Nonperturbative QCD Vacuum Effects in Nonlocal Quark Dynamics

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Abstract

A straightforward calculation reveals the essentially nonlocal character of the leading heavy $Q\bar{Q}$ interaction arising from nonperturbative gluon field correlations in the model of a fluctuating QCD vacuum. In light of this quarkonium spin splitting ratio predictions which have supported the scalar confinement ansatz are reconsidered as a specific example of possible consequences for spectroscopy.
1 Nonlocal Dynamics

A great deal of work has gone into the effort to describe the spectrum and high energy scattering of hadrons as bound states of quarks and gluons. It remains an open and interesting problem. Part of the trouble seems to lie in the historical identification of particle field theory with its familiar perturbation expansion, while nonperturbative phenomena such as confinement are thought to play an important role in the QCD bound state. The controlling aim to obtain an understanding in a formulation of the fundamental Quantum Field Theory has led to the development of nonperturbative phenomenology reflecting the effective degrees of freedom for a given arrangement, where those degrees less relevant are frozen or integrated out of the theory. Unfortunately there is no unique well-defined program by which this is done. Two competing points of view among many are given by potential models on one hand and the method of sum rules on the other. The former assumes a local interaction in terms of quark coordinates, while the gluon field figures in more directly in the later.

It was shown some time ago by Voloshin and Leutwyler separately [1] that large scale fluctuations of the QCD vacuum cannot be described adequately by local potentials. The conclusion found support in the subsequent leading relativistic interaction Hamiltonian of Eichten and Feinberg [2] (see also Lemma in [3]) and refinement at the hands of Marquard and Dosch [4] who considered two extreme cases for the vacuum field’s correlation length relative to those of the heavy quarks bound in a meson. Roughly, for $T_g \ll T_q$ a local description was found to be justified, though for $T_g \gg T_q$ a nonlocality appears making the sum rule approach more suitable.

Potential model builders have taken this to validate the use of local potentials for sufficiently heavy quarks, which is quite right. A quantitative sense of the validity in terms familiar to the language of potential models might be obtained by considering the interaction as a double expansion in the ratios of the two time scales with a common scale, say $\Lambda_{QCD}$, appropriate for the above limits. To any specified order
in $T_g$ then the limits now read, $T_g \to 0$ and $T_g \to \infty$, for local and nonlocal couplings respectively. Finite $T_g \approx 1.0 \text{ GeV}^{-1}$ (from the lattice [5]) then requires a generalization of the analysis in line with intermediate correlations.

This is easily carried out beginning from the expression given in [4] (not to be re-derived here) for the Schwinger function of a singlet $Q\bar{Q}$ pair in an external color field. In the leading dipole approximation [6]

$$G \sim \int [dx] \exp \left[ -\frac{g^2}{36} \int_0^t \int_0^t x(t_1) \cdot x(t_2) \langle E^a(t_1) \cdot E^a(t_2) \rangle_E dt_1 dt_2 \right]$$

$$\approx \int [dx] \exp \left[ \int_0^t dt_1 (x^2 - 2x \cdot \frac{p_m}{m} \zeta) \right]$$

with

$$\zeta = -\frac{g^2}{36} \int_0^\infty d\tau \tau \langle E^a(t_1) \cdot E^a(t_2) \rangle_E, \quad \tau \equiv t_1 - t_2$$

assuming the correlator falls off rapidly for large Euclidean time differences. The integral over $-\frac{g^2}{36} \langle E^a(t_1) \cdot E^a(t_2) \rangle_E$ has been normalized to 1 and $x(t_2) \approx x(t_1) - \tau \dot{x}(t_1)$ to lowest order in quark velocity has been used.

Expression (2) is not difficult to interpret. The nonlocality enters scaled by the time rate of vacuum correlations. For example, should the fields correlate adiabatically with respect to quark motion, corresponding to a stochastic delta correlation, or white noise, so that $\frac{p_m}{m} \zeta \approx 0$, a local potential emerges. But only in this limit. All other correlations lead to nonlocal dynamics the degree of which measured by gluon degrees of freedom as they occur in the fluctuating vacuum. For a general discussion of the effect in the context of the flux tube picture see Isgur [7].

A convenient parameterization for the statistical distribution is provided by the Stochastic Vacuum Model [8]

$$\langle E^a(t_1) \cdot E^a(t_2) \rangle_E = 3\beta[D(\tau) + D_1(\tau) + \tau^2 \frac{\partial^2 D_1}{\partial \tau^2}]$$

with

$$D, D_1 \sim \exp(-|\tau|/T_g)$$

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The model calculation estimates nonlocal contributions to (2) (for $\frac{m}{m} \approx 0.17$) at 50%.

2 Quarkonium Spin Splittings

The qualitative success of the Nonrelativistic Potential Model with linear confinement does not entirely carry over when introduced into relativistic kinematics. The problem has long been thought to be related to a neglect of gluon field momentum. Examples in the study of Regge trajectories are found in [9]. Nonlocal effects might also be relevant to the question of possible Lorentz structures for confinement.

Evaluation of heavy $Q\bar{Q}$ spin splittings for a local potential leads to a widely known argument in favor of dominant scalar confinement. The quantity of interest is the ratio of $\chi$ - state masses [10]

$$r \equiv \frac{M_2 - M_1}{M_1 - M_0} \approx 0.5(exp.) \quad (6)$$

with expected values of less than 0.8 or greater than 2.0 for linear scalar coupling and ranging from 0.8 to 1.4 in the vector case. The other structures are ruled out due to their incompatibility with appropriate nonrelativistic limits arising from pure Lorentz sources. Here the analysis is reconsidered taking nonlocal effects into account.

In the conventional treatment [11] one begins with an expansion of an arbitrary interaction kernel over the five Lorentz invariant amplitudes (scalar, pseudoscalar, vector, axialvector and tensor) in the $q \otimes \bar{q}$ Dirac space

$$K = V_s 1 \otimes 1 + V_{ps} \gamma^5 \otimes \gamma_5 + V_v \gamma^\mu \otimes \gamma_\mu$$

$$+ V_{av} \gamma^\mu \gamma^5 \otimes \gamma_\mu \gamma_5 + \frac{1}{2} V_t \sigma^{\mu\nu} \otimes \sigma_{\mu\nu} \quad (7)$$

A $O(m^{-2})$ potential is then derived in the usual way by a Fourier transform of the transformation matrix or a reduction of the Bethe-Salpeter equation. Both
routes however require the use of free quark propagators of whose adequacy there is some question particularly in the present nonperturbative [12] nonlocal [13] context. An alternative approach possibly more compatible with the presence of the external vacuum field might be a nonlocal minimal coupling, e.g., by way of the instantaneous Salpeter equation. The question will not be gone into here. For comparison with the local result and for the sake of simplicity it is useful to proceed in the usual way as discussed above. The lowest order nonlocal contribution to the amplitudes of (7) in the center of momentum is parameterized as

\[ V_{nl} = \{ \Theta_{ij}(r), p_i p_j \}_{symm} \]  

with

\[ \Theta_{ij}(r) = \theta_1(r)\delta_{ij} + \theta_2(r)\hat{r}_i\hat{r}_j \]  

where \( r \) is the relative coordinate. A similar parameterization is used in [14] where the nonlocality is proposed to resolve the outstanding small baryon splitting puzzle. After adding on the Cornell potential, \(-\frac{4}{3}\alpha_s \frac{a}{r} + ar\), and taking the simplifying, \( \theta_1 = -\theta_2 \equiv \theta \), a straightforward but lengthy calculation yields

\[ r_{scalar} = \frac{1}{5} \frac{8\alpha_s \langle r^{-3} \rangle - \frac{5}{2}a\langle r^{-1} \rangle + 5\langle R_1 \rangle}{2\alpha_s \langle r^{-3} \rangle - \frac{1}{4}a\langle r^{-1} \rangle + \frac{1}{2}\langle R_1 \rangle} \]  

\[ r_{vector} = \frac{1}{5} \frac{8\alpha_s \langle r^{-3} \rangle + 7a\langle r^{-1} \rangle + \frac{5}{2}\langle \mathcal{R}_{(2-1)} \rangle}{2\alpha_s \langle r^{-3} \rangle + a\langle r^{-1} \rangle + \frac{1}{2}\langle \mathcal{R}_{(1-0)} \rangle} \]  

with \( R \) and \( \mathcal{R} \) given in the appendix. It is clearly not possible to establish a numerical range for these expressions without more information on the nonlocality. As it stands the analysis is indeterminate, favoring no Lorentz structure for confinement over another.
3 Summary and Discussion

The main point emphasized here has been that nonperturbative gluon degrees generally arise in the hadronic bound state in the form of nonlocal quark dynamics. This has been shown to follow from a simple variation on the analysis of [4]. A corollary is that the nonlocality appears whenever quark motion is taken into account - e.g., in the leading relativistic contributions to the static local interaction potential. Consequences for quarkonium spin splitting ratios have been considered with a nonlocal parameterization.

It should be added for completeness that that the pseudoscalar, axialvector, and tensor Lorentz structures fail to reduce individually to suitable static limits does not exclude them apriori from participation at higher orders. On the contrary. The nonrelativistic limit suggests nothing beyond what might be said of itself. This observation included leaves the local analysis of [11] indeterminate also. While scalar confinement may or may not be the most simple ansatz ( according to one’s sense of the simple ) it is certainly not an unavoidable conclusion; vector + pseudoscalar, vector + axialvector + tensor among other combination spin structures for a confining local interaction presumably yield equal or better predictions.

What is needed of course is an understanding of the mechanism by which nonperturbative via nonlocal forces enter into the QCD bound state and effective means by which reliable estimates of the effect can be obtained. The Wilson loop occurs naturally in gauge invariant formulations of the state. It (and so interactions derived from it) is manifestly nonlocal for nonstatic quarks. Its evaluation in the Minimal Area Law, the Stochastic Vacuum Model, and Dual QCD have recently been carried out by Brambilla and Vario [15]. These are leading order approximations from three mutually distinct expansions of the Wilson loop - no one contained entirely within another.

Minimal substitution of the QCD inspired relativistic flux tube [16] into the linear Dirac equation [17] is an example of a promising nonlocal model with many
attractive features: correct Regge structure and spin orbit sign, to name two. As a
model however it is to be measured against both observation and the fundamental
theory. On the other hand, differences between evaluations of the Wilson loop in
any one of the above mentioned approximations [18] is amenable to unambiguous,
mathematical resolution. These points seem to have been missed by ref[19].

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4 Appendix

\[ R_{(2-1)} = -6R_1 - \frac{1}{5}R_2 - \frac{1}{10}R_3(2 \frac{\partial}{\partial r} + \frac{3}{r}) + \frac{1}{10}R_4(\frac{1}{r} \frac{\partial}{\partial r} - \frac{4}{r^2}) \] (12)

\[ R_{(1-0)} = -3R_1 + \frac{1}{2}R_2 + \frac{1}{4}R_3(2 \frac{\partial}{\partial r} + \frac{3}{r}) - \frac{1}{4}R_4(\frac{1}{r} \frac{\partial}{\partial r} - \frac{4}{r^2}) \] (13)

with

\[ R_1 = \frac{1}{r^2} \left[ \theta(\frac{\partial^2}{\partial r^2} + \frac{2}{3} \frac{1}{r^2}) + \theta'(\frac{\partial}{\partial r} - \frac{1}{r}) - \frac{1}{3} \theta'' \right] \] (14)

\[ R_2 = \frac{2}{r} \left[ \frac{1}{r} \theta(3 \frac{\partial^2}{\partial r^2} + \frac{5}{r} \frac{\partial}{\partial r} + \frac{8}{3} \frac{1}{r^2}) - 2\theta'(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{2}{3} \frac{1}{r^2}) + \frac{1}{3} \theta'' + \frac{1}{3} \theta''' \right] \] (15)

\[ R_3 = \frac{4}{r} \left[ -\frac{1}{r} \theta(\frac{\partial}{\partial r} + \frac{3}{2} \frac{1}{r}) + \theta' \frac{\partial}{\partial r} \right] \] (16)

\[ R_4 = -\frac{2}{r^2} \theta \] (17)

References


