Coherence and Emergence of Classical Spacetime

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Abstract

Using the coherent-state representation we show that the classical Einstein equation for the FRW cosmological model with a general minimal scalar field can be derived from the semiclassical quantum Einstein equation.

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Canonical quantum gravity has been initiated by DeWitt in the seminal paper [1]. As a methodology to understand the quantum aspects of cosmology, quantum cosmology has been intensively studied, and in particular, as a great conceptual advancement, the boundary conditions have been incorporated for the Universe by Hartle-Hawking [2] and Vilenkin [3]. Semiclassical quantum gravity has also been elaborated as a methodology to include some part of quantum effects into classical gravity [4]. In order to consider the different mass scales between gravity and matter fields and to apply quantum cosmology to the early Universe, one should have the reduction scheme from canonical quantum gravity, $\hat{G}_{\mu\nu} = 8\pi \hat{T}_{\mu\nu}$, to semiclassical quantum gravity, $G_{\mu\nu} = 8\pi \langle \hat{T}_{\mu\nu} \rangle$, and down to classical gravity, $G_{\mu\nu} = 8\pi T_{\mu\nu}$.

In this *Brief Report*, we complete the reduction from semiclassical quantum gravity to classical gravity for a quantum FRW cosmological model with a general minimal scalar field. We find that the coherent-state representation of the semiclassical quantum Einstein equation leads to the classical Einstein equation with a quantum correction. In previous papers [5,6], we showed that in the case of a massive scalar field an exact quantum state of time-dependent Schrödinger equation gives rise to the mean energy density which has the same form as the classical one except that the field intensity is replaced by the absolute value, and that a coherent state exactly gives rise to the classical density plus an additional one from vacuum fluctuation. We extend the result of the massive scalar-field model to the general scalar-field model.

As a quantum cosmological model, we consider the FRW Universe whose Wheeler-DeWitt equation is given by

$$\left[ \frac{2\pi\hbar^2}{3m_P a} \frac{\partial^2}{\partial a^2} - \frac{3m_P k a}{8\pi} - \frac{\hbar^2}{2a^2} \frac{\partial^2}{\partial \phi^2} + a^3 V(\phi) \right] \Psi(a, \phi) = 0. \quad (1)$$

Here, $k$ takes 1, 0, and $-1$ for a closed, spatially flat, and open universe, respectively. The unit system is $c = 1$ and $G = \frac{1}{m_P}$. The corresponding classical Einstein equation is

$$\left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{8\pi}{3m_P} \left( \frac{\dot{\phi}^2}{2} + V(\phi) \right), \quad (2)$$

and the classical field equation is

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\[ \ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} + \frac{dV(\phi)}{d\phi} = 0. \quad (3) \]

Following the reduction scheme, one obtains the semiclassical quantum gravity from the Wheeler-DeWitt equation:

\[ \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{8\pi}{3m_P a^3} \langle \hat{H} \rangle, \quad (4) \]

and

\[ i\hbar \frac{\partial}{\partial t} \Phi(\phi, t) = \hat{H} \Phi(\phi, t) \quad (5) \]

where

\[ \hat{H} = \frac{1}{2a^3} \pi^2 + a^3 V(\hat{\phi}). \quad (6) \]

We now represent the semiclassical Einstein equation in the coherent state. For the case of the massive scalar-field model, the coherent-state representation was given explicitly in terms of classical solutions [6]. For the case of a general scalar-field model, we use the coherent states constructed by Rajagopal and Marshall [7]. We follow their main idea but redefine some of variables to be suitable for the application to quantum field theory in FRW cosmology. We construct the Fock space by introducing the creation and annihilation operators

\[ \hat{\mathcal{A}}^\dagger(t) = u(t) \pi - a^3 \dot{u}(t) \hat{\phi}, \]
\[ \hat{\mathcal{A}}(t) = u^*(t) \pi - a^3 \dot{u}^*(t) \hat{\phi}. \quad (7) \]

As in Ref. [8], if we require that \( \hat{\mathcal{A}} \) and \( \hat{\mathcal{A}}^\dagger \) be the invariant operators

\[ i\hbar \frac{\partial}{\partial t} \{ \hat{\mathcal{A}}^\dagger, \hat{\mathcal{A}} \} - \{ \hat{\mathcal{A}}^\dagger, \hat{\mathcal{H}} \} = 0, \quad (8) \]

then \( u \) satisfies the equation

\[ (a^3 \dot{u} \hat{\phi} + a^3 u V'(\hat{\phi}) = 0. \quad (9) \]
From the usual commutation relation it follows that

\[ \hbar a^3 \left( u \dot{u}^* - u^* \dot{u} \right) = i. \] (10)

The one parameter-dependent vacuum is defined to be annihilated by the annihilation operator

\[ \hat{A}(t) |0, t\rangle = 0. \] (11)

The momentum and position operators are given by

\[ \hat{\phi} = -i\hbar \left( u^* \hat{A}^\dagger - u \hat{A} \right), \]
\[ \hat{\pi} = -i\hbar a^3 \left( \dot{u}^* \hat{A}^\dagger - \dot{u} \hat{A} \right). \] (12)

A coherent state is an eigenstate of \( \hat{A}(t) \):

\[ \hat{A}(t) |\alpha, t\rangle = \alpha |\alpha, t\rangle. \] (13)

It is a unitary transformation of the vacuum by a displacement operator

\[ |\alpha, t\rangle = \exp \left( \alpha \hat{A}^\dagger - \alpha^* \hat{A} \right) |0, t\rangle. \] (14)

The position and momentum operators in the coherent-state representation yield the classical field and momentum

\[ \phi_c \equiv \langle \alpha, t | \hat{\phi} | \alpha, t \rangle = 2\hbar \frac{\left( u^* \alpha^* - u \alpha \right)}{2i}, \]
\[ \pi_c \equiv \langle \alpha, t | \hat{\pi} | \alpha, t \rangle = 2\hbar a^3 \frac{\left( \dot{u}^* \alpha^* - \dot{u} \alpha \right)}{2i}. \] (15)

Thus, in the coherent-state representation, the semiclassical Einstein equation leads to

\[ \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{8\pi}{3m_P a^3} \langle \alpha, t | \hat{H} | \alpha, t \rangle \]
\[ = \frac{8\pi}{3m_P a^3} \left( \frac{\pi_c^2}{2a^3} + a^3 V(\phi_c) + H_q \right) \] (16)

where
\[ H_q = \frac{\hbar^2}{2} a^3 \dot{u}^* \dot{u} + a^3 \left[ \exp\left(\frac{\hbar^2}{2} u^* u \frac{\partial^2}{\partial \phi_c^2}\right) - 1 \right] V(\phi_c) \]  

is a quantum correction to the classical energy density. By identifying \( \pi_u = a^3 \dot{u}^* \) and \( \pi_{u^*} = a^3 \dot{u} \), one may obtain the Hamilton equations for \( H_q \)

\[ (a^3 \dot{u}) + \frac{2}{\hbar^2} a^3 \frac{\partial}{u} \left[ \exp\left(\frac{\hbar^2}{2} u^* u \frac{\partial^2}{\partial \phi_c^2}\right) - 1 \right] V(\phi_c) = 0. \]  

The equation (18) for \( u \) is identical to the equation obtained by differentiating Eq. (9) with respect to \( \hat{\phi} \) and taking the expectation value with the coherent state \( |\alpha, t\rangle \). The coincidence of the invariant equation and the Hamilton equations in the same Fock-space representation follows from the physical principle of the extremization of action [9] for any time-dependent system. It should be noted that the quantum corrections are of the order of \( \hbar^2 \) or higher.

In summary, we showed that for the FRW cosmological model with a general minimal scalar field, the semiclassical Einstein equation (16) can reduce to the classical Einstein equation whose energy density is the sum of classical one and the quantum correction (17). Therefore, we suggest that coherence may be one of the necessary mechanisms for the emergence of classical spacetime from the semiclassical quantum gravity. Though not shown explicitly, the coherence can be understood partially from the dissipation of quantum fields as the Universe expands.

**ACKNOWLEDGMENTS**

JYJ, SPK, and HSS were supported by the Non-Directed Research Fund, Korea Research Foundation, 1996, and KSS was supported by the Center for Theoretical Physics, the Seoul National University.
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