Generalized canonical quantization of bosonic string model in massive background fields

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A method of constructing a canonical gauge invariant quantum formulation for a non-gauge classical theory depending on a set of parameters is advanced and then applied to the theory of closed bosonic string interacting with massive background fields. It is shown that within the proposed formulation the correct linear equations of motion for background fields arise.

1. Introduction

BFV method is the most powerful realization of canonical quantization procedure which provides unitarity at the quantum level and consistency of a theory symmetries and dynamics. Now it has been studied in details \cite{1,2} but formulation of new models in quantum field theory requires investigation of its yet unexplored aspects.

One of these aspects arise from bosonic string theory coupled to background fields \cite{3}. At the quantum level string models should be conformal invariant and this requirement leads to restrictions on spacetime dimension in the case of free string theory and to effective equations of motion for massless background fields in the case of string theories coupled to background \cite{3} (see also the reviews \cite{4}). In terms of covariant functional methods this condition appears as independence of quantum effective action on the conformal factor of two-dimensional metrics or as vanishing of renormalized operator of the energy-momentum tensor trace.

According to the prescription \cite{5} generally accepted in functional approaches to string theory dynamical variables should be treated differently. Namely, functional integration is carried out only over string coordinates $X^{\mu}(\tau, \sigma)$ while components of two-dimensional metrics $g_{ab}(\tau, \sigma)$ are considered as external fields. Then one demands the result of such an integration to be independent on the conformal factor and the integrand over $g_{ab}(\tau, \sigma)$ reduces to finite dimensional integration over parameters specifying string world sheet topologies. This prescription differs from the standard field theory rules when functional integral is calculated over every variable independently. In string theory such an independent integration would lead to appearance at the quantum level of an extra degree of freedom connected with two-dimensional gravity \cite{6}.

This approach can also be applied to string theory interacting with massive background fields which is not classically conformal invariant \cite{7}. One demands that operator of the energy-momentum tensor trace vanish no matter whether the corresponding classical action is conformal invariant or not. As was shown in \cite{8,9} it gives rise to effective equations of motion for massive background fields. Unfortunately, in the case of closed string theory covariant approaches did not reproduce the full set of correct linear equations of motion for massive background fields. Namely, the tracelessness condition on massive
tensor fields has not been obtained neither in standard covariant perturbation approaches [8] nor within the formalism of exact renormalization group [10]. So there exists a problem how to derive the correct equations for massive background fields.

Moreover, from general point of view the requirement of quantum Weyl invariance of string theory with massive background fields means that a non-gauge classical theory depending on a set of parameters is used for constructing of a quantum theory that is gauge invariant under some special values of the parameters. Such a situation occurs in string theory if interaction with massless dilaton, tachyon or any other massive field is turned on. As we consider canonical approach to be the only completely consistent method for constructing quantum theories then every step of any quantization procedure should be justified by an appropriate prescriptions within canonical formulation. So the general problem arising from string theories is how to describe in terms of canonical quantization construction of gauge invariant quantum theory starting with a classical theory without this invariance.

Due to the general BFV method one should construct hamiltonian formulation of classical theory, find out all constraints and calculate algebra of their Poisson brackets. Then one defines fermionic functional \( \Omega \) generating algebra of gauge transformations and bosonic functional \( \Omega \) containing information of theory dynamics. Quantum theory is consistent provided that the operator \( \hat{\Omega} \) is nilpotent and conserved in time. To illustrate how the prescription works we apply it to the theory of closed bosonic string coupled to masssive background fields and show that correct linear equations of motion are produced.

2. General prescription

Consider a system described by a Hamiltonian

\[
H = H_0(a) + \lambda^\alpha T_\alpha(a)
\]

where \( H_0(a) = H_0(q, p, a) \), \( T_\alpha(a) = T_\alpha(q, p, a) \) and \( q \), \( p \) are canonically conjugated dynamical variables; \( a = a_\alpha \) and \( \lambda^\alpha \) are external parameters of the theory.

We suppose that \( T_\alpha(a) \) are some functions of the form

\[
T_\alpha(a) = T^{(0)}_\alpha(a) + T^{(1)}_\alpha(a)
\]

and closed algebra in terms of Poisson brackets is formed by \( T_\alpha^{(0)}(a) \), not by \( T_\alpha(a) \):

\[
\{ T^{(0)}_\alpha(a), T^{(0)}_\beta(a) \} = T^{(0)}_\gamma(a) U^\gamma_{\alpha\beta}(a)
\]

\[
\{ H_0(a), T^{(0)}_\alpha(a) \} = T^{(0)}_\gamma(a) V^\gamma_\alpha(a)
\]

Such a situation may occur, for example, if \( T^{(0)}_\alpha(a) \) correspond to a free gauge invariant theory and \( T^{(1)}_\alpha(a) \) describe a perturbation spoiling gauge invariance. At the quantum level both the algebras of \( T^{(0)}_\alpha(a) \) and \( T_\alpha(a) \) are not closed in general case.

We define quantum operators \( \Omega \) and \( H \) as follows:

\[
\Omega = c^\alpha T_\alpha(a) - \frac{1}{2} U^\gamma_{\alpha\beta}(a) : \mathcal{P}_\gamma c^\alpha c^\beta : \\
H = H_0(a) + V^\gamma_\alpha(a) : \mathcal{P}_\gamma c^\alpha :
\]

where \( : \) stands for some ordering of ghost fields.

In general case \( \Omega^2 \neq 0 \) and \( d\Omega/dt \neq 0 \). However, if there exist some specific values of parameters \( a \) that make the operator \( \Omega \) to be nilpotent
and conserved then the corresponding quantum theory is gauge invariant. Thus it is possible to construct a quantum theory with given gauge invariance that is absent at the classical level.

3. String theory in massive fields

As an example where the described procedure really works and leads to nilpotency and conservation conditions for the operator $\Omega$ with non-trivial solutions for parameters $a$ we consider closed bosonic string theory coupled with background fields of tachyon and of the first massive level. We will restrict ourselves by linear approximation in background fields because an adequate treatment of non-linear (interaction) terms is known to demand non-perturbative methods [15]. This approximation will be enough to establish consistency of background fields dynamics with structure of the corresponding massive levels in string spectrum.

The theory is described by the classical action

$$ S = -\frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{-g} \left\{ \frac{1}{2} g^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu} 
+ g^{ab} g^{cd} \partial_a X^\mu \partial_b X^\nu \partial_c X^\lambda \partial_d X^\kappa F^1_{\mu\nu\lambda\kappa}(X) 
+ g^{ab} \epsilon^{cd} \partial_a X^\mu \partial_b X^\nu \partial_c X^\lambda \partial_d X^\kappa F^2_{\mu\nu\lambda\kappa}(X) 
+ \alpha' R^{(2)} g^{ab} \partial_a X^\mu \partial_b X^\nu W^1_{\mu\nu}(X) 
+ \alpha' R^{(2)} \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu W^2_{\mu\nu}(X) 
+ \alpha^2 R^{(2)} R^{(2)} C(X) + Q(X) \right\}, \quad (5) $$

$\sigma^a = (\tau, \sigma)$ are coordinates on string world sheet, $R^{(2)}$ is scalar curvature of two-dimensional metrics $g^{ab}$, $\eta_{\mu\nu}$ is Minkowski metrics of $D$-dimensional spacetime, $Q$ is tachyonic field and $F$, $W$, $C$ are background fields of the first massive level in string spectrum. As was shown in [8] all other possible terms with four two-dimensional derivatives in classical action are not essential and string interacts with background fields of the first massive level only by means of the terms presented in (5).

Components of two-dimensional metrics $g_{ab}$ should be considered as external fields, otherwise the classical equations of motion $\delta S/\delta g_{ab} = 0$ would be fulfilled only for vanishing background fields. This treatment is similar to covariant methods where functional integral is calculated over $X^\mu$ variables.

After the standard parametrization of metrics

$$ g_{ab} = \epsilon^\gamma \left( \begin{array}{cc} \lambda_1^2 & \lambda_0^2 \\ \lambda_0 \lambda_1 & 1 \end{array} \right) \quad (6) $$

the Hamiltonian in linear approximation in background fields takes the form

$$ H = \int d\sigma \left( \lambda_0 T_0 + \lambda_1 T_1 \right), \quad (7) $$

where

$$ T_0 = T_0^{(0)} + T_0^{(1)}, \quad T_1 = T_1^{(0)} = P^\mu X^\mu, $$

$$ T_0^{(0)} = (1/2) \left( 2\pi\alpha' P^2 + (2\pi\alpha')^{-1} X^2 \right), $$

$$ T_0^{(1)} = (1/2\pi) (\alpha')^{-1} \epsilon^\gamma Q $$

$$ + (\alpha')^{-1} \epsilon^{-\gamma} Y^{+\mu} Y^{+\nu} Y^{-\lambda} Y^{-\kappa} F_{\mu\nu,\lambda\kappa} + R^{(2)} Y^{+\mu} Y^{-\nu} W_{\mu\nu} + \alpha' \epsilon^\gamma R^{(2)} R^{(2)} C \right\} \quad (8) $$

$P^\mu$ are momenta canonically conjugated to $X^\mu$, $X^\mu = \partial X^\mu/\partial \sigma$ and we introduced the following notations:

$$ Y^{\pm\mu} = 2\pi\alpha' P^\mu \mp X^\mu, \quad W_{\mu\nu} = -W^{1\mu}_{\mu\nu} + W^{2\mu}_{\mu\nu}, $$

$$ F_{\mu\nu,\lambda\kappa} = 2 F^{1}_{\mu\nu,\lambda\kappa} + 2 F^{1}_{\mu\nu,\lambda} - 2 F^{2}_{\mu\nu,\kappa} - 2 F^{2}_{\nu\kappa,\lambda}(9) $$

$T_0^{(0)}$ and $T_1^{(0)}$ represent constraints of free string theory and form closed algebra in terms of Poisson brackets. $\lambda_0$ and $\lambda_1$ play the role of external fields and so $T_0$ and $T_1$ cannot be considered as constraints of classical theory. In free string theory conditions $T_0^{(0)} = 0$, $T_1^{(0)} = 0$ result from conservation of canonical momenta conjugated to $\lambda_0$ and $\lambda_1$. According to our prescription in string theory with massive background fields $\lambda_0$ and $\lambda_1$ can not be considered as dynamical variables, there are no corresponding momenta and conditions of their conservation do not appear.

The role of parameters $a$ in the theory under consideration is played by background fields $Q$,
$F$, $W$, $C$ and conformal factor of two-dimensional metrics $\gamma(\tau, \sigma)$. The theory (7) is of the type (1) with $H_0 = 0$, structural constants of classical algebra being independent on time.

Direct calculations up to terms linear in background fields show that the operator $\Omega$ defined according to (4) is nilpotent and conserves provided that the following conditions are fulfilled:

$$D = 26, \quad \gamma = \text{const},$$

$$\left(\partial^2 + 4/\alpha'\right)Q = 0, \quad \left(\partial^2 - 4/\alpha'\right)F_{\mu\nu,\lambda\kappa} = 0,$$

$$\partial^\mu F_{\mu\nu,\lambda\kappa} = 0, \quad \partial^\lambda F_{\mu\nu,\lambda\kappa} = 0,$$

$$F^\mu_{\mu,\lambda\kappa} = 0, \quad F^\mu_{\mu,\lambda} = 0.$$  \hspace{1cm} (10)

The condition $\gamma = \text{const}$ means that string world sheet should be flat $R^{(2)} = 0$ and so the background fields $W$ and $C$ disappear from the classical action (5). The eqs. (10) show that first massive level is described by a tensor of fourth rank which is symmetric and traceless in two pairs of indices and transverse in all indices. This exactly corresponds to the closed string spectrum and so our approach gives the full set of correct linear equations for massive background fields.

The described example demonstrates a possibility to construct canonical formulation of quantum theory invariant under gauge transformations that are absent at the classical level. The proposed method opens up a possibility for deriving interacting effective equations of motion for massive and massless background fields within the framework of canonical formulation of string models and provides a justification of covariant functional approach to string theory.

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