A String Model of Black Hole Microstates

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Abstract

The statistical mechanics of black holes arbitrarily far from extremality is modeled by a gas of weakly interacting strings. As an effective low energy description of black holes the string model provides several highly non-trivial consistency checks and predictions. Speculations on a fundamental origin of the model suggest surprising simplifications in non-perturbative string theory, even in the absence of supersymmetry.

Black holes exhibit thermodynamic properties suggestive of a complicated internal structure. This is somewhat paradoxical within standard general relativity where black holes are absolutely featureless. In the last year or so there has been considerable excitement as it has become clear that string theory can accurately account for the degeneracy implicit in the Bekenstein-Hawking entropy. This development initially concerned the counting of states for extremal [1, 2, 3] and near extremal black holes [4, 5]. The effective string model that emerged apparently describes in much detail the dynamics of near extremal black holes [6, 7, 8, 9]. This no longer leaves any doubt that string theory describes many features of black holes.

Many questions remain however. Current models are restricted to very large wave lengths. In the interesting region with possible information loss the appropriate methods are quite different and they still need further development [10, 11, 12, 13]. The focus of this Letter is another ill-understood

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issue, the generalization far off extremality. Here many new concerns appear, especially because much standard string technology remains untested in a regime that is not even approximately supersymmetric. Some proposals have appeared (including [14, 15, 16] ) but there is still much confusion. Moreover, a large region of parameter space has been identified where the $D$-brane motivated model gives incorrect predictions [17, 18]. In this Letter we shall argue that, nevertheless, the situation is quite hopeful: the thermodynamics of black holes in string theory has non-trivial features suggesting a surprisingly simple improved effective string model that gives a satisfying description of non-extremal black holes. We shall present arguments motivating the string model and note a number of consistency checks, including highly non-trivial ones. We speculate that the model arises in fundamental string theory as the direct product of two chiral sectors, each a $c = 6$ superconformal field theory.

The discussion will assume five-dimensional black holes although the generalization to four dimensions involves no new features. For definiteness we will presume toroidally compactified type II theory. In this context the most general non-rotating black hole is characterized up to duality by its mass $M$ and three conserved $U(1)$ charges $Q_i ; i = 1, 2, 3$ [19]. It is convenient to introduce in intermediate steps the so-called non-extremality parameter $\mu$ and also the “boosts” $\delta_i ; i = 1, 2, 3$. In the resulting parametric form the physically interesting quantities are [20]²

\[ M = \frac{1}{2} \mu \sum_i \cosh 2\delta_i \]  
\[ Q_i = \frac{1}{2} \mu \sinh 2\delta_i \]  
\[ S = 2\pi \mu^\frac{3}{2} \prod_i \cosh \delta_i . \]  

When the black hole solution is embedded in string theory the charges arise from specific sources. For example they may be the winding and momentum charges of a fundamental string as well as solitonic 5-brane charge. Or they may be 1-brane and 5-brane RR-charges along with Kaluza-Klein momentum. The latter representation is particularly popular because the sources of RR-charges are $D$-branes which are accessible to detailed study in string

²We employ units where $G_N = \frac{\pi}{4}$. 

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theory [21]. For the present purposes it will not be necessary to make a specific choice between these embeddings (or many others). In fact, non-perturbative string theory in the absence of supersymmetry remains largely unexplored and offers few concrete tools at this point. We will simply rely on the conjectured duality symmetries to ensure that microscopic degeneracies will be identical in all representations. More importantly, duality also gives some guidance in the arguments. Note that the manifestly symmetric appearance of the three charges in the thermodynamic formulae eq. 1-3 is a necessary condition for this strategy to be consistent.

Physical charges $Q_i$ occur only in quantized units. The constants of proportionality relating $Q_i$ to integers $n_i$ depend on the precise embedding in string theory. Indeed fundamental charges, RR-charges, and solitonic charges depend differently on the microscopic coupling constant, and the geometric moduli of the compactified torus also enter. It is remarkable however that, for the black holes embodied in eqs. 1-3, the moduli cancel in the product of charges so

$$\prod_i Q_i = \prod_i n_i.$$  \hfill (4)

Generically physical parameters transform in covariant but nevertheless complicated ways under the duality symmetry. The right hand side of eq. 4 is very special in this respect because here the full duality symmetry group could easily be restored. Indeed, $\prod_i n_i$ is the manifestation, for our representative class of solutions, of the unique cubic invariant of the $U$-duality group $E_{6(6)}$ [22]. The cancellation of moduli responsible for this simplification is of great importance because it suggests a direct relation between macroscopic quantities and the underlying microscopic theory [2, 23, 24]. This is particularly clear in the extremal limit $\mu \to 0, \delta \to \infty$ with $Q_i$ fixed. Here the entropy becomes

$$S = 2\pi (\prod_i Q_i)^{\frac{1}{2}} = 2\pi (\prod_i n_i)^{\frac{1}{2}}.$$  \hfill (5)

Note that this is also the degeneracy of a chiral string model with central charge $c = 6$ and an effective level $N = \prod_i n_i$. For supersymmetric black holes such a model is very well motivated from fundamental string theory.

In the general non-extremal case we do not a priori know the moduli dependence of the mass eq. 1 and the entropy eq. 3. It is possible that the

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\textsuperscript{3}We have chosen normalizations to avoid a numerical factor in this formula.

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microscopic quantum theory ensures that even among these macroscopic parameters there are moduli independent combinations, interpretable as quantum numbers. We propose that indeed there is exactly one such quantity and it is the entropy. Of course it is the exponential of the entropy that might actually be an integer. The point is however that, by definition, quantum numbers do not depend on moduli. It is a consequence of the proposal that, as moduli vary with quantum numbers fixed, the mass changes in a rather complicated way, as implied by eqs. 1–3. The idea that mass depends on moduli but degeneracies do not is familiar from perturbative string theory where degeneracies follow from central charge and level of the conformal field theory, whereas mass depends on both level and (moduli dependent) physical charges. Our working assumption can be motivated as follows: the semiclassical derivation of the entropy formula shows a remarkable independence of details, such as the low-energy matter content. We interpret this universality as a macroscopic manifestation of the underlying quantum theory. Formally, we note that quantization conditions on the gauge charges derive from topologically non-trivial gauge transformations at infinity [13]; so we expect an analogous condition to follow from general covariance. In fact, the Gibbons-Hawking derivation of the entropy formula from the functional integral appears to realize exactly this [25].

At this point it is instructive to consider briefly the more general case of rotating black holes. In five dimensions the rotation group is $SO(4) \simeq SU(2)_R \times SU(2)_L$; so configurations are characterized by two angular momenta $J_{R,L}$. The entropy is [26]

$$S = 2\pi(\sqrt{N_R} + \sqrt{N_L}) \tag{6}$$

where

$$N_{R,L} = \frac{1}{4} \mu^3 \left( \prod_i \cosh \delta_i \mp \prod_i \sinh \delta_i \right)^2 - J_{R,L}^2 \tag{7}$$

In string theory the space of states is the direct product of two terms because of the world sheet decomposition into left and right moving modes. Entropy deriving from string degeneracy is therefore expected to be the sum of two terms. It is suggestive that eq. 6 takes this characteristic form. Concretely we propose that, in analogy with the supersymmetric case, $N_{R,L}$ arise as effective levels of two independent chiral sectors of a single string theory. Assuming central charge $c = 6$ in each sector eq. 6 expresses a quantitative
microscopic interpretation of the entropy in the most general case. It generalizes in a duality invariant fashion the D-brane motivated effective string model (references include [4, 27]). A consistency requirement on this proposal is that $N_{R,L}$ are separately quantized with all positive integer values as spectrum. Since the angular momentum is quantized it would certainly take a very complicated conspiracy to make the total entropy moduli independent without $N_R$ and $N_L$ being so separately. Moreover, the angular momenta were normalized in the conventional way taking all integer values; so if indeed the first term in eq. 7 is quantized with all integers as spectrum, the full $N_{R,L}$ will also be so for all values of the angular momenta. This is an important test of the proposal. It also implies that, accepting the proposal in the absence of angular momenta, generalization to rotating black holes is automatic, with numerical factors matching correctly [28, 29]. Vanishing angular momenta can therefore be assumed in the following, without loss of generality. Angular momenta nevertheless play an important auxiliary role in the argument because they indicate how the entropy eq. 3 should be divided into left and right moving contributions, and also because they suggest the precise quantization condition. The first point was noted and repeatedly emphasized by Cvetić, most recently in [30].

It was argued from general relativity that, apart from charges, there should be only one quantum number visible in the macroscopic theory, namely the entropy. On the other hand our concrete string model introduces two quantum numbers, the levels of the left and right moving states. The reason that there is no conflict is the constraint

$$N_L - N_R = \mu^3 \prod_i \cosh \delta_i \sinh \delta_i = \prod_i Q_i = \prod_i n_i$$

(8)

that follows from, in turn, eqs. 7,2, and 4. This not only avoids an apparent contradiction but it also relates quantities, purely within the microscopic theory, in a fashion reminiscent of the matching condition in perturbative string theory. It is indeed natural to suspect that in the full non-perturbative theory some condition arises from world sheet reparametrization invariance. Duality is a powerful restriction on its possible form and eq. 8 is the simplest expression consistent with duality. As an independent consistency check note that both sides of eq. 8 have all integers as spectrum. These non-trivial and desired results lend some support to the underlying assumptions.

In the preceding it was argued that eq. 3 should be represented as the
sum of two terms

\[ S_{R,L} = \pi \mu^\frac{3}{2} (\prod_i \cosh \delta_i \mp \prod_i \sinh \delta_i) \]  

(9)

that are separately quantized. The difference of the two terms is

\[ S_\perp = 2\pi \mu^\frac{3}{2} \prod_i \sinh \delta_i . \]  

(10)

It is amusing to note that this is \( \frac{1}{4} \) the area of the inner horizon. Hence, whereas the Bekenstein-Hawking formula suggests that somehow the outer horizon is quantized in Planck units the present proposal implies that also the inner horizon is quantized. This is a tantalizing prospect, although the precise geometric significance escapes us at this point.

In the string model there are separate left and right moving gases of weakly interacting strings. The terminology of left and right moving modes is independent of whether there is a net Kaluza-Klein momentum: all charges are treated democratically. Independent inverse temperatures of left and right moving modes follow from the thermodynamic relations

\[ \beta_{R,L} = \left( \frac{\partial S_{R,L}}{\partial M} \right)_{Q_i} = 2\pi \mu^\frac{1}{2} (\prod_i \cosh \delta_i \pm \prod_i \sinh \delta_i) \]  

(11)

with the physical (Hawking) temperature given by \( \beta = \frac{1}{2} (\beta_R + \beta_L) \). Assuming weak coupling between left and right sectors the emission spectrum is expected to be proportional to the product of the left and right occupation numbers and, specifically, their characteristic statistical occupation factors. For this to be possible while, at the same time, the black hole exhibit the correct overall thermality predicted by Hawking, certain grey body factors must take a very special form. The absorption cross-section of a black hole is calculable in classical field theory and the grey body factor is easily extracted. Remarkably, Maldacena and Strominger found a regime where it has exactly the right form for this interpretation to be consistent [7]. For our purposes it is the precise value of the temperatures that is of interest as they lead to a test of our ideas. The temperatures are only known in the near extremal limit \( \delta_i \gg 1 \) where [17, 18]

\[ \beta_{R,L} \simeq 2\pi \mu^\frac{1}{2} [ (\prod_i \sinh^2 \delta_i + \sum_{i<j} \sinh^2 \delta_i \sinh^2 \delta_j )^{\frac{1}{2}} \pm \prod_i \sinh \delta_i ] . \]  

(12)
In its region of validity eq. 12 agrees with the prediction eq. 11. This is particularly satisfying because the $D$-brane model only accounts for the temperatures when, in addition to $\delta_i \gg 1$, one boost parameter is much smaller than the other two, say $\delta_3 \ll \delta_{1,2}$ [17, 18]. Moreover, the model here provides a precise prediction for the general case: when $\delta_i \sim 1$ the expression under the square root should be augmented with terms that makes it a perfect square $\prod_i (\sinh^2 \delta_i + 1) = \prod_i \cosh^2 \delta_i$.

The black hole also interacts with its surroundings through absorption and emission of charged particles. We consider a particle of the species referred to by index 3. Statistical properties of charged particles are more complicated because they depend on both the energy $\omega$ and the charge $q_3$ of the particle. The combined effect is captured by statistical distributions with characteristic exponent $\beta(\omega - q_3 \Phi_3)$. Potentials follow from eqs. 1-3 and are given by

$$\Phi_i = -\frac{1}{\beta} \left( \frac{\partial S}{\partial Q_i} \right)_{Q_{j\neq i}, M} = \tanh \delta_i .$$

(13)

In the string model the background charge $Q_3$ couples to both left and right sectors; so the effective left and right moving distribution functions have exponents that exhibit intricate structure. Using the left and right entropies, respectively, and proceeding as in eq. 13 we find

$$[\beta(\omega - q_3 \Phi_3)]_{R,L} = \pi \mu^{\frac{1}{2}} (\omega - q_3 \tanh \delta_3) \prod_i \cosh \delta_i \pm (\omega - q_3 \coth \delta_3) \prod_i \sinh \delta_i .$$

(14)

Analogous exponents were inferred from the absorption cross-section in the near extremal limit [18]. They are

$$[\beta(\omega - q_3 \Phi_3)]_{R,L} \approx \pi \mu^{\frac{1}{2}} \prod_i \sinh \delta_i \times$$

$$\times \left( (\omega - q_3 \coth \delta_3)^2 (1 + \frac{1}{\sinh^2 \delta_1} + \frac{1}{\sinh^2 \delta_2}) + \frac{\omega^2 - q_3^2}{\sinh^2 \delta_3} \right)^{\frac{1}{2}} \pm (\omega - q_3 \coth \delta_3) .$$

(15)

It is straightforward to check that only terms of order at most $\sinh^{-4} \delta_i$ for large $\delta_i$ need to be added under the square root in order that the expression becomes a perfect square, the square root can be taken, and eq. 14 follows. Hence eq. 14 agrees with eq. 15 in the regime where eq. 15 is valid. This detailed agreement between complicated functions is a highly non-trivial check.
on the string model. The unusual situation where the approximate expression is much more complicated than the proposed exact one suggests some unrecognized underlying simplicity.

The mass is not an independent parameter, but rather a definite function of the charges and levels given implicitly by eqs. 1-3. In a complete microscopic theory this relation should be derived from arguments internal to the theory. It seems plausible that this should be possible, using dualities and Lorentz invariance. Incidentally, it should be noted that the arguments in the preceding paragraphs relied on no “mass renormalizations” or “redshifts” and indeed none are allowed by the agreement with grey body factors. Therefore it can be argued in the converse direction that the string model parametrize the low energy interaction of the non-extremal black holes.

Up to this point the string model has been presented as an effective theory that embodies the statistical mechanics of non-extremal black holes. It should correctly capture the low energy physics of black holes in string theory. Although the evidence presented in its favor is quite strong the precise relation to more conventional ideas in string theory is certainly not clear. To elucidate the question consider a bound state of $n_1$ $D1$-branes and $n_5$ $D5$-branes, carrying momentum $k$. Strominger and Vafa [3] proposed that, when wrapped on $K3 \times S_1$, the exact degeneracy of this supersymmetric state is described by level $k$ of a superconformal sigma-model with target space

$$C = (K3)^{(n_1n_5+1)}/\Sigma(n_1n_5 + 1)$$

where the quotient by the symmetric group is implemented by orbifolding the product manifold. An unusual feature is that, rather than being a fixed numerical constant, the central charge depends on the quantum numbers of the configuration at hand. The formula is not manifestly duality invariant, as the three charges do not appear symmetrically. An alternative approach, due to Dijkgraaf, Verlinde, and Verlinde [22, 31], was motivated by ideas about second quantized string theory and yields manifestly duality invariant expressions. It was subsequently shown that the proposed degeneracies are in fact identical [32] and specifically the Strominger-Vafa formula is consistent with duality. The rather involved mathematics describing the space of BPS states in various cases is currently being explored [33]. For our purposes we note that, for large charges, the degeneracies derive from a superconformal field theory with a more familiar, state independent, central charge.
$c = 6$, but an unusual relation between the level and spacetime charges, namely $N = \prod_i n_i$. As explained earlier the corresponding string degeneracy accounts for the entropy of extremal black holes. The point here is the following: statistical mechanics of non-extremal black holes suggests that the generalization to the non-supersymmetric context simply involves two such structures with the full space of states realized as the direct product. Each sector describe $BPS$ excitations of some brane configuration wrapped on compact manifolds, not necessarily the same on the two sides. Moreover, each sector is supersymmetric and duality invariant (or covariant, according to the precise context) and the two sectors are related by a matching condition eq. 8 (suitably generalized) that respects these features. As usual the condition on the 0-modes that relates this microscopic physics to a specific spacetime interpretation introduces moduli. Generically duality transformations do not leave moduli invariant; so the duality symmetry is spontaneously broken. In fact it is broken to a compact subgroup and this is quite welcome, as explicitly realized non-compact symmetry is unstable. The novelty in the present proposal is that the introduction of moduli also spontaneously breaks supersymmetry. As the analogous breaking of duality is no cause of concern, it is reasonable to expect that also the breaking of supersymmetry is sufficiently mild that string theory remains under full control.

It should be cautioned that the scenario in the previous paragraph has only been realized in the regime where quantum numbers are large and black hole physics is a guide. Clearly much needs to be done to make the proposal precise in the context of the full algebra of $BPS$ states and to establish the consistency of the resulting vacua in the interacting theory. However, if this endeavor should be successful it could be quite rewarding: it would provide a precise tool for non-perturbative string theory in the absence of supersymmetry.

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References


