The estimation of the noise in cosmic microwave background anisotropy experiments

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ABSTRACT

Even the most sensitive cosmic microwave background anisotropy experiments have signal to noise ratios $\lesssim 5$, so that an accurate determination of the properties of the cosmological signal requires a careful assessment of the experimental noise. Most of the experiments combine simultaneous multi-channel observations in which the presence of correlated noise is likely. This case is common for ground-based experiments in which an important fraction of the noise could be atmospheric in origin. Here, the way to compute and determine the effects produced by this correlated noise is discussed; in particular, the paper considers the Tenerife experiments (three radiometers at 10, 15 and 33 GHz with two independent receivers each) showing how this effect has been taking into account properly in the more recent analysis of these data. It will be demonstrated that for each of the three radiometers of these experiments, the atmospheric noise is equivalent to a Gaussian noise common to both channels with a coherence time smaller than the binning time, the net effect being an enhancement of the error-bars in the stacked scan as compared with the estimation for the case of pure uncorrelated noise. As expected from the spectral index of the atmosphere, the effect is more important at higher frequencies. The formalism is generalized and applied to the general case of simultaneous multi-channel observations.

*Subject headings:* Cosmology: Cosmic Microwave Background - Statistical analysis
1. INTRODUCTION

The sensitivity of the observations of cosmic microwave background (CMB) anisotropies for existing and planned experiments will allow differentiation among theoretical models determining the parameters of cosmological interest with an uncertainty of only a few per cent. However, even the most sensitive experiments have poor signal to noise ratios (Hancock et al. 1994; Bennett et al. 1996, etc.) and are sensitive only to the most intense structures in the CMB signals. The statistical properties of the CMB signal are obtained by using sophisticated techniques (likelihood analysis, etc.), which compute the correlations in the data and compare them with the expectations assuming a given model. This requires an accurate estimation of all the sources of correlated and uncorrelated noise (Wilkinson 1995). Lineweaver et al. (1994) and Dodelson, Kosowsky & Myers (1995) computed the correlated part of the noise in the COBE DMR (Smoot et al. 1992) and Saskatoon data (Wollack et al. 1993), respectively. This paper is mainly concerned with the Tenerife beam-switching radiometers (Hancock et al. 1994, 1996; Gutiérrez et al. 1995; Davies et al. 1996). These experiments cover an angular range (multipoles \( l = 10 - 40 \)) of great interest for establishing the spectral index of the primordial fluctuations and the total density of the Universe and for differentiating between scalar and tensorial modes. No other experiments in this angular range are planned for the near future. Improvements on the results of the Tenerife experiments will require deeper integration time and an extension of the region observed in the sky, but also better estimations of the contaminating foregrounds and the assessment of the atmospheric contribution. This paper focuses on the way in which this atmospheric noise can be evaluated. Section 2 is dedicated to addressing this problem for the Tenerife experiments, whilst Section 3 presents the formalism for the general case of experiments with \( n \) channels at different frequencies. Finally Section 4 presents the conclusions.

2. APPLICATION TO THE TENERIFE EXPERIMENTS

The experiments consist of three radiometers operating at 10, 15 and 33 GHz (two independent receivers each) installed on Tenerife at an altitude of 2400 m. For more than ten years the instruments have collected data in seven bands separated 2.5° from Dec.=+30° to Dec.=+45°, scanning a band at constant declination each day. Detailed descriptions of the instruments, observing technique and data processing can be found in Davies et al. (1996). The analysis presented here is dedicated to the data at Dec.=+40° presented in Hancock et al. (1994, 1996), however the conclusions are similar to those obtained with the more recent data at Dec.=+35° (Gutiérrez et al. 1997). The noise in a beam-sized element (\( \sim 5° \))
in each of the individual observations are $\sim 0.6$, $\sim 0.4$, and $\sim 0.3$ mK at 10, 15 and 33 GHz respectively, whilst the expected amplitudes of the CMB signal for the instrumental configuration is $\sim 30 - 40 \mu$K; this makes it necessary to repeat the observations $\sim 100$ times in order to achieve enough sensitivity in the CMB signals. In the data presented by Hancock et al. (1994) each position in RA correspond to a mean number of 120, 130 and 85 observations at 10, 15 and 33 GHz, respectively. The estimation of the error-bars in the stacking presented by Hancock et al. (1994) was done assuming that the noise in each individual observation were purely uncorrelated. This is correct for the instrumental noise, which has a thermal origin; however, most of the observations in each instrument correspond to measurements with both channels simultaneously (there is a small time delay of 16 ms between them), and therefore any possible atmospheric contribution will introduce correlation between both channels.

This amplitude and temporal scale of coherence of this effect have been evaluated studying the correlation and cross-correlation functions between channels. Figure 1 represents the correlation of the observations as a function of time (which is equivalent to separation in RA). The plots are an extension of Figure 7 in Davies et al. (1996). The top and middle panels indicate that the sources of noise are uncorrelated on scales larger than 4 min (the binning time). The cross-correlations plotted at the bottom show that a part of the noise is common to both channels; it is likely that the atmosphere is the source of this correlated noise. The distribution of the data in each individual scan has been studied using the Kolmogorov-Smirnov test which shows the compatibility with a Gaussian distribution. In summary, these analyses indicate that the effect of the atmosphere on the individual observations can be modelled as a Gaussian noise common to simultaneous measurements of both channels with a time coherence scale smaller than 4 min, and therefore independent between adjacent positions in RA. As expected from the spectral index of atmospheric emission, the largest effect is at 33 GHz. The amplitude of this atmospheric noise can be inferred from the cross-correlation between both channels. I made Monte Carlo simulations of the data and compute the auto-correlation and cross-correlation in the same way as the actual data. The results of these simulations (dashed lines in Figure 1) show good agreement with the actual correlations functions, demonstrating the validity of the model.

Denoting the atmospheric signal by $a$ and the signal due to the instrumental noise by $r_1$ and $r_2$ ($<r_1^2> = <r_2^2>$), in channels 1 and 2 respectively, combining the results of both channels the variance of the stacked scan is

$$<\frac{1}{2}[(a + r_1) + (a + r_2)]^2> = \frac{1}{4} \{4 <a^2> + <r_1^2> + <r_2^2>\}. \quad (1)$$
However, ignoring the correlation due to the atmosphere we would obtain (incorrectly)
\[ \frac{1}{4} \{ 2 < a^2 > + < r_1^2 > + < r_2^2 > \} \] and therefore the noise would be underestimated by a factor

\[ g_+ = \sqrt{1 + \epsilon} , \]

where

\[ \epsilon = \frac{1}{1 + ( < r_1^2 > + < r_2^2 > )/2 < a^2 >} . \]

This is an extension of expression (1) in Davies et al. (1996) for the case in which the instrumental noise in both channels is not exactly the same. In CMB experiments which consist of two channels (A and B) usually the signal is computed comparing the sum \((A + B)/2\) (which contains the signal plus the noise) and the difference \((A - B)/2\) (which only contains the noise see Hancock et al. 1994). In this case, is easy to show that, ignoring the correlated term, there is an overestimation of the noise in the \((A - B)/2\) scan by a factor \( g_+ = \sqrt{1 - \epsilon} \). This factor is a consequence of the subtraction of part of the noise (the atmospheric part) when the difference \((A - B)/2\) scan is computed. In the limit in which the atmospheric noise is negligible as compared with the instrumental noise, we obtain \( g_+ = g_- = 1 \), as expected; in the case in which the atmospheric noise is much more larger than the instrumental noise we obtain \( g_+ \to \sqrt{2} \) and \( g_- \to 1/\sqrt{2} \) and then simultaneous observations with two channels are fully redundant.

For the Tenerife CMB data at Dec=+40°, it is possible to distinguish between the instrumental and the atmospheric noise by analyzing the distribution of the cross-correlation between both channels, as explained above. The deduced \( g_+ \) and \( g_- \) factors have the values quoted in Table 1. As expected, the table indicates that the data which are more affected are the 33 GHz. The mean correlation between channels is larger at 15 GHz than at 10 GHz as can be seen in Figure 1, however in Table 1 the \( g_+,- \) factors are similar for both instruments; the reason for this is that one of the channels at 15 GHz was not operating due to a malfunction most of the time. The data at 15 and 33 GHz were recorded in different campaigns and therefore there are not an extra factor due to the correlated part between data at these two frequencies.

How this re-estimation of the error-bars affects the analyses of the estimation of the CMB signal? Table 1 of Hancock et al. (1994) computed the rms of the astronomical signal \((\sigma_{\text{RMS}})\) present at each frequency from the signals in the \((A + B)/2\) \((\sigma_{A+B}/2)\) and \((A + B)/2\) \((\sigma_{A-B}/2)\) scans, \( \sigma_{\text{RMS}}^2 = \sigma_{A+B}/2 - \sigma_{A-B}/2 \). As has been shown above, these values of \( \sigma_{A+B}/2 \) and \( \sigma_{A-B}/2 \) are affected by the atmospheric signal common to both channels. Following the formalism explained above, I have computed the atmospheric \( \sigma_a \) and astronomical signals \( \sigma_s \) presented in each of the final stacked scans and corrected the values quoted in Table 1 of Hancock et al. (1994). The new and old \((\sigma_{old})\) values are presented in columns 4, 5 and 6 of Table 1.
The correlated noise also affects to the likelihood analysis. The re-estimation of the error-bars presented above is equivalent to increasing the diagonal term in the covariance matrix. Figure 2 presents the likelihood curves for the 15, 33 and combined 15+33 data with the old (a, b and c) and the new (d, e, f) estimation of the noise in the case of a Harrison-Zel’dovich spectrum for the primordial fluctuations. All the cases show evidence of a clear well-defined maximum and similar shapes after the re-estimation of the error-bars. However, the value of the peak is smaller with the new estimation. For instance, in the 15+33 data the likelihood peak normalized with respect to the value for zero signal changes from $\sim 5 \times 10^6$ to $5 \times 10^4$. This indicates a lower significance of the detection. Using a Bayesian analysis for the 15+33 data and assuming a Harrison-Zel’dovich for the spectrum of the primordial fluctuations, a normalization of $Q_{\text{RMS-PS}} = 26 \pm 6 \mu K$ was obtained; with the new estimation of the error-bars in the stacked data it is obtained $Q_{\text{RMS-PS}} = 22^{+10}_{-6} \mu K$. This detection is still very significant and it would be necessary to increase the error-bars artificially by a factor as large as $\sim 2.6$ to make the likelihood curve compatible with pure noise. Figure 2 also presents (panels g, h and i) the likelihood curves of the $(A - B)/2$ with the new error-bars. In the three cases the scans are compatible with noise, emphasizing that the detected signals are common to both channels and frequencies.

3. GENERAL CASE

The formalism presented in last section can be easily generalized for any other experiment with $n$ simultaneous channels operating at different frequencies. Specifically for experiments working at mm wavelengths, it is expected that atmospheric noise would be the main source of noise (Piccirillo et al. 1997). Following the results of the previous section, it will be considered an experiment in which the contribution due to the atmosphere is equivalent to Gaussian noise uncorrelated point to point with a certain dependence in frequency. The result in each channel consists of the signal on the sky plus two components, the first due to thermal uncorrelated noise and the second which is correlated in some degree between the different channels. I assume that the noise in the $i$-th channel is $k_i a + r_i$, where $k_i a$ is the contribution of the atmosphere which has a spectral dependence $k_i$, and $r_i$ is the part due to the thermal noise. When the results of the $n$ channels are combined, the variance of the stacked scan is

\[
< \left( \frac{1}{n} \sum_i (k_i a + r_i)^2 \right) > = \frac{1}{n^2} \sum_i < (k_i a + r_i)^2 > + 2 \sum_{i,j>i} < (k_i a + r_i)(k_j a + r_j) >
\]

\[
= \frac{1}{n^2} \{ < a^2 > \sum_i k_i^2 + \sum_i < r_i^2 > + 2 \sum_{i,j>i} k_i k_j < a^2 > \}.
\]
However, if we ignore the correlation due to the atmosphere we would incorrectly obtain
\[
\frac{1}{n^2} \{ <a^2> \sum_i k_i^2 + \sum_i <r_i^2> \}\]
and therefore we would underestimate the noise by a factor
\[
g_+ = \sqrt{1 + \frac{2 \sum_{i,j>1} k_i k_j}{\sum_i k_i^2 + \sum_i <a^2>}}.
\]
Especially if \(a = 0\), \(g = 1\), and when \(r = 0\), \(g = \sqrt{1 + \frac{2 \sum_{i,j>1} k_i k_j}{\sum_i k_i^2}}\). In the case of two channels operating at the same frequency, the expression of Section 2 is recovered. The factor \(g_+\), when we compute the noise in the difference between channels \(i\) and \(j\), is
\[
g_- = \sqrt{1 - \frac{2k_i k_j <a^2>}{<r_i^2> + <r_j^2> + (k_i^2 + k_j^2) <a^2>}}.
\]

4. CONCLUSIONS

In this paper it has been shown how the presence of correlated noise affects the estimation of the error-bars when data from several channels are combined. This is particularly interesting in cases of poor signal-to-noise-ratios like the CMB observations. It has been shown that for the Tenerife experiments the atmosphere can be modeled like Gaussian noise uncorrelated between adjacent measurements. The net effect, in this case, is an enhancement of the error-bars in the final stacked scans. The maximum effect is at 33 GHz where more than half of the noise is atmospheric in origin. With the new estimation of the noise, the likelihood analysis of the Tenerife data at Dec. = +40° shows the presence of clear CMB signals corresponding to an expected quadrupole \(Q_{\text{RMS-PS}} = 22^{+10}_{-6} \mu\text{K}\), a value slightly smaller in amplitude and in statistical significance than the old estimation presented in Hancock et al. (1994). Obviously, the effect is highly dependent on the observing technique; for instance the preliminary analysis of the data of the Tenerife interferometer at 33 GHz does not show evidence of correlated noise between the sine and cosine channels, even in cases of relatively bad weather. I have assumed here that the origin of the correlated noise between channels is the atmosphere, but the formalism can be applied independently of the physical origin of the noise.
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REFERENCES


This manuscript was prepared with the AAS \LaTeX macros v4.0.
FIGURE CAPTIONS

Fig. 1.— The correlation of the signals in the Tenerife CMB data as a function of the shift in RA. Panels at the top, middle and bottom are for the auto-correlation of channels 1 and 2, and the cross-correlation between them, respectively. The points correspond to the mean of the correlation function and the solid lines enclose the 68 % c.l. along the different observations. The dashed lines correspond to Monte Carlo simulations of the data.

Fig. 2.— Likelihood functions of the results at 15 GHz (left), 33 GHz (middle) and the weighted addition of both 15+33 (right). Panels at the top show the likelihood function of the signal in the \((A + B)/2\) without considering the presence of correlated noise between channels. Plotted in the second line are the new likelihood curves after the re-estimation of the error-bars. The likelihood curves for the \((A - B)/2\) scans are at the bottom.
Table 1: Corrections for the Tenerife results.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>( g_+ )</th>
<th>( g_- )</th>
<th>( \sigma_a ) (( \mu )K)</th>
<th>( \sigma_s ) (( \mu )K)</th>
<th>( \sigma_{old} ) (( \mu )K)</th>
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<td>10 GHz</td>
<td>1.03</td>
<td>0.96</td>
<td>...</td>
<td>36</td>
<td>36</td>
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<tr>
<td>15 GHz</td>
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