LOW-MASS NORMAL-MATTER ATMOSPHERES OF STRANGE STARS
AND THEIR RADIATION

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ABSTRACT

The quark surface of a strange star has a very low emissivity for X-ray photons. I find that a small amount of normal matter at the quark surface with temperature in the range $10^7 \lesssim T_s \ll mc^2/k \simeq 6 \times 10^9$ K is enough to produce X-rays with high luminosity, $L_X \simeq 10^{32} - 10^{34} (\Delta M/10^{-22} M_\odot)^2$ erg s$^{-1}$. For the total atmosphere mass $\Delta M \sim (10^{-20} - 10^{-19}) M_\odot$, this luminosity may be as high as the Eddington limit. The mean energy of X-ray photons which are radiated from such a low-mass atmosphere of a strange star is $\sim 10^2 (T_s/10^8 \text{K})^{0.45} \simeq 30 - 300$ times larger than the mean energy of X-ray photons which are radiated from the surface of both a neutron star and a strange star with a massive normal-matter envelope, $\Delta M \sim 10^{-5} M_\odot$, for a fixed temperature at the stellar core. This raises the possibility that some black hole candidates with hard X-ray spectra are, in fact, such strange stars with a low-mass atmosphere. The X-ray emission from single strange stars is estimated.

Subject headings: stars: atmospheres - X-rays: stars - radiation mechanisms: thermal

1. INTRODUCTION

Strange stars have been proposed by Witten (1984) as a new class of astronomical compact objects. If Witten’s idea is true, at least some part of the compact objects known to astronomers as pulsars, powerful accreting X-ray sources, X-ray and $\gamma$-ray bursters, etc. might be strange stars, not neutron stars as is usually assumed (Alcock et al. 1986; Glendenning 1990; Caldwell & Friedman 1991; Madsen & Olesen 1991; Weber et al. 1996). Strange quark matter with the density of $\sim 4 \times 10^{14}$ g cm$^{-3}$ can exist, by hypothesis, up to the surface of strange stars. The quark surface is a very poor radiator at energies $\varepsilon_\gamma < 20$ MeV (Alcock et al. 1986). But, the presence of ”normal” matter (ions and electrons) at the quark surface may restore the ability of the surface to radiate soft photons (this is like painting with black paint on a silver surface). There is an upper limit to the amount of normal matter at the quark surface, $\Delta M \lesssim 10^{-5} M_\odot$ (e.g., Glendenning & Weber 1992), set by the requirement that the density of the inner layer of normal matter at the quark surface cannot be more than $\rho_n \simeq 4.3 \times 10^{11}$ g cm$^{-3}$. Such a massive envelope of normal matter with $\Delta M \sim 10^{-5} M_\odot$ completely obscures the quark surface.
If a neutron star is not too young (age \( t \gtrsim 10^2 \) yr), the stellar interior may be divided into two regions: the isothermal core with density \( \rho > \rho_e \sim 10^{10} - 10^{11} \text{ g cm}^{-3} \) and the outer envelope with \( \rho < \rho_e \) (e.g., Gudmundsson, Pethick & Epstein 1983; Nomoto & Tsuruta 1987; Schaaf 1990). At the core temperature \( T_c \sim 10^7 - 10^9 \) K, the temperature decreases by a factor of \( \sim 30 - 300 \) in the envelope, from \( T_c \) at the inner boundary of the envelope to \( \sim 10^6 \times (T_c/10^8 \text{ K})^{0.55} \) K at the neutron star surface. Since \( \rho_n \sim \rho_e \), for strange stars with massive envelopes, \( \Delta M \sim 10^{-5} M_\odot \), the temperature variation between the quark surface and the surface of the normal-matter envelope is more or less the same as the core-to-surface temperature variation of neutron stars for a fixed temperature at the stellar center. Besides, if the strange quark matter is superfluid, the cooling behavior of the quark core of strange stars is more or less similar to the cooling behavior of the isothermal core of neutron stars (e.g., Weber et al. 1996). Therefore, at an age \( t \gtrsim 10^2 \) yr, photon radiation from the surface of compact objects is not a good observational signature of strange stars with massive envelopes of normal matter.

It was pointed out (e.g., Haensel, Paczyński & Amstersdamski 1991; Burrows & Hayes 1995; Hartmann & Woosley 1995; Cheng & Dai 1996) that the temperature in the interior of both neutron stars and strange stars at the moment of their formation is very high, \( T_c \sim \) a few \( \times 10^{11} \) K. The mass of gas which is ejected from the surface of such a hot neutron star during \( \sim 10 \) s since its formation is \( \sim 10^{-3} - 10^{-2} M_\odot \) (e.g., Woosley & Baron 1992; Levinson & Eichler 1993; Woosley 1993). This value is considerably larger than the upper limit on the mass of normal-matter envelopes of strange stars. The input physics for calculations of gas outflow from the envelopes is similar for both hot neutron stars and hot strange stars. Therefore, it is natural to expect that if any normal matter remains at the surface of a strange star at \( t \gg 10 \) s, its mass is many orders smaller than the maximum. In the process of gas accretion onto such a strange star, the stellar atmosphere has to pass through the stage with \( \Delta M \ll 10^{-5} M_\odot \) before reaching the maximum, \( \Delta M \sim 10^{-5} M_\odot \), which was usually assumed in all studies of the thermal structure and photon radiation of strange stars. Below, photon radiation of a non-magnetic strange star with a low-mass atmosphere, \( \Delta M \ll 10^{-5} M_\odot \), is considered.

2. STRUCTURE AND PHOTON RADIATION OF THE ATMOSPHERE

We consider here a star consisting of a core of strange quark matter surrounded by a low-density atmosphere of normal matter with the mass \( \Delta M \), which is many orders of magnitude smaller than \( 10^{-5} M_\odot \). The core acts on the atmosphere as a heat reservoir. The thermal structure and photon radiation of the atmosphere can be found by solving the heat transfer problem with \( T = T_s \) as a boundary condition at the inner layer of normal matter. We assume that the temperature of the quark surface is \( T_s \gtrsim 10^7 \) K, and the hot gas of the atmosphere emits mainly due to free-free transitions (Gaetz and Salpeter 1983).

A young strange star cools very rapidly due to intense neutrino emission (e.g., Weber et al. 1996). Therefore, we restrict our consideration to the case of non-relativistic temperatures,
At such temperatures, the energy loss per unit gas volume by bremsstrahlung radiation is

\[ Q_{ff} = C_1 N_i N_e Z^2 T^{1/2} \text{ erg s}^{-1} \text{ cm}^{-3}, \quad (1) \]

where \( N_i = \rho / m_p A \) is the ion density in cm\(^{-3}\), \( N_e = N_i Z \) is the electron density in cm\(^{-3}\), \( m_p \) is the proton mass, \( A \) is the mass number of ions, \( Z \) is their electrical charge, \( T \) is the temperature in K, \( C_1 \approx 1.4 \times 10^{-27} g o(T) \) and \( g_o(T) \) is the frequency averaged Gaunt factor, which is in the range 1.1 to 1.5. Choosing a value of 1.2 for \( g_o(T) \) will give an accuracy to within about 20\% (Karzas and Latter 1961).

At \( T \ll T_0 \), the scale height, \( \Delta x \), of the atmosphere is very small compared to the stellar radius \( R \), and a plane-parallel approximation may be used. In this approximation all parameters depend on only one coordinate, \( x \), which is the distance from the quark surface.

The set of equations which describes the structure of the atmosphere will be the equation of hydrostatic equilibrium and the energy transport equation:

\[ \frac{dP}{dx} = -\frac{\rho GM}{R^2}, \quad \frac{dF}{dx} = -Q_{ff}, \quad (2) \]

where \( P = (N_e + N_i) kT = \rho kT / m_p \mu \) is the gas pressure, \( \mu = A / (1 + Z) \) is the mean molecular weight, \( G \) is the gravitational constant, \( M \) is the stellar mass, and \( F \) is the heat flux due to both thermal conductivity and convection (Schwarzschild 1958). The absorption of radiation is ignored in the energy transport equation because in our case the atmosphere is optically thin for the bulk of radiation (see below).

### 2.1. Isothermal atmosphere

The characteristic time of heat conduction in the atmosphere is

\[ t_{\text{heat}} \simeq \frac{k(1 + Z)N_e(\Delta x)^2}{Z \eta} \text{ s}, \quad (3) \]

and the characteristic cooling time of the hot atmospheric plasma via bremsstrahlung is

\[ t_{\text{cool}} \simeq 1.5 \times 10^{11} \frac{(1 + Z) T^{1/2}}{Z^2 N_e} \text{ s}, \quad (4) \]

where \( \eta \approx 10^{-6} Z^{-1} T^{5/2} \text{ erg s}^{-1} \text{ cm}^{-1} \text{ K}^{-1} \) is the coefficient of heat conductivity for a rarefied totally-ionized plasma (Spitzer 1967).
When \( t_{\text{heat}} \ll t_{\text{cool}} \), the atmosphere is nearly isothermal, \( T \simeq T_S \), and the equation of hydrostatic equilibrium (2) can be integrated immediately:

\[
\rho = \rho_0 \exp \left( -\frac{x}{\Delta x} \right), \quad \text{where} \quad \Delta x = \frac{R^2 k T_s}{G M \mu m_p}.
\] (5)

In this case the photon luminosity of the atmosphere is

\[
L = 4\pi R^2 \int_0^\infty Q_{ff} dx \simeq \frac{4 \times 10^{33} Z^3}{A(1+Z)} \left( \frac{R}{10^6 \text{ cm}} \right)^{-4} \left( \frac{M}{M_\odot} \right) \left( \frac{T_S}{10^8 \text{ K}} \right)^{-1/2} \left( \frac{\Delta M}{10^{12} \text{ g}} \right)^2 \text{ erg s}^{-1},
\] (6)

where \( \Delta M = 4\pi R^2 \rho_0 \Delta x \) is the total mass of the atmosphere.

Using equations (3) and (4), the condition \( t_{\text{heat}} \ll t_{\text{cool}} \) may be written as a limitation of the atmosphere mass: \( \Delta M \ll \Delta M_1 \), where

\[
\Delta M_1 \simeq 7 \times 10^{11} \frac{A}{Z^2} \left( \frac{T_S}{10^8 \text{ K}} \right)^{3/2} \left( \frac{R}{10^6 \text{ cm}} \right)^2 \text{ g}.
\] (7)

### 2.2. Convective atmosphere

The heat flux due to thermal conductivity, which is responsible for the heat transport in the atmosphere when \( \Delta M < \Delta M_1 \), is \( F = -\eta \frac{dT}{dx} \). Using this, we can rewrite the equation of hydrostatic equilibrium (2) in the form:

\[
\frac{d\rho}{dx} = -\frac{\rho}{T} \frac{GM m_p \mu}{R^2 k} \left( \frac{F}{\eta} \right).
\] (8)

In steady state, which we assume in this paper, we have the following boundary condition for \( F \) at \( x = 0 \): \( F|_{x=0} = L/4\pi R^2 \). From this condition and equation (8), the gradient of the density at the quark surface, \( x = 0 \), is

\[
\frac{d\rho}{dx}|_{x=0} = -\frac{\rho_0}{T_s R^2} \left( \frac{GM m_p \mu}{k} - \frac{L}{4\pi \eta} \right).
\] (9)

If the heat transport in the atmosphere is only due to thermal conductivity and the photon luminosity of the atmosphere is higher than

\[
L_1 = \frac{4\pi GM m_p \mu}{k} \eta|_{T= T_s} \simeq 3 \times 10^{33} \frac{\mu}{Z} \left( \frac{M}{M_\odot} \right) \left( \frac{T_S}{10^8 \text{ K}} \right)^{5/2} \text{ erg s}^{-1},
\] (10)
then the density of normal matter near the quark surface should increase with the distance from the surface, \((d\rho/dx)\big|_{x=0} > 0\). However, such a density behaviour is unstable, and convection develops in the atmosphere at \(L > L_1\). The value of \(L_1\) coincides with the photon luminosity \(L_1\) after substitution \(\Delta M = \Delta M_1\) from equation (7) into equation (6).

If convection takes place in the stellar atmosphere, it is a good approximation, as a rule, to say that the temperature gradient is equal to the adiabatic one, i.e. \(PT^{\gamma/(1-\gamma)} = \text{constant}\), where \(\gamma\) is the ratio of the specific heats at constant pressure and at constant volume. In this case, from equation (2), the temperature and density distributions are

\[
T = T_s \left[ 1 - \frac{(\gamma - 1)x}{\gamma \Delta x} \right], \quad \rho = \rho_0 \left( \frac{T}{T_s} \right)^{1/(\gamma - 1)},
\]

where \(\Delta x\) is determined by equation (5).

Using equations (1) and (11), one can get the photon luminosity of the convective atmosphere

\[
\tilde{L} = \frac{4\gamma L}{(3\gamma + 1)},
\]

where \(L\) is the photon luminosity of the isothermal atmosphere which is given by equation (6). Since \(\gamma \geq 1\), the value of \(\tilde{L}\) is in the range \(L \leq \tilde{L} < \frac{4}{3}L\). For a rarefied totally-ionized plasma we have \(\gamma = \frac{5}{3}\) and \(\tilde{L} = \frac{16}{9}L\). As noted earlier, the accuracy of equation (1) for the energy loss \(Q_{ff}\) is about 20%. Moreover, we did not take into account the general relativity effects which are \(\sim 20\%\). Therefore, the accuracy of our calculations of bremsstrahlung radiation from the atmospheres of strange stars is several times ten of percent. We can see that the difference between \(L\) and \(\tilde{L}\) is within the accuracy of our calculations.

The adiabatic approximation may be used to estimate the photon luminosity of the convective atmosphere only if the characteristic time of convection \(t_{\text{conv}} \simeq \Delta x/v_{\text{conv}}\), which is the main process of heat transport at \(\Delta M > \Delta M_1\), is smaller than the characteristic cooling time \(t_{\text{cool}}\) for the atmospheric plasma, where \(v_{\text{conv}}\) is the convective velocity which is limited by the velocity of sound, \(v_{\text{conv}} \lesssim c_s = (\gamma P/\rho)^{1/2}\). Using this and equation (4), the condition \(t_{\text{conv}} < t_{\text{cool}}\) may be written in the form: \(\Delta M < \Delta M_2\), where

\[
\Delta M_2 \simeq \frac{4 \times 10^{12} A}{Z^2 \mu^{1/2}} \left( \frac{T_s}{10^8 \text{ K}} \right) \left( \frac{R}{10^6 \text{ cm}} \right)^2 \text{ g}.
\]

At \(\Delta M = \Delta M_2\), the photon luminosity is

\[
L_2 \simeq 6 \times 10^{34} \frac{(1 + Z)^2}{Z^3} \left( \frac{M}{M_\odot} \right) \left( \frac{T_s}{10^8 \text{ K}} \right)^{3/2} \text{ erg s}^{-1}.
\]

For \(M \simeq 1.4M_\odot, T_s \simeq 2 \times 10^8 \text{ K}\) and \(Z = 1\) the value \(L_2\) is \(\sim 10^{36} \text{ erg s}^{-1}\) that is only two orders of magnitude smaller than the Eddington limit \(L_{\text{Edd}} \simeq 1.3 \times 10^{38} (A/Z)(M/M_\odot) \text{ erg s}^{-1}\).

The mean free-free optical depth of the atmosphere is
\[ \tau_0 \simeq \alpha_{ff} \Delta x \sim 10^{-9} \left( \frac{L}{10^{34} \text{erg s}^{-1}} \right) \left( \frac{T_s}{10^8 \text{K}} \right)^{-4}, \]  

where \( \alpha_{ff} \simeq 10^2 T_s^{-4} Q_{ff} \) is the Rosseland mean of the free-free absorption coefficient. At \( T_s > \text{a few} \times 10^7 \text{ K} \), the atmosphere is optically thin, \( \tau_0 \ll 1 \), up to \( L \sim L_{\text{Edd}} \).

For \( \Delta M > \Delta M_2 \), both thermal conductivity and convection are not able to account for the cooling of atmospheric matter, and a thermal instability develops in the atmosphere. As a result of this, the atmosphere cannot be in hydrostatic equilibrium during a time larger than \( \sim t_{\text{cool}} \), and it is strongly variable on a timescale of a few \( \times (2R^2 \Delta x/GM)^{1/2} \sim 10^{-4}(T_s/10^8 \text{K})^{1/2} \) s. Consideration of this variability and estimation of the photon luminosity at \( \Delta M > \Delta M_2 \) are under way and will be published elsewhere. Here, it is worth noting only that at \( T_s > \text{a few} \times 10^7 \text{ K} \) and \( \Delta M > \Delta M_2 \) the tendency of the photon luminosity to increase with increase of \( \Delta M \) has to be held up to \( L = L_{\text{Edd}} \).

### 3. CONCLUSIONS AND DISCUSSION

The photon luminosity of a strange star with a low-mass atmosphere, \( \Delta M < \Delta M_2 \), is given by equation (6) with the accuracy of several times ten of percent irrespective of the atmosphere structure. The photon luminosity may be very high, up to \( \sim L_{\text{Edd}} \). It is very important for discovery of strange stars that the mean energy of X-ray photons which are radiated from such stars is substantially larger than the mean energy of X-ray photons which are radiated from the surface of both a neutron star and a strange star with a massive envelope of normal matter, \( \Delta M \sim 10^{-5} M_\odot \), for a fixed temperature at the stellar core.

A source of X-rays may be a strange star if it meets the following criteria:

1. The X-ray emission (or at least one of its components) may be fitted by thermal emission of optically-thin plasma at \( kT \) up to \( \sim 10^2 \) keV.

2. The X-ray flux is variable at the X-ray luminosity \( L_X > L_2 \simeq (0.05 - 5) \times 10^{35}(T/10^8 \text{ K})^{3/2} \) erg s\(^{-1}\) depending on the composition of the atmosphere. This variability is either irregular or quasi-periodic.

3. The mass of the compact X-ray source is on the high side for neutron star masses because conversion of a neutron star to a strange star requires a very high density at the center of the neutron star (e.g., Alcock et al. 1986).

A few enigmatic X-ray sources which are considered as black hole candidates (e.g., Cherepashchuk 1996) answer these criteria and may be, in fact, strange stars with a low-mass atmosphere. They are 1E 1740.7 - 2942; GRS 1915 - 105, GRO J0422 + 32, GX 339 - 4 and SS 433. Some other powerful X-ray sources which are black hole candidates as well, for example Cyg
X-1, have both hard X-ray spectra which may be fitted by emission of optically-thin plasma and strong X-ray flux variability. The existent lower limits to the mass of these X-ray sources are substantially higher than the Oppenheimer-Volkoff limit for a strange star (Cherepashchuk 1996). However, these objects may be, in principle, a triple system with strange star (cf. Bahcall et al. 1974).

The thermal energy of a strange star itself is not enough for the star to be a powerful X-ray source, $L_X \sim (0.1 - 1)L_{\text{Edd}}$, for a long time, $t \geq 10^4 - 10^5$ yr, and accretion of gas onto the strange star is necessary to account for such a strong prolonged X-ray emission. The kinetic energy of the accreted gas may be transformed into emission of the strange star atmosphere in the following way. Let the magnetic field at the stellar surface be $B_0 \sim (0.3 - 1) \times 10^{12}$ Gauss. This field canalizes the gas motion along the field lines to the quark surface. In this case, the kinetic energy of ions at the surface is about hundred MeV per nucleon that is about 5 times more than the Coulomb barrier at the quark surface (Alcock et al. 1986). Accreted particles penetrate through the quark surface (cf. below), and they are dissolved into quark matter. As a result, the quark core is heated at the magnetic poles. The process of heat transport through the core is very fast due to a very high heat conductivity of quark matter, and therefore, the quark core is nearly isothermal. Then, the energy which is released in the process of gas accretion is radiated from the normal-matter atmosphere more or less isotropically just as it is discussed above.

The total atmosphere mass which restores the ability of the quark surface to radiate X-ray photons is extremely small. At first sight, even if the rate of gas accretion onto a hot strange star is very small, the atmosphere mass increases rapidly up the the value when the photon luminosity is $L_{\text{Edd}}$. But this does not necessarily happen. The point is that ions of accreted gas in the process of their motion through the atmosphere collide with the atmosphere ions and draw them into the quark surface. As a result, a steady state of the atmosphere may be achieved before the photon luminosity is $L_{\text{Edd}}$. Let us estimate the photon luminosity at such a steady state. For definiteness, we assume that a single strange star with $M \simeq 1.4M_\odot$ is at rest in a uniform ionized gas of pure hydrogen. We shall assume that the gas is at rest at infinity with the density of $n_a \simeq 1 \text{ cm}^{-3}$ and the temperature of $T_h \simeq 10^4$ K. This situation will correspond to accretion in a typical interstellar H II region. In this case, the accretion rate is $\dot{M} \simeq 2 \times 10^{10}$ g s$^{-1}$ (e.g., Shapiro & Teukolsky 1983). The efficiency for radiation of the gas during its accretion is very low, i.e. the gas motion is nearly adiabatic. The mean kinetic energy of accreted protons at the surface is $E_{\text{kin}} \simeq 100$ MeV. The temperature of accreted gas near the stellar surface is $T_h \simeq 10^{11}$ K ($kT_h \simeq 10$ MeV). The bulk of accreted protons passes through the quark surface. Protons can be reflected from the quark surface back into the atmosphere only if in the frame of the star the kinetic energy of their radial motion to the stellar surface is smaller than the Coulomb barrier. Taking this into account and assuming that the accreted protons are Maxwellian, the rate of the atmosphere mass accumulation is $\dot{M}_a \simeq \exp(-E_{\text{kin}}/kT_h)\dot{M} \simeq \exp(-10)\dot{M}$. The characteristic time of the atmosphere mass decrease due to bombardment by the accreted protons is $\Delta t_b \simeq (n_a\sigma_v)^{-1} \simeq (4\pi R^2 m_p/\dot{M}\sigma)$, where $n_a \simeq \dot{M}/(4\pi R^2 v_0 m_p)$ is the density of
accreted protons at the surface, $v_a$ is their velocity and $\sigma \simeq 10^{-26} \text{ cm}^2$ is the cross section for proton-proton collision at energies of $\sim 100 \text{ MeV}$. The atmosphere mass at the steady state is $\Delta M_{st} \simeq \dot{M}_a \Delta t_b \simeq 4\pi \exp(-E_{\text{kin}}/kT_b)R^2m_p\sigma^{-1} \simeq 10^{12} \text{ g}$ which does not depend on the accretion rate. The steady state may be reached in $\sim \Delta t_b \sim 10^5 \text{ s}$. From equation (6), for $T_s \sim 10^7 - 10^8 \text{ K}$ the expected X-ray luminosity of a single strange star is $L_X \sim 10^{34} \text{ erg s}^{-1}$, that is of the order of X-ray luminosity of a single neutron star in $\sim 10^2 - 10^4 \text{ yr}$ after its formation. But, in the case of such a strange star the expected X-ray spectrum is much harder than the X-ray spectrum of the thermal radiation from a single neutron star with the same luminosity.

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