Direct Measurement of the Top Quark Mass

The DØ Collaboration*

*Authors listed on the following page.

Fermi National Accelerator Laboratory, Batavia, Illinois 60510
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Abstract

We measure the top quark mass $m_t$ using $t\bar{t}$ pairs produced in the DØ detector by $\sqrt{s} = 1.8$ TeV $p\bar{p}$ collisions in a 125 pb$^{-1}$ exposure at the Fermilab Tevatron. We make a two constraint fit to $m_t$ in $t\bar{t} \to bW^+\bar{b}W^-$ final states with one $W$ decaying to $q\bar{q}$ and the other to $e\nu$ or $\mu\nu$. Events are binned in fit mass versus a measure of probability for events to be signal rather than background. Likelihood fits to the data yield $m_t = 173.3 \pm 5.6$ (stat) $\pm 6.2$ (syst) GeV/c$^2$. 

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(DØ Collaboration)
Universidad de los Andes, Bogotá, Colombia
University of Arizona, Tucson, Arizona 85721
Boston University, Boston, Massachusetts 02215
Brookhaven National Laboratory, Upton, New York 11973
Brown University, Providence, Rhode Island 02912
Universidad de Buenos Aires, Buenos Aires, Argentina
University of California, Davis, California 95616
University of California, Irvine, California 92697
University of California, Riverside, California 92521
LAFEX, Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, Brazil
CINVESTAV, Mexico City, Mexico
Columbia University, New York, New York 10027
Delhi University, Delhi, India 110007
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
Florida State University, Tallahassee, Florida 32306
University of Hawaii, Honolulu, Hawaii 96822
University of Illinois at Chicago, Chicago, Illinois 60607
Indiana University, Bloomington, Indiana 47405
Iowa State University, Ames, Iowa 50011
Korea University, Seoul, Korea
Kyungsung University, Pusan, Korea
Lawrence Berkeley National Laboratory and University of California, Berkeley, California 94720
University of Maryland, College Park, Maryland 20742
University of Michigan, Ann Arbor, Michigan 48109
Michigan State University, East Lansing, Michigan 48824
Moscow State University, Moscow, Russia
University of Nebraska, Lincoln, Nebraska 68588
New York University, New York, New York 10003
Northeastern University, Boston, Massachusetts 02115
Northern Illinois University, DeKalb, Illinois 60115
Northwestern University, Evanston, Illinois 60208
University of Notre Dame, Notre Dame, Indiana 46556
University of Oklahoma, Norman, Oklahoma 73019
University of Panjab, Chandigarh 16-00-14, India
Institute for High Energy Physics, 142-284 Protvino, Russia
Purdue University, West Lafayette, Indiana 47907
Rice University, Houston, Texas 77005
Universidade Estadual do Rio de Janeiro, Brazil
University of Rochester, Rochester, New York 14627
CEA, DAPNIA/Service de Physique des Particules, CE-SAACLAY, Gif-sur-Yvette, France
Seoul National University, Seoul, Korea
State University of New York, Stony Brook, New York 11794
Tata Institute of Fundamental Research, Colaba, Mumbai 400005, India
University of Texas, Arlington, Texas 76019
Texas A&M University, College Station, Texas 77843
We also demand \( \vec{G} \) the vector of measured (fit) variables and exhibited in Fig. ?? only in the calorimeter. As signatures of the \( e \) and one isolated \( \mu \) than is possible for the lighter quarks, which decay after they form hadrons. Since our best estimate of \( m_t \) is obtained from the best match between MC samples and the data. From the 90-event distribution shown in Fig. ??(a), reduces the difference in \( \Delta R \equiv (\Delta \phi^2 + \Delta \eta^2)^{1/2} = 0.5 \), having \( E_T > 15 \) GeV and \(|\eta| < 2\).

Within \( \Delta R = 0.5 \) of a jet axis, additional muons (\( \mu \) tags) satisfying \( p_T^\mu > 4 \) GeV/c and \(|\eta| < 1.7 \) arise mainly from \( b \) and \( c \) quark semileptonic decay. These occur in \( \approx 20\% \) of \( t\bar{t} \) events but only \( \approx 2\% \) of background events [?]. In untagged events, to suppress background we require \( E_T^c \equiv (|E_T^e| + |E_T^\mu|) > 60 \) GeV and \(|\eta_W| < 2 \) for the \( W \rightarrow \ell \nu \). The latter cut, exhibited in Fig. ??(a), reduces the difference in \( \eta_W \) distributions between data and Monte Carlo (MC) simulated background. We use the HERWIG MC [?] to simulate top signal, and the VECBOS MC [?] (with HERWIG fragmentation of partons into jets) to simulate (but not to normalize) the dominant \( W+\text{multijet} \) background. The \( \approx 20\% \) of background events from non-\( W \) sources are modeled by multijet data that barely fail the lepton identification criteria.

To each event passing the above cuts, we make a two constraint (2C) kinematic fit [?] to the \( t\bar{t} \rightarrow \ell+\text{jets} \) hypothesis by minimizing a \( \chi^2 = (\mathbf{v} - \mathbf{v}^*)^T G (\mathbf{v} - \mathbf{v}^*) \), where \( \mathbf{v} \) (\( \mathbf{v}^* \)) is the vector of measured (fit) variables and \( G^{-1} \) is its error matrix. Both reconstructed \( W \) masses are constrained to equal the \( W \) pole mass, and the same fit mass \( m_{\text{fit}} \) is assigned to both the \( t \) and \( \bar{t} \) quarks. If the event contains \( >4 \) accepted jets, only the four jets with highest \( E_T \) are used. In \( \approx 50\% \) of MC top events, these jets correspond to the \( b, \bar{b}, q, \) and \( \bar{q} \). With (without) a \( \mu \) tag in the event, there are \( 6 \) (12) possible fit assignments of these jets to the quarks, each having two solutions to the \( \nu \) longitudinal momentum \( p_x^\nu \). We use \( m_{\text{fit}} \) only from the permutation with lowest \( \chi^2 \), the correct choice for \( \approx 20\% \) of MC top events. Because of the ambiguities, \( m_{\text{fit}} \) is not the same as \( m_t \), though they are strongly correlated. Our best estimate of \( m_t \) is obtained from the best match between MC samples and the data.

From the 90-event distribution shown in Fig. ??(b) we select 77 events with a 2C fit satisfying \( \chi^2 < 10 \). Of these, 5 are \( \mu \) tagged and \( \approx 65\% \) are background. Further separation of signal and background events is based on four kinematic variables \( x \equiv \{ x_1, x_2, x_3, x_4 \} \).
chosen to have small correlation with \( m_{\text{fit}} \). On average, all are larger for MC top events than for background events, selected to have the same \( \langle m_{\text{fit}} \rangle \) as the top events [?]. The simpler variables are \( x_1 \equiv E_T \) and \( x_2 \equiv A \), where aplanarity \( A \) is \( \frac{3}{2} \) times the least eigenvalue of the normalized laboratory momentum tensor of the jets and the \( W \) boson. The third variable \( x_3 \equiv H_{T2}/H_z \) measures the event’s centrality, where \( H_z \) is the sum of \( |p_z| \) of \( \ell, \nu \), and the

![Graphs showing events per bin vs. event selection variables defined in the text, plotted for (a–b, g–h) top quark mass analysis samples, and (c–f) \( W+3 \) jet control samples. Histograms are data, filled circles are expected top + background mixture, and open triangles are expected background only. Solid arrows in (a–b) show cuts applied to all events; the open arrow in (g) illustrates the LB cut. The nonuniform bin widths in (g–h) are chosen to yield uniform bin populations.](image)
jets, and $H_{T2}$ is the sum of all jet $|E_T|$ except the highest. Finally, $x_4 \equiv \Delta R_{jj}^{\text{min}} E_T^{\text{min}} / E_T^L$ measures the extent to which jets are clustered together, where $\Delta R_{jj}^{\text{min}}$ is the minimum $\Delta R$ of the six pairs of four jets, and $E_T^{\text{min}}$ is the smaller jet $E_T$ from the minimum $\Delta R$ pair. As shown for the background dominated $W+3$ jet sample in Fig. ??(c–f), $x_1$–$x_4$ are reasonably well modeled by MC; this is true also for the $W+2$ jet and top mass samples (not shown).

We bin events in a two-dimensional array with abscissa $m_{\text{fit}}$ and ordinate $D(x)$, where $D$ is a multivariate discriminant. To show that our results are robust, we use two methods for which the definition of $D$, the granularity with which it is binned, and the additional requirements are different. In our “low bias” (LB) method, we first parametrize $\mathcal{L}_i(x_i) \equiv s_i(x_i)/b_i(x_i)$, where $s_i$ and $b_i$ are the top signal and background densities in each variable, integrating over the others. We form the log likelihood $\ln \mathcal{L} \equiv \sum \omega_i \ln \mathcal{L}_i$, where the weights $\omega_i$ are adjusted slightly away from unity to nullify the average correlation (“bias”) of $\mathcal{L}$ with $m_{\text{fit}}$, and for each event we set $D_{\text{LB}} = \mathcal{L} / (1 + \mathcal{L})$. Finally, we divide the ordinate coarsely into signal- and background-rich bins according to whether the LB cut is passed. This cut is satisfied if a $\mu$ tag exists; otherwise it is not satisfied if $D_{\text{LB}} < 0.43$ (Fig. ??(g)) or if $H_{T2} < 90$ GeV.

Our neural network (NN) method is sensitive to the correlations among the $x_i$ as well as to their individual densities. We use a three layer feed-forward NN with 4 input nodes fed by $x$, 5 hidden nodes, and 1 output node, trained on samples of top signal (background) with

![Diagram](image_url)

**FIG. 2.** Events per bin (x areas of boxes) vs. $D_{\text{NN}}$ (ordinate) and $m_{\text{fit}}$ (abscissa) for (a) expected 172 GeV/c^2 top signal, (b) expected background, and (c) data. $D_{\text{NN}}$ is binned as in Fig. ??(h).
density \( s(x)/(b(x)) \) \[^3\]. For a given event, the network output \( D_{\text{NN}} \) approximates the ratio \( s(x)/(s(x) + b(x)) \). We divide the ordinate finely into ten bins in \( D_{\text{NN}} \), independent of \( H_{T2} \) or \( \mu \) tagging. Figure \( ?(g-h) \) shows that \( D_{\text{LB}} \) and \( D_{\text{NN}} \) are distributed as predicted and provide comparable discrimination, as we expect when the \( \omega_i \) are close to unity and the \( L_i \) are not strongly correlated. Figure \( ?? \) exhibits the arrays for the NN method. Little correlation between \( D_{\text{NN}} \) and \( m_{\text{fit}} \) is evident in the expected signal or background distributions, which are distinct; the data clearly reveal contributions from both sources. Figure \( ?? \) shows the distributions of \( m_{\text{fit}} \) for data (a) passing and (b) failing the LB cut.

To each \( m_t \) for which we have generated MC, we assign a likelihood \( L \) which assumes

![Graph showing distributions](image)

FIG. 3. (a–b) Events per bin vs. \( m_{\text{fit}} \) for events (a) passing or (b) failing the LB cut. Histograms are data, filled circles are the predicted mixture of top and background, and open triangles are predicted background only. The circles and triangles are the average of the LB and NN fit predictions, which differ by \(<10\%\). (c) Log of arbitrarily normalized likelihood \( L \) vs. true top quark mass \( m_t \) for the LB (filled triangles) and NN (open squares) fits, with errors due to finite top MC statistics. The curves are quadratic fits to the lowest point and its 8 nearest neighbors. In MC studies, 7\% (27\%) of simulated experiments yield a smaller LB (NN) maximum likelihood.
TABLE I. Results of fits to data and MC events. Fits to data yield values and errors $\sigma$(stat) for $m_t$, $n_s$, and $n_b$ (described in the text). Systematic errors are combined in quadrature. The resulting $m_t$ and its statistical error $\sigma_m$ are the combined LB and NN values. Fits to MC use ensembles of 10,000 simulated experiments composed of top + background, with $m_t$, $\langle n_s \rangle$, and $\langle n_b \rangle$ as listed. They yield a mean result $\langle m_t \rangle$, a mean statistical error $\langle \sigma_m \rangle$, and a range $\pm \delta m$ within which 68% of the results fall. Using the LB (NN) method, 6% (25%) of the simulated experiments produce a $\sigma_m$ which is smaller than we obtain. For an “accurate subset” of the MC ensembles with mean $\sigma_m/m_t$ that matches our value, $\delta m$ is smaller.

<table>
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<th>Fits to data</th>
<th>---LB fit---</th>
<th>---NN fit---</th>
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</thead>
<tbody>
<tr>
<td>Quantity fit</td>
<td>value</td>
<td>value</td>
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<tr>
<td>$m_t$ (GeV/c$^2$)</td>
<td>174.0 ± 5.6</td>
<td>171.3 ± 6.0</td>
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<tr>
<td>$n_s$</td>
<td>23.8 +8.3 $-$7.8</td>
<td>28.8 +8.4 $-$9.1</td>
</tr>
<tr>
<td>$n_b$</td>
<td>53.2 +10.7 $-$9.3</td>
<td>48.2 +11.4 $-$8.7</td>
</tr>
<tr>
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<td>energy scale $\pm$ 4.0</td>
<td>generator $\pm$ 4.1</td>
</tr>
<tr>
<td>Resulting $m_t$ (GeV/c$^2$)</td>
<td>173.3 ± 5.6 (stat) ± 6.2 (syst)</td>
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</table>

<table>
<thead>
<tr>
<th>Fits to MC type</th>
<th>---input---</th>
<th>---output---</th>
</tr>
</thead>
<tbody>
<tr>
<td>(top + background) of fit $m_t$, $\langle n_s \rangle$, $\langle n_b \rangle$, $\langle \sigma_m \rangle$, $\langle m_t \rangle$, $\delta m$</td>
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<td>full ensemble</td>
<td>LB 175 24 53 9.9 175.0 8.7</td>
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<tr>
<td>&quot;</td>
<td>NN 172 29 48 8.5 171.6 8.0</td>
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</tr>
<tr>
<td>accurate subset</td>
<td>LB 175 24 53 5.5 175.3 4.6</td>
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<tr>
<td>&quot;</td>
<td>NN 172 29 48 5.8 172.0 6.0</td>
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</tbody>
</table>

that all samples obey Poisson statistics. Bayesian integration [?] over possible true signal and background populations in each bin yields

$$L(m_t, n_s, n_b) = \prod_{i=1}^{M} \sum_{j=0}^{n_i} \left( \frac{n_{si} + j}{j} \right) \left( \frac{n_{bi} + k}{k} \right) p_s^j(1 + p_s)^{-n_{si} - j - 1} p_b^k(1 + p_b)^{-n_{bi} - k - 1},$$

where $n_s$ ($n_b$) is the expected number of signal (background) events in the data; $n_i$, $n_{si}$, and $n_{bi}$ are the actual number of data, MC signal, and MC background events in bin $i$; $k \equiv n_i - j$; $p_s,b \equiv n_{s,b}/(M + \sum_i n_{s,b,i})$; and $M = 40$ (200) bins for the LB (NN) methods. Maximizing $L$ for each $m_t$ gives the best estimates $n_s^*(m_t)$ and $n_b^*(m_t)$ for $n_s$ and $n_b$. Figure ??(c) displays $\ln L(m_t, n_s^*(m_t), n_b^*(m_t))$ vs. $m_t$, where the curves determine the best fit $m_t$ and its statistical error $\sigma_m$.

Table ?? presents the fit results, which are consistent with Ref. [?] and with recent reports [?]. The LB and NN results $m_t^{LB}$ and $m_t^{NN}$ are mutually consistent; in 21% of MC experiments they are further apart. Nevertheless we include half of $m_t^{LB} - m_t^{NN}$ in the systematic error. To obtain our result, shown in Table ??, we combine $m_t^{LB}$ and $m_t^{NN}$ allowing for their $(88 \pm 4)$% correlation (determined by MC experiments). Figures ??(a–b)
show that this result represents the data well. From the MC experiments summarized in Table ?? we measure the interval ±δm within which 68% of the MC estimates fall. For the full ensemble, δm is larger than σm from our data. However, for “accurate subsets” of the ensemble for which the average σm/mt is the same as we observe, δm is close to σm [?].

A principal systematic error in mt arises from uncertainty in the jet energy scale, which is calibrated in three steps. In step 1, applied before events are selected, the summed energy Ejet of particles emitted within the jet cone is related [?] to the measured energy Em by Ejet = (Em − O)/R(1 − S). Here the calorimeter response R is calibrated using Z → ee decays and ET balance in γ+jet events, the fractional shower leakage S out of the jet cone is set by test beam data, and the energy offset O due to noise and the underlying event is determined using events with multiple interactions. Steps 2 and 3 are applied only to jet energies used to find mfit. In step 2, top MC is used to correct Ejet to the parton energy in both data and MC. This sharpens the resolution in mfit. Step 3 is a final adjustment based on more detailed study of γ+jet events in data and MC, particularly focused on the dependence of the ET balance upon η of the jet. We assign a jet-scale error of ±(2.5% + 0.5 GeV) based on the internal consistency of step 3, on variations of the γ+jet cuts and the model for the underlying event, and on an independent check of the ET balance in Z+jet events. This leads to an error on mt of ±4.0 GeV/c².

We estimate the uncertainties in modeling of QCD by substituting the isajet MC generator [?] for herwig, independently for top MC and for vecbos fragmentation, and by changing the vecbos QCD scale from jet ⟨pt⟩² to M² W. The resulting systematic error due to the generator is ±4.1 GeV/c². Other effects including noise, multiple pp interactions, and differences in fits to ln L contribute ±2.2 GeV/c². All systematic errors (Table ??) sum in quadrature to ±6.2 GeV/c². Therefore our direct measurement of the top quark mass is

mt = 173.3 ± 5.6 (stat) ± 6.2 (syst) GeV/c².

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REFERENCES

* Visitor from IHEP, Beijing, China.
† Visitor from Universidad San Francisco de Quito, Quito, Ecuador.

[10] We have varied our analysis procedures (e.g. the binning of $D$) in ways which have little systematic effect on MC results. From data we observe little change in $m_t$, together with variations in $\sigma_m$ which are of the same order as those of $\delta m$ in Table ?? . We interpret the variations in $\sigma_m$ as stochastic effects to which the MC studies in Table ?? are relevant.