We study the definition of perturbations in the presence of a submanifold, like e.g. a brane. In the standard theory of cosmological perturbations, one compares quantities at the same coordinate points in the non-perturbed and the perturbed manifolds, identified via a (non-unique) mapping between the two manifolds. In the presence of a physical submanifold one needs to modify this definition in order to evaluate perturbations of quantities at the submanifold location. As an application, we compute the perturbed metric and the extrinsic curvature tensors at the brane position in a general gauge.