase between these two is very small suppressing the SM value of $\tau$) can be quite different from the phase of the left-right contribution $M_{12}^{LR}$. Consequently, the convention independent parameter $\text{Re}(\bar{\epsilon})$, which measures the amount of $\Delta B = 2$ CP violation, can be enhanced by a factor of four or more for $B^0_d$ and by almost two orders of magnitude for $B^0_s$ systems, if compared with the SM prediction. In terms of the requirements on the event multiplicity in the future $B$ factories and hadron facilities this would mean that about an order of magnitude less $B_d$ mesons should be produced to measure indirect CP violation in the $B$ system. Therefore, unlike the SM predictions, the left-right $\text{Re}(\bar{\epsilon})$ could be measured in the future experiments providing a definite signal of physics beyond the SM. According to the kaon system analyses, no significant suppression is expected due to the phase $\sin \sigma$ in $\bar{\epsilon}$. An important observation is that CP violating effects for $B^0 - \bar{B}^0$
can be significant no matter whether the left-right and the SM contributions to the $\epsilon_K$
interfere constructively or destructively.

In conclusion, stringent tests of the left-right symmetric model can be carried out in
the $B$ meson system when measuring the leptonic asymmetries.

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References


J.F. Gunion, J. Griñols, A. Mendez, B. Kayser and F. Olness,

Z. Phys. C.


A. Ali and D. London, preprint DESY 95-148, UDEM-GGP-TH-95-32, hep-
ph/9508272.

Ph/95-104, FERMILAB-PUB-95/305-T, SLAC-PUB 7009, hep-ph/9512380, and refer-
ences therein, to appear in Rev. Mod. Phys.


Figure captions

Fig. 1. Feynman diagrams contributing to $|\Delta B| = 2$ transition in the left-right model.

Fig. 2. Function $F$ plotted against the right-handed gauge boson mass.

Fig. 3. $2\text{Re}(\tau)$ as a function of the right-handed gauge boson mass for the fixed $M_H = 12$ TeV and $\sin \sigma = 1$.

Fig. 4. $2\text{Re}(\tau)$ as a function of the flavour changing Higgs boson mass for the fixed $M_2 = 1.6$ TeV and $\sin \sigma = 1$. 
Figure 1:
Figure 2:
Figure 3:
Figure 4:

$B^0 - B^0$

$M_2 = 1.6 \text{ TeV}$