The hard scattering amplitude for the higher helicity components of the pion form factor

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For obtaining the spin space wave function of the pion meson in the light-cone formalism from the naive quark model, it is necessary to take into account Wigner rotation. Consequently there are higher helicity ($\lambda_1 + \lambda_2 = \pm 1$) components in the light-cone spin space wave function of the pion besides the usual helicity ($\lambda_1 + \lambda_2 = 0$) components. For the pion electromagnetic form factor, we calculate the hard-scattering amplitude for the higher helicity components in the light-cone perturbation theory. It is found that the hard-scattering amplitude for the higher helicity components is of order $1/Q^4$, which is vanishingly small compared to that of the ordinary helicity component at very high $Q^2$ but should be considered in the $Q^2$ region where experimental data are available.


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I. INTRODUCTION

There have been a lot of discussions about whether perturbative QCD (PQCD) is applicable to exclusive processes at currently available experimental energies [1-12]. In the example of the pion and proton form factors, Isgur and Llewellyn Smith [2] noticed that in the energy region of a few GeV the main contributions come from the end-point region $x \to 0, 1$ ($x$ is the fractional momentum carried by the parton) where the running coupling constant $\alpha_s$ becomes large. Thereby perturbation expansion might be illegal. Recently this problem has been attacked and it is suggested that PQCD might be still applicable to the exclusive processes at currently experimental accessible regime of momentum transfer ($Q^2 \sim \text{a few GeV}^2$) by using some techniques to cure the end-point problem [5-10]. Huang and Shen [5] pointed out that the applicability of PQCD to the hadronic form factors is questionable only as momentum transfers being $Q^2 \leq 4\text{GeV}^2$ by reanalyzing the contributions from the end-point region for the pion form factor. Li and Sterman [6] proposed a modified perturbation expression for the pion form factor by taking into account the customarily neglected partonic transverse momentum as well as Sudakov correction. They obtain a similar conclusion as [5]: PQCD begins to be self-consistent at about $Q \sim 20\Lambda_{QCD}$. More recently, Ji, Pang, and Szczepaniak [12] pointed out that the usual factorization perturbation expression for the pion form factor is derived from the light-cone time-order perturbative expansion, and the natural variable to make a separation of perturbative contributions from contributions intrinsic to the bound-state wave function itself is the light-cone energy rather than the gluon virtuality of the hard scattering amplitude $T_H$. They find that the “legal” PQCD contribution defined by the light-cone energy cut becomes self-consistent at even much smaller $Q^2$ region as compared to that defined by the gluon four-momentum square cut.

Nevertheless, we notice that although most of the recently calculations [5-12] show that perturbative QCD is self-consistent and applicable to the exclusive processes at currently experimental accessible energy regions, the numerical predictions for the pion form factor are
much smaller than the experimental data. There are two possible explanations: one is that the non-perturbative contributions will dominate in this region; the other is that the non-leading order contributions in perturbative expansions may be also important in this region. To make choice between the two possible explanations one needs to analyse the non-leading contributions which come from higher-twist effect, higher order in $\alpha_s$, and higher Fock states. Field, Gupta, Otto, and Chang [13] pointed out that the contribution from the next-leading order in $\alpha_s$ is about $20% \sim 30%$ to the perturbative pion form factor. Employing the modified factorization expression for the pion form factor proposed by Li and Sterman [6], Refs. [8,9] considered the transverse momentum effect in the wave function and found that the transverse momentum in the wave function play the role to suppress perturbative prediction. Thus it is necessary to calculate the other non-leading contributions such as that from higher twist effect and higher Fock states.

One of the other sources which may provide non-leading perturbative contribution is the higher helicity components in the light-cone wave function [14–16]. The effects from higher helicity components (or Wigner rotation effect) have been investigated in the description of pion properties at high energies [16,17] as well as at low energies [15,18-21] and the same effect has been also applied to explain the “proton spin crisis” [22,23]. However, the calculations for the contributions coming from higher helicity components to the pion form factor in the high energy region are conflicting in literatures [16,17]. Ma and Huang [16] pointed out that the higher helicity components provide a large enhancement for the perturbation prediction of the pion form factor and thus may provide the other fraction which is needed to fit the experimental data around $Q^2 \geq 2$ GeV$^2$. More recently, Wang and Kisslinger [17] also analysed this effect based on the modified perturbative approach. In their approach this effect gives a large suppression for the pion form factor as compared to the prediction obtained in the original hard-scattering model in the $Q^2$ domain where experimental data are available. Thereby they concluded that non-perturbative contributions dominate in this region. Refs. [16] and [17] gave very different conclusions concerning the question whether the perturbative QCD contributions dominate or not in the available experimental energy region.
We point out that the conflict between the above works is due to the difference between the hard-scattering amplitudes for the higher helicity components adopted in Refs. [16] and [17]. It is assumed in Ref. [16] that the hard-scattering amplitude for the higher helicity components is the same as that for the ordinary helicity component,

\[ T_H^{(\lambda_1+\lambda_2=\pm 1)} = T_H^{(\lambda_1+\lambda_2=0)} = \frac{4g^2C_F}{x_2y_2Q^2}. \]  

(1)

But the hard-scattering amplitude employed in Ref. [17] is

\[ T_H^{(\lambda_1+\lambda_2=\pm 1)} = -T_H^{(\lambda_1+\lambda_2=0)} = -\frac{4g^2C_F}{x_2y_2Q^2 + (k_\perp - l_\perp)^2} \approx \frac{4g^2C_F}{x_2y_2Q^2}. \]  

(2)

It can be seen that the asymptotic \((Q^2 \to \infty)\) behaviors of Eqs. (1) and (2) are with opposite signs. That is the reason that Refs. [16] and [17] gave opposite conclusions concerning the PQCD contributions from the higher helicity components in the experimental available \(Q^2\) region.

The purpose of this paper is to analyze the effect from the higher-helicity components of the pion wave function in the light-cone perturbative QCD and address the conflict between Refs. [16] and [17]. We first review and analyze the spin structure for the pion light-cone wave function and the necessity to take into account the higher helicity components in Sec. II. Then in Sec. III we calculate the hard-scattering amplitude for the higher helicity components of the pion form factor. We first explicitly show that the hard-scattering amplitude for the higher helicity components vanishes in the leading order \(O(1/Q^2)\) As the parton intrinsic transverse momentum is taken into account, it is found that the asymptotic behavior of the hard-scattering amplitude for the higher helicity components is of order \(1/Q^4\) which differs from either Eq. (1) or Eq. (2). We conclude that the higher helicity components, though provide vanishingly small contributions to the perturbative pion form factor in the asymptotic limit \(Q^2 \to \infty\), they should be considered in the available experimental energy region since they are next-to-leading order contributions. Sec. IV is served as a summary.
II. THE LIGHT-CONE WAVE FUNCTION OF THE PION AND ITS HIGHER HELICITY COMPONENTS

The light-cone (LC) formalism [24] provides a convenient framework for the relativistic description of hadrons in terms of quark and gluon degrees of freedom, and the application of PQCD to exclusive processes has mainly been developed in this formalism (light-cone PQCD) [1,25,26]. The essential feature of light-cone PQCD application to exclusive processes is that the amplitudes for these processes can be written as a convolution of hadron light-cone wave functions (or quark distribution amplitudes) for every hadron involved in the process with a hard-scattering amplitude $T_H$. Thus light-cone wave function is an essential part: It determines the distributions of the quark and gluons entering the short distance sub-processes and provides the link between the long-distance non-perturbative and short distance perturbative physics. In principle, light-cone wave function can be computed from rigorous light-cone QCD. Unfortunately this task is very complex and difficult, and there is no exact solution up to now. More practical and more convenient way is to connect light-cone wave function with the instant-form wave function which can be obtained by solving the Bethe-Salpeter equation with some approximations [26]. The connection for the spin space wave functions between the two formalisms are accomplished [14–16] by the use of Wigner rotation [27]. The connection for the momentum space wave functions become possible with the help of some ansatz such as the Brodsky-Huang-Lepage prescription [26].

It should be emphasized that in order to connect the spin structures in two different frames correctly, it is necessary to consider Wigner rotation effect. As it is known, spin is essentially a relativistic notion associated with the space-time symmetry of Poincaré. The conventional 3-vector spin $\mathbf{s}$ of a moving particle with finite mass $m$ and 4-momentum $p_\mu$ can be defined by transforming its Pauli-Lubanski 4-vector $\omega_\mu = 1/2 J^{\rho\sigma} P^\rho \epsilon_{\nu\rho\sigma\mu}$ to its rest frame via a non-rotation Lorentz boost $L(p)$ which satisfies $L(p)p = (m, 0)$, by $(0, \mathbf{s}) = L(p) \omega/m$. Under an arbitrary Lorentz transformation, a particle state with spin $\mathbf{s}$ and 4-momentum $p_\mu$ will transform to the state with spin $\mathbf{s}'$ and 4-momentum $p'_\mu$. 
\[ s' = R_\omega(A, p)s, \quad p' = \Lambda p, \]

where \( R_\omega(A, p) = L(p')\Lambda L^{-1}(p) \) is a pure rotation known as Wigner rotation. When a composite system is transformed from one frame to another one, the spin of each constituent will undergo a Wigner rotation. These spin rotations are not necessarily the same since the constituents have different internal motion. In consequence, the sum of the constituent’s spin is not Lorentz invariant. Hence, although the pion has only \( \lambda_1 + \lambda_2 = 0 \) spin components in the rest frame of the pion, it may have \( \lambda_1 + \lambda_2 = \pm 1 \) spin components in the infinite-momentum frame (light-cone formalism)\(^4\), where \( \lambda_1 \) and \( \lambda_2 \) are the quark and anti-quark helicities respectively. One advantage of light-cone dynamics is that Wigner rotation relating spin states in different frames is unity under a kinematic Lorentz transformation.

To obtain light-cone spin space wave function of the pion one can transform the ordinary instant-form SU(6) quark model spin space wave function of the pion into light-cone dynamics [14–16]. In the pion rest frame \((q_1 + q_2 = 0)\), the instant-form spin space wave function of the pion is

\[ \chi_T = (\chi_1^\uparrow \chi_2^\downarrow - \chi_1^\downarrow \chi_2^\uparrow) / \sqrt{2}, \]

in which \( \chi_1^\uparrow, \chi_2^\downarrow \) are the two-component Pauli spinors and \( q^\mu_i = (q^0, q), q^\mu_2 = (q^0, -q) \) are 4-momenta for the two quarks respectively with \( q^0 = (m^2 + q^2)^{1/2} \). The instant-form spin states \(|J, s\rangle_T\) and the light-cone form spin states \(|J, s\rangle_F\) are related by a Wigner rotation \( U^J \) [15-21],

\[ |J, \lambda\rangle_F = \sum_s U^s_{\lambda} |J, s\rangle_T. \]  

This rotation is called as Melosh rotation [28] for spin-1/2 particles. Applying transformation Eq. (5) on the both sides of Eq. (4) one can obtain the spin space wave function of the pion

\(^4\)Notice that the instant-form dynamics in the infinite-momentum frame is equivalent to light-front dynamics in an ordinary frame.
in the infinite-momentum frame. Transforming for the left side \((i.e., \text{the pion})\) is simple since Wigner rotation is unity. For the right side \((i.e., \text{the two spin-1/2 partons})\), each particle instant-form and light-cone form spin states are related by the Melosh transformation,

\[
\chi^\uparrow(T) = w[(q^+ + m)\chi^\uparrow(F) - q^R\chi^\downarrow(F)];
\]

\[
\chi^\downarrow(T) = w[(q^+ + m)\chi^\downarrow(F) + q^L\chi^\uparrow(F)],
\]

where \(w = [2q^+(q^0 + m)]^{-1/2}, q^{R,L} = q^1 \pm iq^2\), and \(q^+ = q^0 + q^3\). Then the light-cone spin space wave function of the pion reads

\[
\chi_F(x, k_\perp) = \sum_{\lambda_1, \lambda_2} C^F_0(x, k_\perp, \lambda_1, \lambda_2)\chi_1^{\lambda_1}(F)\chi_2^{\lambda_2}(F).
\]

When expressed in terms of the equal-time momentum \(q^\mu = (q^0, q)\), the spin component coefficients \(C^F_0\) have the forms,

\[
C^F_0(x, q, \uparrow, \downarrow) = w_1w_2[(q^+_1 + m)(q^+_2 + m) - q^2_\perp]/\sqrt{2};
\]

\[
C^F_0(x, q, \downarrow, \uparrow) = -w_1w_2[(q^+_1 + m)(q^+_2 + m) - q^2_\perp]/\sqrt{2};
\]

\[
C^F_0(x, q, \uparrow, \uparrow) = w_1w_2[(q^+_1 + m)q^L_2 - (q^+_2 + m)q^L_1]/\sqrt{2}; \tag{8}
\]

\[
C^F_0(x, q, \downarrow, \downarrow) = w_1w_2[(q^+_1 + m)q^R_2 - (q^+_2 + m)q^R_1]/\sqrt{2}.
\]

The equal-time momentum \(q = (q^3, q_\perp)\) and the light-cone momentum \(k = (x, k_\perp)\) can be connected according to the Brodsky-Huang-Lepage prescription \[26\] which is obtained by equating the off-shell propagators in the two frames,

\[
xM \leftrightarrow (q^0 + q^3); \tag{9}
\]

\[
k_\perp \leftrightarrow q_\perp,
\]

in which \(M\) is defined as

\[
M^2 = \frac{k^2_\perp + m^2}{x(1 - x)}. \tag{10}
\]

From (9) we have

\[
\frac{k^2_\perp + m^2}{4x(1 - x)} - m^2 = q^2. \tag{11}
\]
From Eqs. (8), (9) and (10) the coefficients $C_0^F$ can be expressed in the light-cone momentum $\mathbf{k} = (x, \mathbf{k}_\perp)$,

$$
C_0^F (x, \mathbf{k}_\perp, \uparrow, \downarrow) = \frac{m}{[2(m^2 + k^2_\perp)]^{1/2}};
$$

$$
C_0^F (x, \mathbf{k}_\perp, \downarrow, \uparrow) = -\frac{m}{[2(m^2 + k^2_\perp)]^{1/2}};
$$

$$
C_0^F (x, \mathbf{k}_\perp, \uparrow, \uparrow) = -\frac{(k_1 - ik_2)}{[2(m^2 + k^2_\perp)]^{1/2}};
$$

$$
C_0^F (x, \mathbf{k}_\perp, \downarrow, \downarrow) = -\frac{(k_1 + ik_2)}{[2(m^2 + k^2_\perp)]^{1/2}}.
$$

$C_0^F$ satisfy the relation

$$
\sum_{\lambda_1, \lambda_2} C_0^F (x, \mathbf{k}_\perp, \lambda_1, \lambda_2) C_0^F (x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = 1.
$$

It can be seen explicitly from Eqs. (4), (7) and (12) that the light-cone spin space wave function of the pion $\chi_F$ has higher helicity ($\lambda_1 + \lambda_2 = \pm 1$) components besides the ordinary helicity ($\lambda_1 + \lambda_2 = 0$) component, while the instant-form spin space wave function of the pion, $\chi_T$ has only the ordinary helicity component. Notice that $\chi_F$ is also an eigen-state of the total spin operator $(\hat{S}^F)^2$ in the light-cone formalism [16].

Now the light-cone wave function for the lowest valence state of the pion can be expressed as [16]

$$
|\psi^{\pi}_{q\bar{q}}\rangle = \psi(x, \mathbf{k}_\perp, \uparrow, \downarrow) |\uparrow\downarrow\rangle + \psi(x, \mathbf{k}_\perp, \downarrow, \uparrow) |\downarrow\uparrow\rangle
$$

$$
+ \psi(x, \mathbf{k}_\perp, \uparrow, \uparrow) |\uparrow\uparrow\rangle + \psi(x, \mathbf{k}_\perp, \downarrow, \downarrow) |\downarrow\downarrow\rangle,
$$

where

$$
\psi(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = C_0^F (x, \mathbf{k}_\perp, \lambda_1, \lambda_2) \varphi(x, \mathbf{k}_\perp).
$$

Here $\varphi(x, \mathbf{k}_\perp)$ is the momentum space wave function in the light-cone formalism.

The above result means that the light-cone spin of a composite particle is not directly the sum of its constituents’ light-cone spins but the sum of Wigner rotated light-cone spins of the individual constituents. A natural consequence is that in light-cone formalism a hadron’s
helicity is not necessarily equal to the sum of the quark’s helicities, i.e., \( \lambda_H \neq \sum_i \lambda_i \). This result is important for understanding the proton “spin puzzle” \([22]\). It has been shown \([23]\) that the relativistic SU(6) quark model of the nucleon, supplemented with Wigner rotation effect \([22]\) and the flavor asymmetry generated by the spin-spin interaction of the valence spectator quarks, could reproduce the observed ratio \( F_n^2 / F_p^2 \) and the proton, neutron, and deuteron polarization asymmetries, \( A_{p1}, A_{n1}, A_{d1} \). If the intrinsic quark-antiquark pairs generated by the non-perturbative meson-baryon fluctuations in the nucleon sea are further taken into account, we could arrive at a consistent framework \([29]\) to understand a number of anomalies observed in the proton’s structure: the origin of polarized strange quarks implied by the violation of the Ellis-Jaffe sum rule; the flavor asymmetry of the nucleon sea implied by the violation of Gottfried sum rule; and the conflict between two different measurements of strange quark distributions.

III. THE HARD SCATTERING AMPLITUDE FOR THE HIGHER HELICITY COMPONENTS IN THE PION FORM FACTOR

The pion electromagnetic form factor can be expressed by the Drell-Yan-West formula \([30]\),

\[
F(Q^2) = \sum_{n,\lambda_i} \sum_j e_j \int [dx] \int [d^2k_\perp] \psi^*_n(x, k_{\perp,i}, \lambda_i) \psi_n(x, k'_{\perp,i}, \lambda_i),
\]

where \( k'_\perp = k_\perp - x_i q_\perp + q_\perp \) for the struck quark, \( k'_\perp = k_\perp - x_i q_\perp \) for the spectator quarks, and \( e_i \) is the electric charge of the struck quark. At higher momentum transfer, the pion form factor in the leading order can be given by \([6,12,16]\)

\[
F_\pi(Q^2) = \int [dx] \int [dy] \int [d^2k_\perp] \int [d^2l_\perp] \psi^{((1-x)Q)}(x, k_{\perp}, \lambda_i) T_H(x, y, q_{\perp}, k_{\perp}, l_{\perp}) \psi^{((1-y)Q)}(y, l_{\perp}, \lambda_i)
\]

\[
= \int [dx] \int [dy] \int [d^2k_\perp] \int [d^2l_\perp] \varphi^{((1-x)Q)}(x, k_{\perp}) \left[ \mathcal{W}_1 T_H^{(\lambda_1+\lambda_2=0)}(x, y, q_{\perp}, k_{\perp}, l_{\perp}) \right] + \mathcal{W}_2 T_H^{(\lambda_1+\lambda_2=\pm 1)}(x, y, q_{\perp}, k_{\perp}, l_{\perp}) \varphi^{((1-y)Q)}(y, l_{\perp})
\]

where \([dx] = dx \delta(1 - x_1 - x_2), [d^2k_\perp] = d^2k_\perp / (16 \pi^3), \varphi^{((1-x)Q)}(x, k_{\perp}) \) is the light-cone momentum space wave function of the valence Fock state with a cut-off \( k_{\perp}^2 = (1 - x)Q \), \( T_H \).
are the hard-scattering amplitudes which can be calculated from the time-ordered diagrams in light-cone PQCD, and \( \mathcal{W}_1 \) and \( \mathcal{W}_2 \) are the factors from Wigner rotation,

\[
\mathcal{W}_1 = m / \left[ (m^2 + \mathbf{k}_\perp^2)(m^2 + \mathbf{l}_\perp^2) \right]^{1/2} ;
\]

\[
\mathcal{W}_2 = \mathbf{k}_\perp \cdot \mathbf{l}_\perp / \left[ (m^2 + \mathbf{k}_\perp^2)(m^2 + \mathbf{l}_\perp^2) \right]^{1/2} .
\]  

(18)

In the derivation for Eq. (17) we have applied the relations

\[
T^*_H(\downarrow \rightarrow \uparrow) = T_H(\uparrow \rightarrow \downarrow), \quad T^*_H(\downarrow \rightarrow \downarrow) = T_H(\uparrow \rightarrow \uparrow) .
\]  

(19)

After summing over all helicities, only the real part of each hard-scattering amplitude survives. Thereby there are only two independent hard-scattering amplitudes:

\[
T^{(\lambda_1 + \lambda_2 = 0)}_H = \frac{1}{2} \left[ T_H(\uparrow \rightarrow \uparrow) + T_H(\downarrow \rightarrow \downarrow) \right] ;
\]

\[
T^{(\lambda_1 + \lambda_2 = \pm 1)}_H = \frac{1}{2} \left[ T_H(\uparrow \rightarrow \uparrow) + T_H(\downarrow \rightarrow \downarrow) \right] .
\]  

(20)

As \( \mathbf{k}_\perp = \mathbf{l}_\perp = 0 \), Wigner rotation factors \( \mathcal{W}_1 = 1, \mathcal{W}_2 = 0 \), and Eq. (17) reduces to the ordinary perturbation expression for the pion form factor. In more general situation, there is also contribution from the higher helicity components \( T^{(\lambda_1 + \lambda_2 = \pm 1)}_H \) besides the hard-scattering amplitude \( T^{(\lambda_1 + \lambda_2 = 0)}_H \) from the ordinary helicity component of the pion. Notice that quark helicity is conserved at each vertex in \( T_H \) in the limit of vanishing quark mass, since both photon and gluon are vector particles [1,31]. Hence there is no hard-scattering amplitude with quark and antiquark helicities being changed. \( T^{(\lambda_1 + \lambda_2 = 0)}_H \) has been calculated in cases when the intrinsic transverse momenta are neglected (see for example [1,25,26]) and taken into account [12]. The purpose of this paper is to calculate \( T^{(\lambda_1 + \lambda_2 = \pm 1)}_H \), i.e., the contribution from the higher helicity components of the pion light-cone wave function.

In the light-cone perturbative QCD, there are six time-order diagrams as shown in Fig. 1 which contribute to \( T_H(\uparrow \rightarrow \uparrow) \) and \( T_H(\downarrow \rightarrow \downarrow) \). The calculation rules for the light-cone PQCD can be found in literatures [1,25,26]. First, we neglect the intrinsic transverse momenta \( \mathbf{k}_\perp \) and \( \mathbf{l}_\perp \). The contribution of diagram (a) can be written as,

\[
T_H^{(a)} = \text{Tr} \frac{1}{D_{11} D_{12}} \frac{1}{y_1 - x_1} \theta(y_1 - x_1) + \text{Inst.},
\]  

(21)

9
where \( D_{11} \) and \( D_{12} \) are the “energy denominators”,
\[
D_{11} = -\frac{y_1 x_2^2}{x_1 (y_1 - x_1)} q_\perp^2, \quad D_{12} = -\frac{x_2 q_\perp^2}{x_1}, \tag{22}
\]
and \( \text{Tr} \) is the sum of some spinors and \( \gamma \)-matrix in light-cone PQCD,
\[
\text{Tr} = \bar{u}^\uparrow(y_1, y_1 q_\perp) ig \gamma^\mu u^\uparrow(x_1, q_\perp) d_{\mu\nu} \bar{v}^\uparrow(x_2, q_\perp) ig \gamma^\nu v^\uparrow(y_2, y_2 q_\perp). \tag{23}
\]
By using Eqs. (19) and (20), we need to calculate only the real part of \( \text{Tr} \) which reads,
\[
\text{RTr} = -g^2 \frac{2x_2(x_1 y_2 + y_1 x_2)}{x_1 (y_1 - x_1)^2} q_\perp^2. \tag{24}
\]
The “Inst.” part in Eq. (21) represents the contribution from instantaneous diagram which is one feature of light-cone PQCD,
\[
\text{Inst.} = g^2 \frac{4x_1 \theta(y_1 - x_1)}{x_2(y_1 - x_1)^2 q_\perp^2}. \tag{25}
\]
Then the contribution from diagram (a) reads
\[
T^{(a)}_H = g^2 \frac{2x_1 \theta(y_1 - x_1)}{x_2^2 y_1 q_\perp^2}. \tag{26}
\]
It is known that the contribution from each diagram, for example \( T^{(a)}_H \), is itself not gauge-invariant, but the gauge-invariance will be satisfied when summing over all time-order diagrams (a)-(f). The contributions from the other diagrams can be calculated in a similar way. Observing that the term “Tr” is the same for the diagrams (a), (b) and (c), and employing the following relations for the “energy denominators”,
\[
D_{22} = D_{12}, \quad D_{31} = D_{11},
\]
\[
D_{11} = D_{32} + D_{12}, \quad D_{21} = -D_{32} = \frac{x_2 y_2}{y_1 - x_1} q_\perp^2, \tag{27}
\]
we can sum over the contributions from diagrams (a), (b) and (c),
\[
T^{(a+b+c)}_H = \text{Tr} \frac{1}{D_{12}} \frac{1}{x_1 - y_1} \frac{1}{D_{21}} - g^2 \frac{1}{D_{12}} \frac{4}{(x_1 - y_1)^2}
\]
\[
= \frac{2g^2}{x_2 y_2 q_\perp^2} \frac{1}{x_1 - y_1}. \tag{28}
\]
We point out that under transformation \((x \leftrightarrow y)\) there is symmetry for the six diagrams,

\[
\text{Diagrams } (a, b, c) \iff \text{Diagrams } (d, e, f) \quad \text{under } (x \leftrightarrow y).
\]

(29)

Thus the contributions form diagrams (d), (e), and (f) are,

\[
T_H^{(d+e+f)} = \frac{2g^2}{x_2 y_2 q^2} \frac{1}{y_1 - x_1}.
\]

(30)

From Eqs. (28) and (30) we can obtain the hard-scattering amplitude for the higher helicity components in the approximation neglecting parton intrinsic transverse momenta,

\[
T_H^{(\lambda_1 + \lambda_2 = \pm 1)}(x, y, q_\perp) = T_H^{(a+b+c)} + T_H^{(d+e+f)} = 0.
\]

(31)

Eq. (31) shows that there is no contribution from the higher helicity components for the pion form factor in the leading order \(O(1/Q^2)\) (or the intrinsic transverse momenta being neglected); which is in agreement with the early result obtained by Brodsky and Lepage [1,31].

Now we take into account the parton intrinsic transverse momenta \(k_\perp\) and \(l_\perp\). Then “Tr” means

\[
\text{Tr} = \frac{\bar{u}(y_1, y_1 q_\perp + l_\perp) i g \gamma_\mu u(x_1, q_\perp + k_\perp)}{\sqrt{y_1}} \frac{\bar{v}(x_2, -k_\perp) i g \gamma_\nu v(y_2, y_2 q_\perp - l_\perp)}{\sqrt{x_2}}
\]

and

\[
\text{RTr} = \frac{[y_1(x_2 q_\perp + k_\perp) - x_1 l_\perp] \cdot [y_2(x_2 q_\perp + k_\perp) - x_2 l_\perp]}{x_1 x_2 y_1 y_2 (x_1 - y_1)^2} \frac{2(x_1 y_2 + y_1 x_2)}{2(x_1 y_2 + y_1 x_2)}.
\]

(32)

The “energy denominators” are

\[
D_{11} = -\frac{(x_2 q_\perp + k_\perp)^2}{x_1 x_2}, \quad D_{12} = -\frac{(x_2 q_\perp + k_\perp)^2}{x_1 x_2};
\]

\[
D_{21} = -\frac{l_\perp^2}{y_1 y_2} + \frac{[y_2(x_2 q_\perp + k_\perp) - x_2 l_\perp]^2}{x_2 y_2 (y_1 - x_1)}, \quad D_{22} = D_{12};
\]

\[
D_{32} = -\frac{k_\perp^2}{x_1 x_2} - \frac{[y_2(x_2 q_\perp + k_\perp) - x_2 l_\perp]^2}{x_2 y_2 (y_1 - x_1)}, \quad D_{31} = D_{11}.
\]

(34)

Using the symmetry
Diagrams \((a, b, c) \iff \text{Diagrams } (d, e, f)\) under \[
\begin{aligned}
x &\iff y \\
k_\perp &\iff -l_\perp
\end{aligned}
\] (35)

we get,

\[
T_H^{(\lambda_1+\lambda_2=\pm 1)}(x, y, q_\perp, k_\perp, l_\perp) = \text{RTr} \left[ \left( \frac{1}{D_{11}D_{12}} + \frac{1}{D_{31}D_{32}} \right) \frac{\theta(y_1 - x_1)}{y_1 - x_1} + \frac{1}{D_{21}D_{22}} \frac{\theta(x_1 - y_1)}{x_1 - y_1} \right] 
\]

\[
+ \frac{4}{D_{12}(y_1 - x_1)^2} \begin{cases}
\begin{aligned}
x &\iff y \\
k_\perp &\iff -l_\perp
\end{aligned}
\end{cases}
\] (36)

In the above calculation we have neglected the quark masses since it is “current quark masses” that should appear in perturbative calculation. The pionic mass can also be neglected in PQCD calculation.

To simplify Eq. (36), we adopt the following two prescriptions: 1) It is pointed out in Ref. [12] that as one concerns with the effect from intrinsic transverse momenta the terms proportional to the “bound energies” of the pions in the initial and final states \(i.e. \sim k_\perp^2/(x_1 x_2)\) and \(\sim l_\perp^2/(y_1 y_2)\) can be ignored to avoid the involvement of the higher Fock states contributions\(^5\). Neglecting these terms in the “energy denominators”, we have,

\[
T_H^{(\lambda_1+\lambda_2=\pm 1)}(x, y, q_\perp, k_\perp, l_\perp) = \frac{y_2(x_2q_\perp + k_\perp) - x_2l_\perp}{(x_2q_\perp + k_\perp)^2/y_1(x_1 - y_1)} \begin{cases}
\begin{aligned}
x &\iff y \\
k_\perp &\iff -l_\perp
\end{aligned}
\end{cases}
\] (37)

2) Notice that in the factorization expression for the pion form factor Eq. (17), we have \(k_\perp^2 \ll q_\perp^2\) and \(l_\perp^2 \ll q_\perp^2\). Hence when calculating to the next-to-leading order in \(1/Q\) for \(T_H\), we can neglect the terms such as \(k_\perp^2/q_\perp^2\), \(k_\perp^2/q_\perp^2\) and \((k_\perp \cdot l_\perp)/q_\perp^2\) in the both the “energy denominators” and “RTr”. Then we get

---

\(^5\)As the transverse momenta \(k_\perp\) and \(l_\perp\) are included, it is necessary to take into account the contributions from higher Fock states to satisfy the gauge-invariance, since the covariant derivative \(D_\mu = \partial_\mu + igA_\mu\) makes both transverse momenta \(k_\perp, l_\perp\) and the transverse gauge degree \(gA_\perp\) be of the same order [12].
\[ T_{H}^{(\lambda_1 + \lambda_2 = \pm 1)}(x, y, q, k_\perp, l_\perp) = \frac{[y_1 y_2 (x_2 q_\perp^2 + 2 q_\perp \cdot k_\perp) + (x_1 - y_1) q_\perp \cdot l_\perp]}{(x_2 q_\perp^2 + 2 q_\perp \cdot k_\perp)[y_2 (x_2 q_\perp^2 + 2 q_\perp \cdot k_\perp) - 2 x_2 q_\perp \cdot l_\perp]} \times \frac{2g^2}{y_1 y_2 (x_1 - y_1)} + \begin{cases} x \leftrightarrow y \medskip \kern 2cm \text{ or } \medskip \kern 2cm k_\perp \leftrightarrow -l_\perp \end{cases}. \]

As the intrinsic transverse momenta \( k_\perp \) and \( l_\perp \) are neglected (or in the asymptotic limit \( Q^2 \to \infty \)), Eqs. (36), (37), and (38) reduce to Eq. (31), i.e., the hard-scattering amplitude for the higher helicity components goes to zero. It can be found from Eqs. (37) and (38) that the leading contribution of the hard-scattering amplitude for the higher helicity components is of order \( 1/Q^4 \) which is next-to-leading contribution compared to the contribution coming from the ordinary helicity component, but it may give sizable contributions to the pion form factor in the intermediate energy region. We also notice that (37) and Eq. (38) differ to either Eq. (1) or Eq. (2), hence the calculations in neither Ref. [16] nor Ref. [17] is reliable. It is necessary to re-consider the PQCD contributions from the higher helicity components based on proper hard-scattering amplitude derived from theory at the energy scale where the current experiments are accessible. The quantitative predictions depend on numerical calculation which involves 6-dimensional integral with tedious technical details, and will be given elsewhere.

**IV. SUMMARY**

The light-cone formalism provides a convenient framework for the relativistic description of hadrons in terms of quark and gluon degrees of freedom, and the application of perturbative QCD to exclusive processes has mainly been developed in this formalism. In order to obtain correct spin structure for the hadron wave function in the light-cone formalism from the instant-form wave function, the relativistic effect due to Wigner rotation should be taken into account. Consequently, in the light-cone formalism, there are higher helicity \((\lambda_1 + \lambda_2 = \pm 1)\) components in the spin space wave function besides the usual helicity \((\lambda_1 + \lambda_2 = 0)\) components. We give the hard scattering amplitude for the higher helicity
components in the perturbative calculation for the pion form factor. It is found that the
hard-scattering amplitude for the higher helicity components is of order $1/Q^4$, which is van-
ishingly small compared to that of the ordinary helicity components at very high $Q^2$ but
should be considered in the $Q^2$ region where experimental data are available.
REFERENCES


FIGURE CAPTION

Fig. 1. Leading order time-order diagrams contributing to the hard scattering amplitude for the higher helicity ($\lambda_1 + \lambda_2 = \pm 1$) components of the pion in the perturbative calculation for the pion form factor, where $k_1 = (x_1, k_\perp)$, $k_2 = (x_2, -k_\perp)$, $l_1 = (y_1, y_1 q_\perp + l_\perp)$, and $l_2 = (y_2, y_2 q_\perp - l_\perp)$, and the momenta are expressed in the light-cone variables ($+, \perp$). As usual the momentum of the pion in the initial state is taken to be $P = (1, 0_\perp)$ and the momentum of the photon is $q = (0, q_\perp)$ with $q^- = q_\perp^2$. 