An Inverse Cherenkov Accelerator Using a Dielectric Channelled Waveguide

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Abstract: A cylindrical dielectric structure based inverse Cherenkov, laser driven accelerator scheme is proposed. The scheme uses the inverse process of a charged particle radiating inside a partially filled dielectric wave guide. Due to the efficient coupling of the laser beam to the accelerating fields, a very modest amount of laser power can produce GV/m scale gradients. Numerical examples are given for several cases. One particular parameter set demonstrates that a net acceleration of 1 GeV can be achieved using a 200 kW, 3 ns, radially polarized 10.6 μm laser drive beam. A proof of principle prototype experiment is also discussed.

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New methods of particle acceleration have been a subject of intense study in the past
decade. One of the more appealing possibilities is the use of a laser beam to efficiently accelerate
electrons. Laser-plasma interaction based techniques such as laser wake-field [1,2] and laser
plasma beatwave[3,4] acceleration have already been successfully demonstrated. In principle,
any reverse process of a charged particle radiation can also be used for particle acceleration.
Recently, there is increasing interest in non-plasma based laser particle acceleration schemes
[5,6]. For example, Fabry-Perot type resonance acceleration structures have been proposed[7],
and W. Kimura et al. have successfully demonstrated inverse Cherenkov acceleration in a high
pressure H₂ gas by using a radially polarized CO₂ laser beam[8]. In this paper we propose a new
scheme which uses a hollow dielectric fiber in which light propagates as a travelling TM₀₁ wave.
Advantages of our proposed scheme are: 1) efficient coupling of the laser power to the electron
beam; 2) it overcomes the problem of electron beam scattering in the gas, and 3) the acceleration
distance can be sustained due to the nature of the travelling wave structures (hollow fiber).

The concept of charged particle acceleration by the inverse Cherenkov effect is not new.
Pantell and Fontana proposed a scheme using pressurized gas as the medium to support an
acceleration field[9] and a subsequently experiment has been carried out [8]. The Pantell et al.
scheme is based on the creation of an accelerating electric field component due to the wavefront
tilt when a radially polarized is focused by an axicon onto the direction of the charged particle
beam. They also discussed the possibility using an infinite medium with a small hole to accelerate
the charged particles based on the same principle. In their proposal, the total acceleration
distance is limited due to the finite transverse size of the driving laser beam. Therefore, sustained
high gradient acceleration can not be achieved unless the laser beam can propagate in the same
direction as the electron beam. The required laser power for the original ICA scheme is high (>> GW) in order to achieve the high gradient. In this paper, we propose a new method using the inverse process of Cherenkov radiation, in which the supporting structure is a cylindrically symmetric hollow dielectric channeled waveguide with a total reflective outer wall.

The Cherenkov radiation of a charged particle travelling inside the dielectric channel has been studied in detail [10]. The radiation pattern is dominated by $TM_{on}$ ($n$ is the radial mode number), which has a strong longitudinal, $E_z$, component. Previously we proposed using Cherenkov radiation emitted by an intense electron beam in a dielectric structure as a means of particle acceleration, a dielectric wakefield accelerator (DWA)[11]. The electromagnetic wave frequency in the DWA scheme is in the GHz range rather than THz as would be inverse Cherenkov acceleration using a CO$_2$ laser, but the fundamental physics is identical. If dimensions of the dielectric channeled waveguide are chosen properly, a particular $TM_{on}$ mode wavelength $\lambda$ can be matched to a laser wavelength. Therefore, a laser beam with the same wavelength as $TM_{on}$ mode will propagate inside the dielectric channel and be used to as the power source to accelerate charged particles (inverse Cherenkov process). An attractive way to power such a dielectric channeled waveguide would be to use the similar radially polarized laser beam as proposed by Pantell et al., the reason being that the azimuthal magnetic and radial electric fields in such a beam are similar to the corresponding $TM_{on}$ fields in the dielectric channeled waveguide. This permits the laser power to be efficiently coupled into the dielectric channeled waveguide. Once coupled, the laser beam will propagate in the $TM_{on}$ mode. We can therefore treat this problem as a travelling wave linear accelerator made of dielectric channeled waveguide with a laser as the power source. Coupling of the laser beam into the dielectric tube
can be done using a tapered transition section, a technique commonly used in the optical fiber technology.

The acceleration structure under the study has inner vacuum radius $a$ and a layer of dielectric ($\varepsilon$) with radius $b$. A conductor tube is used to guide the laser beam. The radially polarized laser beam propagates through a focusing lens and then transition section in which the beam interferes with itself to satisfy proper boundary condition and thus establishes the correct mode for synchronous particle acceleration in the acceleration section. This proposed scheme is shown in the Figure 1.

Let us first review some important characteristics of Cherenkov radiation in dielectric structures. A charged particle passing through a medium of refractive index $\sqrt{\varepsilon}$ with velocity $v$ will radiate photons when the Cherenkov radiation condition is satisfied ($v > c/\sqrt{\varepsilon}$), where $c$ is velocity of the light in vacuum. The intensity of the radiation per unit length in an infinite medium is

$$\frac{dW}{dl} = \frac{e^2}{c^2 \sqrt{\varepsilon} \omega \beta} \int_{\varepsilon > 1} (1 - \frac{1}{\varepsilon \beta^2}) \omega d\omega$$

which is proportional to $\sin^2(\theta_c) = (1 - 1/\varepsilon \beta^2)$, and $\theta_c$ is commonly known as the "Cherenkov angle". Because the dielectric constant of a pressurized gas system is very close to 1, the radiated power is very small compared to the radiation in a solid medium. Cherenkov radiation inside a dielectric loaded waveguide has some additional and very important characteristics which distinguish it from that in infinite media. First, whereas in infinite media the radiation spectrum is continuous, radiation inside a dielectric loaded waveguide is constrained to
accessible eigenmodes of the guide which are, for an axially moving charge, pure transverse magnetic (TM). Furthermore, the effective Cherenkov angle can be quite large due to the relatively large value of the dielectric constant of the lining compared to gaseous media such as those used in "traditional" ICA experiments. Together, these result in there being strong coupling between the excited laser fields and a charge particle to be accelerated. Numerical examples verify this point and are presented later in this paper.

The axial electric fields inside the acceleration structure shown in Figure 1 can be described by

\[
\begin{align*}
E_z^{(1)}(\omega, r, z, t) & = E_0 J_0(\beta r) e^{i(kz - \omega t)} \\
E_z^{(2)}(\omega, r, z, t) & = [B_1 J_0(s_1 r) + D_1 N(s_1 r)] e^{i(kz - \omega t)}
\end{align*}
\]  

where \( E_0, B_1, \) and \( D_1 \) are the field amplitudes in the region 0 (vacuum) and 1 (dielectric) respectively and are related by boundary conditions, and

\[
\begin{align*}
k^2 & = \frac{\omega^2}{\nu^2}(1 - \beta^2) \\
s^2 & = \frac{\omega^2}{\nu^2}(\beta^2 \epsilon - 1)
\end{align*}
\]  

where \( \beta c = \nu = \frac{\omega}{k} \) is phase velocity of the wave travelling inside the tube. Therefore \( \beta \) determines the synchronism of the wave and the accelerated particles. The transverse electric field can be written as [12]

\[
E_r = i \frac{\omega/\nu}{\omega^2/\nu^2(\beta^2 \epsilon - 1)} \frac{\partial E_z}{\partial r}
\]
and magnetic field \( H_\phi = \varepsilon E_r \) everywhere inside the tube. By matching the boundary conditions at \( a \) and \( b \) (\( E_r \) and \( D_r \) continuous), all the components in the field can be expressed in terms of \( E_0 \). The electric field inside the hole described by equation 2 and 4 have very interesting characteristics. When \( k \sim 0 \), i.e., the phase velocity of the wave is \( c \), \( E_r \) is constant across the vacuum hole. When apply the Panofsky and Wenzel theorem [13] to this case, which relates the longitudinal and transverse forces exerted on a charged particle as,

\[
F_r(r, z) = e \int \frac{\partial E_z(r, z)}{\partial r} dz
\]

Thus there are no focusing and de-focusing forces. Although it is possible for the laser beam to couple the energy into other deflecting modes, such as \( \text{HEM}_{1a} \), it can be easily shown that the phase velocities of those modes are not synchronized with the accelerated beam.

The stored energy per unit length defined as \( U \) in the tube is the sum of contribution from both regions, and can be expressed as,

\[
U = \sum_{n=1}^{n_{\text{max}}} 2\pi \frac{1}{2} \int (\varepsilon_\infty E_z^2 + \mu_0 B^2) r dr = \frac{E_0^2}{u}
\]

where \( u \) is a geometric factor which solely depends on the structure geometry and dielectric constant. For a given laser beam power, the axial electric field in the center region of the tube can be expressed as,

\[
E_0 = \left( \frac{P}{u \beta \gamma c} \right)^{1/2}
\]
where $\beta_s$ is the group velocity. The total acceleration distance $L = \frac{\nu \tau \beta_s}{1 - \beta_s}$ depends on the laser pulse length. Where $\tau$ is the laser pulse duration and $\nu$ is the velocity of the accelerated particles.

We numerically calculated the field distributions and energy densities for the boundary value problem discussed above (equations (2) (3) (4) (5) and (6)) for various geometries. For simplicity and ease of comparison, we fix the dielectric constant at 3 in these calculation. Other values yield the similar results provided the geometry is adjusted accordingly.

The ideal way to accelerate a charged particle is in the lowest radial $TM_{01}$ mode. However, at the optical frequencies being considered, the size of the structure would be prohibitively small. By using a higher order mode $TM_{0n}$ ($n > 1$), we can still have a uniform axial electric field in the center region and the field inside the dielectric layer would still governed by equation 2. Figure 2 shows the $E_z$ vs the radius for different radial modes. Hollow glass fibers with inner diameters of 10 microns can be fabricated and in fact, have been used for laser guided atoms experiment [14].

The following table gives several example cases illustrate that a very strong longitudinal field can be established by a relatively modest laser power. Here we are using a CO$_2$ laser ($\lambda=10.6 \, \mu m$) as the power source. Example 1 shows that for $n=1$, a structure with $a=4.7 \, \mu m$ and $b=6 \, \mu m$ is required for a dielectric of constant of 3, and less than 200 kW power is required to achieve 1 $GV/m$ acceleration. The group velocity in this case is 0.522 $c$. The power required is considerably less than that needed to achieve a correspondingly high gradient in the gas ICA
scheme. In order to have gain of 1 GeV net acceleration, a total acceleration distance of 1 m is required. Thus, at least 3 ns laser pulse length is needed. Example 2 gives the result for the radial mode \( n = 2 \). The structure has 15 \( \mu m \) inner radius and 20 \( \mu m \) outer radius. This is significantly larger than the example 1. In this case, 10 MW laser power will produce 1 GV/m gradient. One can even operate at still higher order modes. Example 3 shows that for \( n=11 \), a structure size of \( \sim 100 \mu m \) can be used. Approximately 1.3 GW of laser power is necessary to produce 1 GV/m in this case.

<table>
<thead>
<tr>
<th>Example</th>
<th>Power</th>
<th>( a(\mu m) )</th>
<th>( b(\mu m) )</th>
<th>( E_z )</th>
<th>radial modes</th>
<th>( \beta_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2 MW</td>
<td>4.7</td>
<td>6</td>
<td>1.1 GV/m</td>
<td>1</td>
<td>0.522 c</td>
</tr>
<tr>
<td>2</td>
<td>10 MW</td>
<td>15</td>
<td>20</td>
<td>1.0 GV/m</td>
<td>2</td>
<td>0.78 c</td>
</tr>
<tr>
<td>3</td>
<td>1.3 GW</td>
<td>60</td>
<td>100</td>
<td>1.0 GV/m</td>
<td>11</td>
<td>0.7</td>
</tr>
</tbody>
</table>

We would like to point out that a small, gas filled gap between the dielectric fiber and the outer conducting wall could be used to fine tune the phase velocity by adjusting the gas pressure. (serving in a sense as do the tuning stubs in some conventional linac structures). Fields inside the gap have interesting properties, though longitudinal electric field is very small, the transverse electric and magnetic fields are large. Therefore, a large portion of the laser power will flow through this vacuum section. In a case similar to that of example 3, except a 50 \( \mu m \) tuning gap, the axial electrical field is lowered by 20\%, although group velocity and geometries remain very close to example 3.

A key questions which must be addressed for all structure supported laser accelerator
schemes is the laser damage threshold. For materials commonly used in CO$_2$ laser applications, the damage threshold is typically about 0.5 J/cm$^2$ with laser pulse length in the range 10 ps - 1 ns [15]. Continuing research on high damage threshold materials may found a much better material to support the higher gradient which is much more desirable for our applications. For a proof of principle experiment currently considered at the Accelerator Test Facility of Brookhaven National Laboratory, using the geometry of example 2 (above), this would impose a limit of 1 μJ. For a 10 ps, 0.1 MW laser beam, an acceleration gradient of ~ 100 MV/m would be established over an acceleration distance of 13 mm due to the high group velocity of the wave. This would be feasible and interesting demonstration.

The transport of a charged particle beam through these small dielectric structures presents a major technical challenge. However, Wang et al [16] have studied the Brookhaven ATF low emittance beam line and have shown that in an emittance of $10^{-10}$ m-rad the electron beam contains $10^6$ particles, and can be focused to < 1 μm and transported through a distance of a few tens of centimeter. In principle this should be sufficient for a proof of principle experiment.

Next we would like to discuss collective beam effects, i.e. wakefields and beam break-up. As in any slow wave structure, the longitudinal wake function and frequency strongly depend on the size of structure. For the structures considered here (to be used at optical frequencies), the wake function can be very large. Calculation of the longitudinal wake functions for the structure considered here have been made. For radial mode $n=1$ structures, the wake field is on the order of TV/m (assuming 1 nC beam and 1μm rms bunch length). If the
ratio of wake field amplitude / acceleration field is to be kept below 10% to control the 
momentum spread of the beam at 1 GV/m acceleration gradient, the maximum charge which 
could be accelerated is about 0.1 pC which corresponding to a wake field amplitude of 100 
MV/m. However, for a 100 μm structure such as example 3 above, the wake field would be on 
the order of GV/m for 1 nC beam therefore, 10 - 100 pC beam can be accelerated through the 
dielectric channeled waveguide.

As in any accelerating structure, misaligned beam travelling in the dielectric structure 
will excite transverse wake-fields, with possible serious in head-tail instabilities. As stated 
above, in the relativistic limit, there is no direct focusing effect by the laser beam. External 
focusing by solenoids or quadrupoles with BNS damping [17] must be used. Design and 
fabrication of miniature strong magnets is already a subject of much current study[18], and more 
work is needed to explore the various options for controlling the beam break up effects.

In conclusion, we propose a new laser driven accelerator technique which promises an 
efficient way to couple laser energy to a charged particle beam. The scheme has all the 
properties of a travelling wave linac structure. Numerical examples show that a very modest 
amount of laser energy is required to achieve high gradient acceleration, much less that in gas 
based inverse Cherenkov acceleration in pressured gas system. Beam loading has been 
calculated and beam break-up issues have been discussed. An outline of a proof of principle 
experiment has been proposed. The Accelerator Test Facility (ATF) at Brookhaven National 
Laboratory may be a desirable place to do such an experiment because a suitable, radially 
polarized laser beam and low emittance e' beam are already available.
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References


Figure 1. Schematic diagram of a dielectric based "inverse Cherenkov Accelerator. A radially polarized laser beam propagates through an axicon type lens and is subsequently focus into a hollow dielectric structures. The laser beam interferes with itself and the resulting patterns are TM0n modes with strong longitudinal electrical fields. The structure has inner radius a and outer radius b partially filled with dielectric material. A transition section serves the purpose of matching the laser beam into the waveguide with minimum reflection.
Figure 2

Three graphs are shown, each labeled with different parameters:

1. For the first graph:
   - $a = 4.7 \, \mu m$
   - $b = 6 \, \mu m$
   - $n = 1$

2. For the second graph:
   - $a = 16 \, \mu m$
   - $b = 20 \, \mu m$
   - $n = 2$

3. For the third graph:
   - $a = 60 \, \mu m$
   - $b = 100 \, \mu m$
   - $n = 11$

The graphs are labeled 'Normalized Axial Electric Field' and show variations in the electric field with radial position (μm).
Figure 2 Caption. Longitudinal electric field distribution vs transverse positions. Where $a$ is inner radius of the vacuum region and $b$ is the radius of dielectric layer. $n$ is a mode number corresponding the phase velocity $= 1$. The longitudinal field in the vacuum region is constant. Field inside the dielectric layer varies in the form of zero's order bessel function.