Wall fluctuation modes and tensor CMB anisotropy in open inflation models

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We calculate the spectrum of large angle cosmic microwave background (CMB) anisotropies due to quantum fluctuations of the gravitational wave modes in one-bubble open inflation models. We find the bubble-wall fluctuation modes, which had been thought to exist discretely in previous analyses, are actually contained in the continuous spectrum of gravitational wave modes when the gravitational coupling is correctly taken into account. Then we find that the spectrum of the tensor CMB anisotropy can be decomposed into the part due to the wall fluctuation modes and that due to the usual gravitational wave modes in a way which is almost model-independent, even when the gravitational coupling is strong. We also discuss observational constraints on the model parameters. We find that an appreciable portion of the parameter space is excluded but the remaining allowable region is still wide enough to leave the one-bubble scenario viable.

I. INTRODUCTION

The inflationary universe scenario is one of the most successful scenarios that can explain the homogeneity and isotropy of the universe on very large scales as well as the inhomogeneous structure on smaller scales such as galaxies and clusters of galaxies. The standard model of inflation predicts that our universe is spatially flat with $\Omega_0 = 1$. However, there are increasing observational evidences that indicate $\Omega_0 < 1$ [1]. Consequently the standard model of inflation needs to be modified to account for $\Omega_0 < 1$. As one of such modifications, the one-bubble open inflationary scenario has attracted much attention recently.

The basic idea of the one-bubble open inflationary universe scenario was proposed by Gott III [2] fifteen years ago and was revived recently by several authors [3–5]. In this scenario, there are two stages of inflation. The universe is in the false vacuum initially and this first stage of inflation is assumed to last long enough so that the universe becomes sufficiently homogeneous and isotropic and becomes well-approximated by a pure de Sitter space. Then the nucleation of a vacuum bubble occurs through quantum tunneling. This process is described by a Euclidean bounce solution, which is a non-trivial classical solution of the field equation in Euclidean spacetime having $O(4)$-symmetry [6,7]. Then the expanding bubble after nucleation is described by the classical solution obtained by analytic continuation of the bounce solution to Lorentzian spacetime. Owing to the $O(4)$-symmetry of the bounce solution, the expanding bubble has $O(3,1)$-symmetry. This implies that the system is homogeneous and isotropic on the hyperbolic time slicing inside the bubble and that the nucleation of a bubble can be regarded as the creation of an open universe [2,8]. Then it is assumed that the vacuum energy inside the bubble is non-zero and the second stage of inflation commences. One assumes this inflation is of slow rollover type and it lasts just enough to make the present day density parameter $\Omega_0$ appreciably smaller than unity.

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There are essentially two types of one-bubble open inflation models: single field models \([3,4]\) and two field models \([5,9,10]\). In the former case, the false vacuum decay and the subsequent stage of inflation inside the bubble are both governed by a single scalar field, while in the latter case they are driven by two different fields.

In both cases, the quantum fluctuations of the inflation field at the second stage of inflation give rise to curvature perturbations of the universe which account for the present large scale structure of the universe. The spectrum of these fluctuations was calculated and the resulting CMB anisotropy was evaluated by several authors \([3,4,11-15]\). Through these works, it was clarified that there could appear discrete modes which have a coherence scale greater than the spatial curvature scale of the universe (the so-called de Sitter supercurvature modes) \([16,14]\). Their existence could be harmful because of their large contribution to the CMB anisotropy on largest angular scales \([11]\). In fact, the simplest two field model which assumes two decoupled scalar fields \([5]\) was shown to be unsuccessful in this respect \([17]\). Fortunately, however, in most of the other models the mass of the inflation field at the false vacuum is large compared with the Hubble mass scale there and in such cases these discrete modes were found to disappear.

In the case of a single field model, it was argued that there exists yet another type of discrete supercurvature modes \([18,12,13,3]\). They correspond to the fluctuations of the bubble-wall. A peculiar nature of these modes is that they can be regarded as tensor-type modes which are spatially transverse-traceless by a suitable coordinate transformation. It was not clear if these fluctuations contribute also in the case of a two field model, since the wall fluctuations are associated with the scalar field which mediates the false vacuum decay but not with the inflaton field inside the bubble.

Recently, a method to calculate the spectrum of quantized gravitational waves has been developed \([19,20]\). Then it has been shown that the spectrum shows infrared divergence on pure de Sitter space but it disappears once the existence of the bubble wall is appropriately taken into account. It has been also shown that the spectrum of gravitational wave modes is continuous and there exists no discrete modes \([19]\). At the same time, it has been shown that the discrete bubble-wall fluctuation modes, which had been found in previous analyses, cease to exist once the coupling between the scalar field and the metric perturbation is correctly taken into account \([19]\). Thus the situation has become rather confusing.

In this paper, using the spectrum of gravitational wave modes obtained in Ref. \([19]\), we calculate the tensor CMB anisotropy on large angular scales. The tensor CMB anisotropy in one-bubble inflation models has been calculated recently by Hu and White \([21]\). However, the gravitational wave spectrum they have assumed for their calculations seems to be rather ad hoc and they have not discussed constraints on the model parameters. Here we investigate the dependence of the tensor CMB anisotropy on the model parameters in detail. In doing so, we resolve the problem of the wall fluctuation modes. We then discuss observational constraints on the parameters of one-bubble inflation models. The paper is organized as follows. In section 2, we review the solution of the bubble configuration with gravity under the thin-wall approximation and consider the wall fluctuation modes. We find the wall fluctuation modes are transmuted into a part of the continuous spectrum of the gravitational wave modes when the gravitational coupling is taken into account, thus resolving the problem of the wall fluctuation modes. In section 3, we calculate the spectrum of the tensor CMB anisotropy and show that it can be cleanly separated into two parts; one due to the wall fluctuation modes and the other due to the usual gravitational wave modes. In section 4, using the results of section 3, we consider constraints on the model parameters from the observed CMB anisotropy by COBE. We find the allowable region of the parameter space is relatively small but still large enough to maintain the viability of one-bubble inflation models for an open universe. Finally, section 5 is devoted to summary and discussion.

## II. Thin-Wall Bubble and Wall Fluctuation Modes

In this section, we first review an \(O(4)\)-symmetric bubble solution under the thin-wall approximation \([22,13]\) and how the bubble wall fluctuations are described when the coupling between the scalar-type and tensor-type perturbations is neglected. Then we consider vacuum fluctuations of gravitational wave modes and show that these vacuum fluctuations exactly coincide with the bubble wall fluctuations in the limit of small vacuum energy difference between the true and false vacua or in the limit of small bubble radius.

An \(O(4)\)-symmetric bubble configuration that mediates the false vacuum decay is described by the Euclidean metric,

\[
\frac{ds^2}{g_E} = d\tau^2 + a_E^2(\tau) \left( d\rho^2 + \sin^2 \rho \, d\Omega^2 \right),
\]

and by the scalar field \(\sigma\) which depends only on \(\tau; \sigma = \sigma(\tau)\). The Euclidean action for this configuration is

\[
S_E = 2\pi^2 \int d\tau \frac{a_E^3}{\kappa} \left[ \frac{3}{2} \left( \frac{\dot{a}_E}{a_E} \right)^2 + \frac{1}{2} \dot{\sigma}^2 + V(\sigma) \right],
\]

\[
= 2\pi^2 \int d\tau \left[ \frac{a_E^2}{\kappa} \dot{\sigma}^2 - \frac{3}{2} \kappa \dot{a}_E^2 \right],
\]

where \(\kappa = 8\pi G = 8\pi / M_{Pl}^2\) \(\dot{} = d/d\tau\) and the second equality follows from the constraint equation (or Euclidean Friedmann equation).
\[ \left( \frac{\dot{a}_E}{a_E} \right)^2 - \frac{1}{a_E^2} = \frac{\kappa}{3} \left( \frac{1}{2} \sigma^2 - V \right). \]  

(2.3)

Assuming that the bubble wall is infinitesimally thin, we have

\[ \dot{a}_E^2 = \begin{cases} 
1 - \frac{\kappa}{3} V_F a_E^2 =: 1 - H_L^2 a_E^2 & \text{in false vacuum,} \\
1 - \frac{\kappa}{3} V_T a_E^2 =: 1 - H_R^2 a_E^2 & \text{in true vacuum,}
\end{cases} \]

(2.4)

where \( V_T \) and \( V_F \) are the vacuum energies at the true and false vacua, respectively. The above equation can be easily solved to give

\[ a_E = \begin{cases} 
\frac{1}{H_L} \cos H_L \tilde{\tau}, & -\frac{\pi}{2H_L} \leq \tilde{\tau} < \tilde{\tau}_W, \\
\frac{1}{H_R} \cos H_R \tau, & \tau_W < \tau \leq \frac{\pi}{2H_R},
\end{cases} \]

(2.5)

where \( \tilde{\tau} = \tau - \tau_W + \tilde{\tau}_W \) and \( \tau_W \) is the value of \( \tau \) at the wall. Note that the continuity of \( a_E \) at the wall implies

\[ a_E(\tau_W) = \frac{1}{H_L} \cos H_L \tilde{\tau}_W = \frac{1}{H_R} \cos H_R \tau_W, \]

(2.6)

and hence \( \cos H_L \tilde{\tau}_W > \cos H_R \tau_W \) since \( H_L > H_R \). Note also that \( \tau_W, \tilde{\tau}_W > 0 \) because the maximum of the scale factor \( a_E \) should be on the false vacuum side if the bubble configuration describes the false vacuum decay. We should mention, however, that this condition on \( \tau_W \) and \( \tilde{\tau}_W \) comes from the limit of applicability of our formalism [23]. Hence models which violate this condition are not excluded a priori. We do not consider such models here simply because we do not know the outcome.

Inserting the solution (2.5) to Eq. (2.2), we then obtain the reduced action for the thin-wall bubble:

\[ S_{TW} = 2\pi^2 \left[ S_1 R^3 - \frac{2}{\kappa} \left( \frac{(1 - H_L^2 R^2)^{3/2} - 1}{H_L^2} - \frac{(1 - H_R^2 R^2)^{3/2} - 1}{H_R^2} \right) \right], \]

(2.7)

where \( R \) and \( S_1 \) are the radius and the surface tension of the wall, respectively:

\[ R := a_E(\tau_W), \quad S_1 := \int \sigma^2 d\tau. \]

(2.8)

Then the wall radius is determined by putting \( dS_{TW}/dR = 0 \), which gives

\[ \frac{\kappa}{2} RS_1 = (1 - H_R^2 R^2)^{1/2} - (1 - H_L^2 R^2)^{1/2}. \]

(2.9)

This can be solved for \( R \) by elementary algebra. We find

\[ R = \frac{\kappa S_1}{\sqrt{(H_L^2 - H_R^2 + (\frac{\kappa}{2} S_1)^2) + H_R^2 \kappa^2 S_1^2}} = \frac{3S_1}{\sqrt{(\Delta V + 6\pi GS_1^2)^2 + 24\pi GV_T S_1^2}}, \]

(2.10)

where \( \Delta V = V_F - V_T \). We note that one must have

\[ \Delta V > 6\pi GS_1^2, \]

(2.11)

which comes from the condition \( \tau_W, \tilde{\tau}_W > 0 \). For our present purpose, however, it is convenient to return to Eq. (2.9) and rewrite it as

\[ \frac{\kappa}{2} RS_1 = \Delta s := \sin H_R \tau_W - \sin H_L \tilde{\tau}_W. \]

(2.12)

Note that \( \frac{\kappa}{2} RS_1 < 1 \) for any bubble configuration which describes the \( O(4) \)-symmetric false vacuum decay. One could consider a configuration in which \( \tilde{\tau}_W < 0 \), in which case \( \frac{\kappa}{2} RS_1 \) could be greater than unity. However, such a configuration would not describe the false vacuum decay as mentioned before.
In order to clarify the parameter-dependence of the bubble configuration, it is convenient to introduce the following non-dimensional parameters:

\[ \alpha := \frac{\Delta V}{6\pi G S_1^2}, \quad \beta := \frac{V_T}{6\pi G S_1^2}. \]  

(2.13)

Note that \( \alpha > 1 \) because of the condition (2.11). In terms of these, we have

\[ \cos H_R \tau_W = \frac{2\sqrt{\beta}}{\sqrt{(\alpha + 1)^2 + 4\beta}}, \quad \sin H_R \tau_W = \frac{\alpha + 1}{\sqrt{(\alpha + 1)^2 + 4\beta}}, \]

\[ \cos H_L \tilde{\tau}_W = \frac{2\sqrt{\alpha + \beta}}{\sqrt{(\alpha + 1)^2 + 4\beta}}, \quad \sin H_L \tilde{\tau}_W = \frac{\alpha - 1}{\sqrt{(\alpha + 1)^2 + 4\beta}}, \]  

(2.14)

and

\[ \frac{\kappa}{2} R S_1 = \Delta s = \frac{2}{\sqrt{(\alpha + 1)^2 + 4\beta}}. \]

(2.15)

We readily see that \( \Delta s \to 0 \) as \( \alpha \to \infty \) \( (H_L R = \cos H_L \tilde{\tau}_W \to 0) \) or \( \beta \to \infty \) \( (\Delta V/V_T \to 0) \). Now let us consider the wall fluctuation modes. Assuming that the coupling of the scalar field perturbation to the metric perturbation can be neglected, it has been shown that quantum fluctuations of the bubble wall induced by the false vacuum decay give rise to the scalar field perturbation on the time constant hypersurface of an open universe inside the bubble, and consequently give rise to the curvature perturbation on the comoving hypersurface on which \( \sigma \) is constant \[12–14\].

The background metric for the open universe inside the bubble is given by analytic continuation of the Euclidean metric (2.1) as

\[ ds^2 = -dt^2 + a(t)^2 \left[ \left( -\frac{\pi}{2 H_R} \right) + \sinh^2 r \right] d\Omega^2 \]

where

\[ t = i \left( \tau - \frac{\pi}{2 H_R} \right), \quad r = i \rho, \quad a(t) = \frac{\sinh H_R t}{H_R}, \]

(2.17)

and \( \gamma_{ij} \) is the metric on the unit hyperboloid. On these coordinates, the spatial curvature perturbation due to the wall fluctuation modes is expressed as

\[ \mathcal{R}_c = \sum_{lm} \mathcal{R}_W(t) \mathcal{Y}_{2lm}(r, \Omega), \]

(2.18)

where \( \mathcal{Y}_{2lm} \) are the spatial harmonics given by

\[ \mathcal{Y}_{2lm} = \sqrt{\frac{\Gamma(l + 3)\Gamma(l - 1)}{2 \Gamma_3/2}} \frac{F_{3/2 - 1/2}^{-1} \left( \cosh r \right)}{\sinh^2 r} \mathcal{Y}_{lm}(\Omega). \]

(2.19)

The mean square value of \( \mathcal{R}_W \) has been found to be \[14\]

\[ |\mathcal{R}_W|^2 = \frac{H_R^2}{R S_1} = \frac{\kappa H_R^2}{2\Delta s}. \]

(2.20)

Note that this diverges for \( RS_1 \to 0 \), i.e., in the limit of small vacuum energy difference or small wall radius.

An important property of this curvature perturbation is that it can be regarded as a tensor-type perturbation which is spatially transverse-traceless \[18,12,14\]. Namely it is equivalent to the metric perturbation,

\[ H_{ij} = -\frac{\dot{a}}{a} H_R^{-1} \mathcal{R}_W \mathcal{Y}_{ij} = : H_W \mathcal{Y}_{ij}, \]

(2.21)

where \( H_{ij} \) is the perturbation of the spatial metric \( \gamma_{ij} \), \( \mathcal{Y}_{ij} \) is defined by

\[ \mathcal{Y}_{ij} := \mathcal{Y}_{ij} - \gamma_{ij} \mathcal{Y}, \]

(2.22)

and we have suppressed the indices \( \{2lm\} \) of \( \mathcal{Y}_{2lm} \). The spatial tensor \( \mathcal{Y}_{ij} \) is transverse-traceless and corresponds to an even parity tensor harmonic with the eigen value \( p^2 = 0 \). Thus the bubble wall fluctuation modes can be regarded as even parity gravitational wave modes which exist discretely at \( p^2 = 0 \). The mean square value of \( H_W \) is given by
\[ |H_W|^2 \rightarrow 4|R_W|^2 = \frac{2\kappa H_R^2}{\Delta s}, \quad (2.23) \]

for \( H_R t \gg 1 \).

Let us now turn to the quantum fluctuations of gravitational wave modes. It has been shown that the spectrum of gravitational wave modes consists of continuous modes only and there appears no supercurvature mode [19]. As for even parity \( p^2 = 0 \) modes, for which there appears degeneracy between the scalar-type and tensor-type perturbations, it has been shown that the discrete wall fluctuation modes cease to exist once the coupling between the scalar field perturbation and the gravitational wave perturbation is fully taken into account [19]. Then an immediate question is where the wall fluctuation modes go. A peculiar feature of the gravitational wave spectrum is that it diverges in the limit \( p \rightarrow 0 \) in the pure open de Sitter space but it becomes finite once the presence of the bubble wall is taken into account [19,20]. Thus one suspects if this feature is related to the problem of wall fluctuation modes. Below we show that the wall fluctuation modes are in fact contained in the continuous spectrum of even parity gravitational wave modes in the vicinity of \( p = 0 \).

We confine our attention to the even parity gravitational wave perturbation and express it as

\[ H_{ij} = \int_{-\infty}^{\infty} dp \sum_{lm} U_{p,lm}(t) Y_{ij}^{plm}(r, \Omega), \quad (2.24) \]

where \( Y_{ij}^{plm} \) are the even parity tensor harmonics which are normalized as

\[ \int d\Sigma \gamma_{ii'} \gamma_{jj'} Y_{ij}^{plm} Y_{ij'}^{p'l'm'} = \delta(p - p') \delta_{l,l'} \delta_{m,m'}, \quad (2.25) \]

With this normalization, it can be shown that \( Y_{ij} \) given by Eq. (2.22) coincides with \( Y_{ij}^{plm} \) evaluated at \( p = 0 \) [19]:

\[ Y_{ij} = Y_{ij}^{plm} \big|_{p = 0} := Y_{ij}^0. \quad (2.26) \]

By quantizing \( H_{ij} \) and assuming that the state is in the Euclidean vacuum, the spectrum of \( U_{p,lm} \) has been found to be [19]

\[ |U_p|^2 := \sum_{\pm \pm p,lm} (U_{\pm p,lm}^2) = \frac{4\kappa H_R^2 \coth \pi p}{2p(1 + p^2)} (1 - y), \quad (2.27) \]

where

\[ 1 - y = 1 - \frac{(\Delta s)^2 \cos bp + 2p \Delta s \sin bp}{(4p^2 + (\Delta s)^2) \cosh \pi p}, \quad (2.28) \]

with \( \Delta s \) being defined in Eq. (2.12) and

\[ b := \ln \left( \frac{1 + \sin H_R \tau_W}{1 - \sin H_R \tau_W} \right) = \ln \left( \frac{\sqrt{(\alpha + 1)^2 + 4\beta} + \alpha + 1}{4\beta} \right). \quad (2.29) \]

In the limit \( \Delta s \ll 1 \), i.e., \( \alpha \) or \( \beta \) is large, assuming \( b \lesssim \Delta s^{-1} \), which holds except for an unrealistic case of exponentially small \( \beta \), the spectrum (2.27) is sharply peaked around \( p = 0 \) and can be approximated as

\[ |U_p|^2 \approx \frac{2\kappa H_R^2}{\pi} \frac{1}{p^2 + (\Delta s/2)^2}. \quad (2.30) \]

Then we can regard the gravitational wave perturbation as being dominantly given by the \( p = 0 \) modes only with the mean square amplitude given by

\[ \int_0^{\infty} dp |U_p|^2 \approx \frac{2\kappa H_R^2}{\Delta s}. \quad (2.31) \]

We find this mean square amplitude exactly coincides with that of the wall fluctuation modes, Eq. (2.23). Therefore, since \( Y_{ij} = Y_{ij}^0 \), we conclude that the gravitational wave perturbation in this case is actually the one due to the wall fluctuations. This is a reasonable result. Since the limit of small \( \Delta s (= (\kappa/2)RS_1) \) can be regarded as the limit of weak
gravity (i.e., $\kappa \to 0$), we should obtain the same result as the one obtained by neglecting the gravitational degrees of freedom.\footnote{The form of Eq. (2.30) suggests that the pole at $p = i\Delta s/2$ may have some physical significance for description of the wall fluctuation modes. In fact, a detailed analysis of the behavior of mode functions in the region outside the lightcone emanating from the origin $t = r = 0$ (the region $C$ in Ref. [14] or [19]) shows that the spacetime with the bubble wall admits a quasi-normal mode of the metric perturbation at $p = i\Delta s/2$ which satisfies the purely outgoing boundary condition there. It will be extremely interesting if the contribution of the wall fluctuation modes to the spectrum can be described as a quantum excitation of the quasi-normal mode.}

Here we comment on an important fact. In the previous analyses of the wall fluctuation modes [12–14], in which the coupling to gravity was neglected, it was not clear if these modes would contribute to the temperature anisotropy in the case of two field models of open inflation. The reason is that the wall fluctuations are associated with a scalar field which causes the false vacuum decay but not with another scalar field which drives the subsequent inflation, and the fluctuations of the latter field are thought to be responsible for the curvature perturbation of the universe. Now it has become clear that the wall fluctuations do contribute to the temperature anisotropy even in the case of two field models. This implies that the wall fluctuation modes give rise to a constraint not only on a single field model but also on a two field model.

We also note that $|U_p|^2 p^3 \to 2 s H_r^2$ for $p \gg 1$. Hence the usual scale-invariant spectrum is recovered on scales smaller than the curvature scale. The spectrum consists of two distinguishable parts. What happens in the present case is that the part due to the wall fluctuation modes dominate over the scale-invariant part by a factor $1/\Delta s \gg 1$. Then it is natural to speculate that the gravitational wave spectrum can be decomposed into two parts even if $\Delta s$ is not small; one due to the wall fluctuations and the other due to the usual vacuum fluctuations. As a supporting evidence, we show the spectrum $|U_p|^2$ for various choices of the model parameters in Fig. 1. One sees that all the spectra are identical for $p \gtrsim 1$ while they exhibit similar behavior except for the position and the amplitude of the maximum for $p \lesssim 1$. In the next section, we will evaluate the large angle CMB anisotropy due to the tensor modes and show that this speculation indeed turns out to be true.

III. LARGE ANGLE TENSOR CMB ANISOTROPY

The CMB temperature anisotropy due to a tensor-type perturbation is given by

$$\frac{\delta T}{T}(\gamma^i) = -\frac{1}{2} \int^{\eta_{LSS}}_{\eta_0} \frac{d\eta}{d\eta} H'_{ij}(\eta, x^i(\eta)) \gamma^i \gamma^j = -\frac{1}{2} \int^{\eta_{LSS}}_{\eta_0} \frac{d\eta}{d\eta} H'_{rr}(\eta, x^i(\eta)),$$

where $\eta$ is the conformal time; $d\eta = dt/a(t)$, $H_{ij}$ is the transverse-traceless metric perturbation as given by Eq. (2.24), $H_{rr}$ is the $(r, r)$-component of $H_{ij}$, $H'_{ij}$ is the $\eta$ derivative of $H_{ij}(\eta, x^i)$, $x^i = x^i(\eta)$ is the photon trajectory, $\gamma^i$ is the

$\text{FIG. 1. The gravitational wave spectra for various model parameters. The lines show, from top to bottom, the cases } (\alpha, \beta) = (10^2, 10^{-2}), (10^2, 1), (10^2, 10^{-2}), (10, 1), (10, 10^{-2}), (1, 1) \text{ and } (1, 10^{-2}). \text{ The first three cases degenerate into a single line in the figure.}$
where $\cos \theta$ and $\eta_{LSS}$, and that at present, $\eta_0$, are given respectively as

$$\eta_{LSS} = 2 \text{arccosh} \sqrt{1 + \frac{\Omega_0^{-1} - 1}{1 + z_{LSS}}}, \quad \eta_0 = 2 \text{arccosh} \sqrt{\Omega_0^{-1}},$$

(3.2)

where $z_{LSS}$ is the redshift of the last scattering surface. For simplicity, we adopt the value $z_{LSS} = 1100$ in the following calculations, but the results are insensitive to different choices of $z_{LSS}$.

The evolution equation for $U_{plm}$ is given in terms of the conformal time as [24]

$$U''_{plm} + 2 \frac{a'}{a} U'_{plm} + (p^2 + 1) U_{plm} = 0.$$  

(3.3)

Since we are interested in the anisotropy on large angular scales, we consider only those modes that come inside the Hubble horizon after the universe becomes matter-dominated. The scale factor in the matter-dominated universe is given by $a(\eta) = \cosh \eta - 1$. Then with the initial condition that $U_{plm}$ approaches a constant as $\eta \to 0$, the above equation can be solved exactly to give

$$U_{plm}(\eta) = U_{plm}(0) \frac{3 \left( \cosh \frac{\eta \sin p\eta}{2} - \sinh \frac{\eta}{2} \cos p\eta \right)}{(1 + 4p^2) \sinh^3 \frac{\eta}{2}} =: U_{plm}(0) G_p(\eta),$$

(3.4)

where $U_{plm}(0)$ has the spectrum given by Eq. (2.27). Then the temperature anisotropy is expressed as

$$\left( \frac{\delta T}{T} \right)_{p,l} Y_{lm} = - \frac{1}{2} U_{plm}(0) \int_{\eta_{LSS}}^{\eta_0} d\eta G'_p(\eta) Y_{rr}^{plm}(\eta_0 - \eta, \Omega),$$

(3.5)

where we have decomposed the anisotropy as

$$\left( \frac{\delta T}{T} \right) = \int_0^\infty dp \sum_{lm} \left( \frac{\delta T}{T} \right)_{p,l} Y_{lm}.$$  

(3.6)

The explicit form of $Y_{rr}^{plm}$ is

$$Y_{rr}^{plm} = \sqrt{\frac{(l - 1)(l + 1)(l + 2)}{2(1 + p^2)}} \frac{\Gamma(l + 1 + ip)}{\Gamma(1 + ip)} \frac{P^{-l - 1/2}_l (\cosh r)}{\sinh^{5/2} r} Y_{lm}(\Omega),$$

(3.7)

where $P^\mu_l$ is the Legendre function of the first kind.

As customarily done, we describe the temperature anisotropy in terms of the multipole moments $C_l$ of the temperature autocorrelation function:

$$C(\theta) := \left< \frac{\delta T}{T} (\gamma^i) \frac{\delta T}{T} (\bar{\gamma}^i) \right> = \frac{1}{4\pi} \sum_l (2l + 1) C_l P_l (\cos \theta),$$

(3.8)

where $\cos \theta = \gamma_{ij} \bar{\gamma}^i \bar{\gamma}^j$. Then the moment for the tensor-type perturbation, $C_l^{(T)}$, is given by

$$C_l^{(T)} = \int_0^\infty dp \left< \left( \frac{\delta T}{T} \right)^2 \right>_{p,l}$$

$$= \frac{(l - 1)(l + 1)(l + 2)}{8} \int_0^\infty dp \frac{U_p^2}{1 + p^2} \left| \frac{\Gamma(l + 1 + ip)}{\Gamma(1 + ip)} \right|^2 \left| \int_{\eta_{LSS}}^{\eta_0} d\eta G'_p(\eta) \frac{P^{-l - 1/2}_l (\cosh r)}{\sinh^{5/2} r} \right|^2, \quad (3.9)$$

where $r = \eta_0 - \eta$. 

7
equally valid if Eq. (3.10) holds. The only reason for our choice of
in the choice of $A$
where, except for the dependence on $\Omega_0$
integral of the gravitational wave spectrum.

A
gravitational wave modes and the model-dependence is conveniently characterized by the factor $C_{\alpha,\beta}$, where the first three cases, the fourth and fifth cases, and the sixth and seventh cases, respectively, are almost degenerate.

The anisotropy spectra $\tilde{C}_l := l(l+1)C_{l}^{(T)}/(4\kappa H_R^2)$ for various different model parameters in the $\Omega_0 = 0.3$ universe are shown in Fig. 2. We see that all the spectra converge down to the same constant value as $l$ increases, which presumably corresponds to the part due to the conventional scale-invariant spectrum. This suggests that the enhancement on smaller $l$ indeed represents the contribution from the wall fluctuation modes. A further evidence that the spectrum can be decomposed into the two parts is found by inspecting the behavior of the integrand of Eq. (3.9) as a function of $p$. We show in Fig. 3 the integrand times $p$ for $l = 10$ in the case of the model parameters $\alpha = 10^2$ and $\beta = 1$. The wide peak on the side of smaller $p$ comes from the peak in the original spectrum of gravitational wave modes $|U_p|^2$ which is due to the wall fluctuation modes, while the peak on the side of larger $p$ arises from the coherence between the oscillations of $P_{l p^{-1/2}}(\cosh r)$ and $C_p'(\eta)$, which may be regarded as the contribution from the scale-invariant part of the gravitational wave spectrum. To examine this conjecture, we pose the following ansatz as the form of the CMB spectrum:

$$l(l+1)C_{l}^{(T)} = 4\kappa H_R^2 \left( A \tilde{C}_l^{(W)} + \tilde{C}_l^{(R)} \right),$$

(3.10)

where, except for the dependence on $\Omega_0$, $\tilde{C}_l^{(W)}$ and $\tilde{C}_l^{(R)}$ are assumed to be model-independent and represent contributions from the wall fluctuation modes and the residual continuous spectrum of gravitational wave modes, respectively. Thus the only model-dependent part is the factor $A$, which we assume to be given by the total integral of the gravitational wave spectrum:

$$A = \frac{1}{4\kappa H_R^2} \int_0^\infty dp |U_p|^2.$$

(3.11)

Note that $A \approx 1/(2\Delta s)$ for $\Delta s \ll 1$ from Eq. (2.31). It should be also kept in mind that there always exists freedom in the choice of $A$ by a constant shift: Replacing $A$ with $A + C$ where $C$ is a model-independent constant will be equally valid if Eq. (3.10) holds. The only reason for our choice of $A$ is that it is simplest and seems most natural.

If the above ansatz (3.10) works, $\tilde{C}_l^{(W)}$ will be given by the spectrum for the case $\Delta s \to 0$, which we show in Fig. 4 for various values of $\Omega_0$. As expected, we find $\tilde{C}_l^{(W)}$ precisely agrees with the spectrum due to the wall fluctuation modes calculated previously (See Fig. 4 in [14]). Then the residual part $\tilde{C}_l^{(R)}$ can be easily calculated. We plot the results for various cases of model parameters in Fig. 5. We find the parameter-dependence is surprisingly small, which is exhibited in the narrow thickness of each curve for each value of $\Omega_0$. Thus we conclude that the tensor CMB anisotropy can be decomposed into the part due to wall fluctuation modes and the part due to continuous gravitational wave modes and the model-dependence is conveniently characterized by the factor $A$ which is the total integral of the gravitational wave spectrum.
FIG. 4. The normalized tensor CMB anisotropy in the limit $\Delta s \to 0$. The lines show, from top to bottom (at larger $l$), the cases of the universe with $\Omega_0 = 0.1, 0.2, 0.3, 0.4, 0.5$ and 0.6.

FIG. 5. The residual, almost model-independent part of the tensor CMB anisotropy spectra. The top bunch of lines at $l = 2$ are the universe with $\Omega_0 = 0.5$, the second bunch with $\Omega_0 = 0.3$ and the third bunch with $\Omega_0 = 0.1$. Each bunch contains the nine cases of $\alpha = 10^{-2}, 1, 10^2$ and $\beta = 10^{-2}, 1, 10^2$, respectively. The bottom line in each bunch that is slightly away from the other ones corresponds to the case $\alpha = \beta = 1$.

IV. CONSTRAINTS ON THE MODEL PARAMETERS

In this section, we consider constraints on the model parameters by comparing the tensor CMB anisotropy calculated in the previous section with the scalar CMB anisotropy which is due to the quantum fluctuations of the inflaton field.

For definiteness, we consider either a single field or two field model of open inflation in which the potential of the inflaton field has the form $V = \lambda \phi^{2n}$ inside the bubble. We also assume that the mass of the inflaton field at the false vacuum is much greater than the Hubble parameter there so that there is no contribution from the de Sitter supercurvature modes to the scalar CMB anisotropy. Then it has been shown that the CMB anisotropy spectrum is almost model-independent and indistinguishable from the one for the Bunch-Davies vacuum or the conformal vacuum [14]. Therefore, as a reference spectrum for the scalar CMB anisotropy, we take the one calculated for the case of the conformal vacuum. We express it as

$$l(l+1)C^{(S)}_l = \left(\frac{3H_R^2}{5\phi}\right)^2 \tilde{C}^{(S)}_l.$$  (4.1)

The normalized spectra $\tilde{C}^{(S)}_l$ for various values of $\Omega_0$ are plotted in Fig. 6.

As seen from Fig. 4, the anisotropy spectrum due to the wall fluctuation modes rises rather steeply towards lower $l$. Hence if it dominates over the scalar CMB anisotropy at $l \lesssim 10$, it will contradicts with the anisotropy spectrum observed by COBE which is relatively flat there [25]. Hence we consider the ratio,

$$r_l := \frac{l(l+1)\tilde{C}^{(W)}_l}{l(l+1)\tilde{C}^{(S)}_l}_{|l=10} = \left(\frac{3H_R^2}{5\phi}\right)^{-2} 4\kappa H_A^2 A \frac{\tilde{C}^{(W)}_l}{\tilde{C}^{(S)}_{10}} = \frac{25}{18\pi} \frac{A}{\zeta^2} \frac{\tilde{C}^{(W)}_l}{\tilde{C}^{(S)}_{10}},$$  (4.2)

where

$$\zeta := \frac{V}{V^\prime M_{pl}} = \frac{M_{pl}}{2n\phi} \approx \sqrt{\frac{N(\phi)}{8n\pi}},$$  (4.3)

and $N(\phi)$ is the e-folding number of expansion from the time the inflaton has the value $\phi$ until the end of inflation. In the present case, a relevant value of $N$ will be $\sim 60$ as we are interested in the present Hubble horizon scale. Then $\zeta^2 \sim 3$ for $n = 1$. The ratio $\tilde{C}^{(W)}_l/\tilde{C}^{(S)}_l$ is a factor which can be calculated model-independently. The result is shown in Fig. 7 for $\Omega_0 = 0.1 \sim 0.6$. Then a bound on the value of $r_l$ translates to that on $A$, which in turn constrains the model parameters. Although it is hard to tell precisely the maximum allowable value of $r_l$, we tentatively require $r_l < 1$ as a conservative bound. Then we have
\[ A \lesssim 7 \left( \frac{\zeta^2}{3} \right) \left( \frac{C_{l}^{(W)}}{C_{10}^{(S)}} \right)^{-1}. \] (4.4)

A contour plot of \( A \) on the \((\alpha, \beta)\)-plane is shown in Fig. 8, where \( \alpha \) and \( \beta \) are the non-dimensional parameters of a model defined in Eqs. (2.13). Then taking account of the ratio \( \bar{C}_{l}^{(W)}/\bar{C}_{10}^{(S)} \) shown in Fig. 7, we find the model parameters are most severely constrained when \( \Omega_{0} = 0.3 \sim 0.4 \). In this case, the constraint (4.4) gives \( \alpha \lesssim 70 \) and \( \beta \lesssim 10^3 \). On the other hand, the constraint is weakest when \( \Omega_{0} = 0.1 \), for which we obtain \( \alpha \lesssim 140 \) and \( \beta \lesssim 3 \times 10^3 \). In any case, we conclude that the constraint is not too tight but it is necessary to tune the model parameters to some degree.

Finally, we consider another important condition for a one-bubble open inflation model to be successful. Since the nucleated bubbles should not collide with each other, the nucleation rate must be exponentially suppressed. In other words, the exponent \( B \) of the bubble nucleation rate should be much greater than unity. In the present case, \( B \) is given by the action for the \( O(4) \)-symmetric bubble minus the action for the purely false vacuum configuration. In terms of \( \alpha \) and \( \beta \), it is expressed as

\[ B = \frac{M_{pl}^4}{\Delta V} f(\alpha, \beta), \] (4.5)
where

\[ f(\alpha, \beta) = \frac{3\alpha}{16\gamma^3} \left[ 8 - \frac{(\alpha - 1)^3}{\alpha + \beta} + \frac{(\alpha + 1)^3}{\beta} + \frac{\gamma^3}{\alpha + \beta} - \frac{\gamma^3}{\beta} \right], \quad (4.6) \]

and \( \gamma = \sqrt{(\alpha + 1)^2 + 4\beta} \). A contour plot of \( f \) is given in Fig. 9. We find that \( f \gtrsim 10^{-5} \) in the whole region of our interest. Thus we have \( B \gg 1 \) as long as \( \Delta V \ll 10^{-5}M_{pl}^4 \), which is easily satisfied for any reasonable choice of \( \Delta V \).

V. SUMMARY AND DISCUSSION

We have calculated the tensor CMB anisotropy on large angular scales in the one-bubble open inflationary universe scenario. We have found that the wall fluctuation modes, which were previously regarded as discrete supercurvature modes of the tunneling scalar field, are transmuted into a part of continuous spectrum of the gravitational wave modes when the coupling between the scalar field and the metric perturbations is correctly taken into account and the resulting CMB anisotropy is identical in the limit of weak gravity. We have then shown that the tensor CMB anisotropy spectrum can be conveniently decomposed into two parts; one due to the wall fluctuation modes and the other due to the usual gravitational wave modes, even in the case when the gravitational coupling is non-negligible.

We have then considered constraints on the model parameters by comparing the calculated tensor CMB anisotropy with the scalar CMB anisotropy. Assuming a typical chaotic inflation type potential for the inflaton field inside the bubble, and requiring that the tensor anisotropy does not dominate the spectrum at \( l < \sim 10 \), we have found that the allowable region in the parameter space is large enough, though some tuning of the parameters is necessary. Specifically, we have found that the constraints are most stringent for the universe with \( \Omega_0 = 0.3 \sim 0.4 \), for which we have obtained \( \alpha = \Delta V/(6\pi G S_1^2) < \sim 70 \) and \( \beta = V_T/(6\pi G S_1^2) < \sim 10^3 \). The condition on the parameter \( \beta \) is not tight at all. However, the parameter \( \alpha \) is relatively tightly bound because we must have \( \alpha > 1 \), which comes from the existence condition of the bubble configuration that describes false vacuum decay.

As noted before, however, it is worthwhile to keep in mind that the violation of the condition \( \alpha > 1 \) does not directly imply a failure of the model. We have excluded such models simply because we cannot predict the outcome within the present formalism. In this connection, we mention an interesting possibility. Namely, if one considers a scenario in which the universe is created with a bubble from nothing, just like the creation of a universe discussed in the quantum cosmological context [26,27], the homogeneity and isotropy of the classical background universe inside the bubble will be guaranteed irrespective of the value of \( \alpha \). In this case, \( \alpha \) can take any positive value as long as \( \alpha < \sim 70 \).

It has been pointed out that the scalar and tensor contributions to the CMB anisotropy can be separated by measuring the polarization pattern of CMB [28,29]. These works assume the plane wave decomposition of the metric perturbation. However, it will not be valid for the present case in which the tensor contribution is dominated by the wall fluctuation modes whose coherence scale is of the order of curvature scale. It is a challenging issue to formulate a method to describe the CMB polarization due to the curvature scale modes and to see if there appears a feature characteristic to the one-bubble inflationary scenario.

Finally, we should mention that we have assumed the inflaton potential to be of chaotic inflation type inside the bubble, which is of course not the only possibility. It is interesting to see how the other types of potential models are constrained by the tensor CMB anisotropy.

Acknowledgments

Y.Y. thanks Prof. S. Ikeuchi for his continuous encouragement. This work was supported in part by Monbusho Grant-in-Aid for Scientific Research No. 07304033.